Breakup and Structure of 1n Halo Nuclei

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- ✗ **Non-perturbative treatment of breakup**
  - *Eikonal approximation*
- ✗ **Projection versus SuSy transformation of potential**
  - *Exact treatment of the Pauli principle*
  - *Application for excitation processes*
- ✗ **Outlooks**

Angela Bonaccorso (Pisa)
David Brink (Oxford)

Philippe Chomaz (Ganil)

Coulomb break-up to all orders

- Breakup amplitude in perturbation theory

$$g_{fi}^{C} = g_{fi}^{C\,pert\,1} + g_{fi}^{C\,pert\,2} + g_{fi}^{C\,pert\,3} + \cdots$$

- Sudden approximation for all orders

$$g_{fi}^{C\,sudd}(k,d) = \phi_i(k_{t=+\infty}) - \phi_i(k_{t=-\infty})$$

with $k_{t=+\infty} = k_f + Q\frac{\boldsymbol{x}}{x}$ and $k_{t=-\infty} = k_f$

- BBM amplitude : high order terms treated at the sudden approx.

$$g_{fi}^{C\,BBM} = g_{fi}^{C\,pert\,1} + \overbrace{g_{fi}^{C\,pert\,2} + g_{fi}^{C\,pert\,3} + \cdots}^{sudd}$$

$$g_{fi}^{C\,BBM} = g_{fi}^{C\,pert\,1} + \left| g_{fi}^{C\,sudd} - g_{fi}^{C\,sudd\,pert1} \right|$$

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Non-perturbative Coulomb + nuclear breakup

BBM amplitude

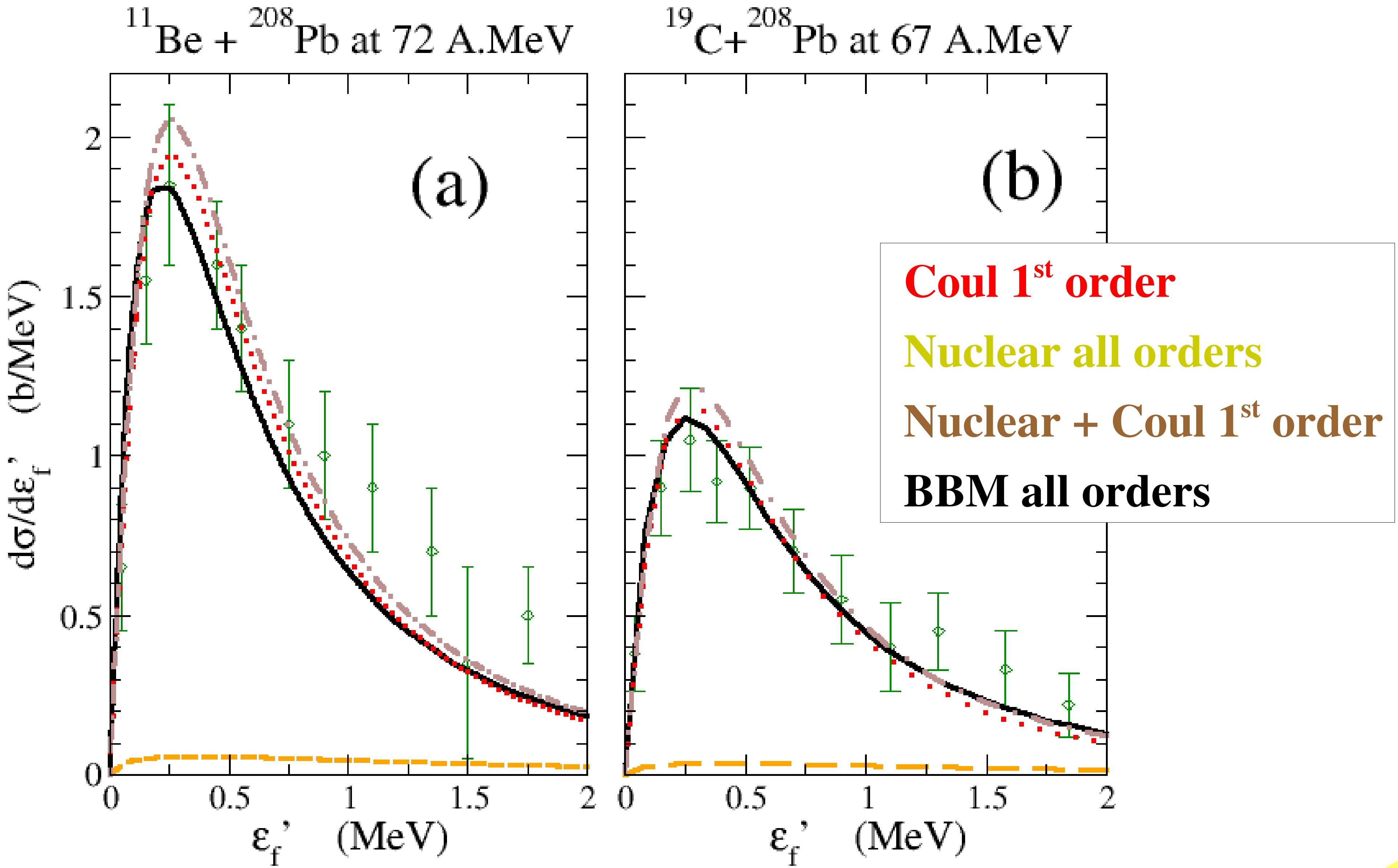
$$g_{fi}^{CN\,BBM} = g_{fi}^{C\,pert\,1} + g_{fi}^{N\,pert\,1} + \left(g_{fi}^{CN\,sudd} - g_{fi}^{C\,sudd\,pert\,1} - g_{fi}^{N\,sudd\,pert\,1} \right)$$

$$g_{fi}^{CN\,BBM} = g_{fi}^{C\,pert\,1} + \left(g_{fi}^{CN\,sudd} - g_{fi}^{C\,sudd\,pert1} \right)$$

J.M., A. Bonaccorso & D. Brink, Nucl.Phys. A720 (2003) 337-353

Results

❑ Energy distribution



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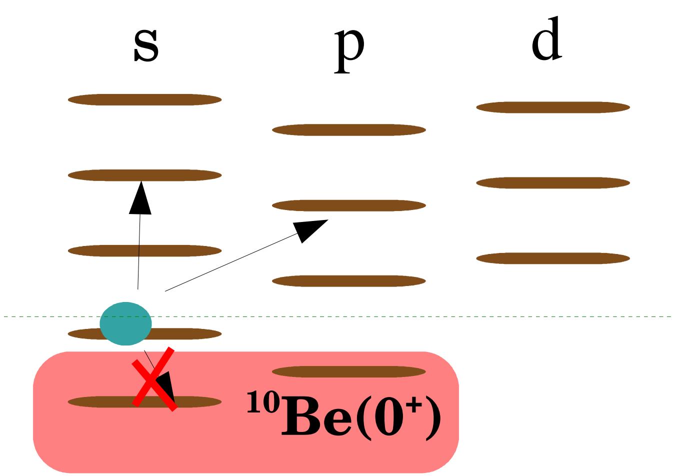
- ✗ Non-perturbative treatment of breakup
  - *Eikonal approximation*
- ✗ **Projection versus SuSy transformation of potential**
  - ***Exact treatment of the Pauli principle***
  - ***Application for excitation processes***
- ✗ Outlooks

Halo Nuclei : Core + Valence space

GS(^{11}Be) : [^{10}Be(0^+) + n(2s½)]+...



The orbitals of the core are Pauli blocked...

Projector method : $\hat{p} = \hat{1} - \sum_{i \in core} |\phi_i\rangle\langle\phi_i|$

Schrödinger eq. : $\hat{p}\hat{H}\hat{p}|\phi(t)\rangle = i\hbar \frac{d}{dt}|\phi(t)\rangle$

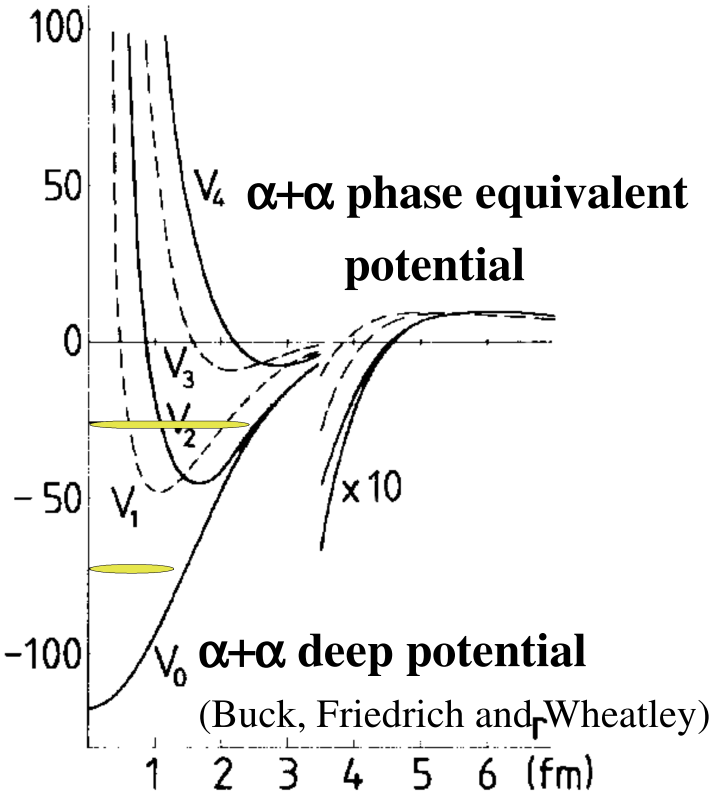
Quantum Mechanics SuperSymmetry ...

From Toy Model to Nuclear Physics

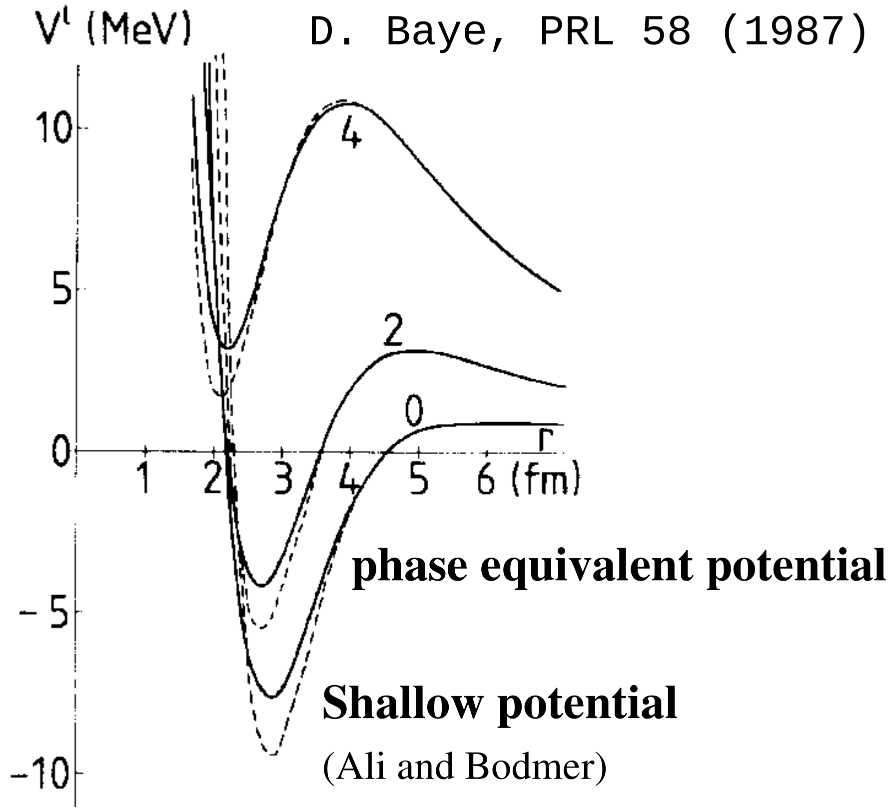
Short History

Early 80ies: Theoretical model (Witten, Sukumar,...)

Link between Deep and Shallow Nucleus-Nucleus Potential



D. Baye, PRL 58 (1987) 2738



Recently...

Application to Coulomb breakup of ^{11}Be:

Capel, Baye, Melezhik, PLB 552 (2003) 145

Comparison between projection method and SUSY

Thompson et al., PRC 61 (2000) 24318

Descouvemont et al., PRC 67 (2003) 44309

Quantum Mechanics Supersymmetry

Spherical coordinates: $|\psi\rangle = \frac{1}{r}|\phi_l\rangle \otimes |Y_{lm}\rangle$

Radial Hamiltonian: $\hat{h}_0^l = \hat{p}^2/2m + \hat{v}_0^l$ with $\hat{v}_0^l = \frac{\hbar^2}{2m}\frac{l(l+1)}{\hat{r}^2} + \hat{v}_0$

Introduce the operators : $\hat{a}_0^{\pm} = \frac{1}{\sqrt{2m}}\{\hat{w}_0 \mp i\hat{p}\}$

Factorize $\hat{h}_0^l = \hat{a}_0^+ \hat{a}_0^- + e_0^l$ ← **Ground State Energy**

From : Schrödinger eq. destruction operator

$$\hat{h}_0^l|\phi_0^l(e_0^l)\rangle = e_0^l|\phi_0^l(e_0^l)\rangle \Leftrightarrow \hat{a}_0^-|\phi_0^l(e_0^l)\rangle = 0$$

$$\Rightarrow w_0(r) = \frac{d}{dr}\ln\phi_0^l(e_0^l, r)$$

← **Ground State w.f.**

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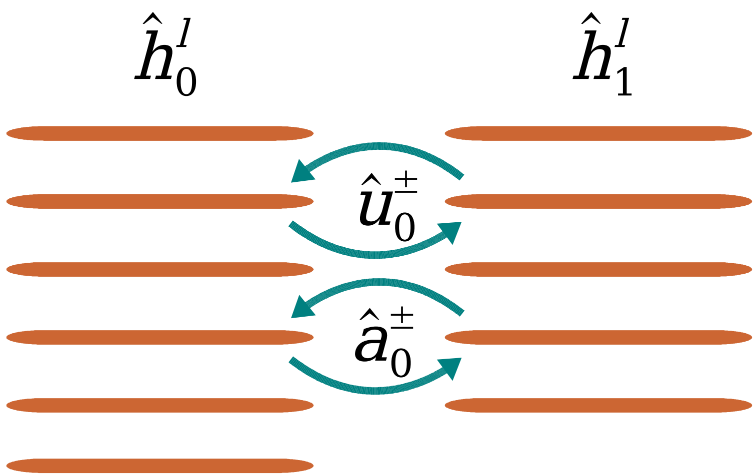
Quantum Mechanics Supersymmetry

$$\hat{h}_0^l = \hat{a}_0^+ \, \hat{a}_0^- + e_0^l \qquad \text{with} \qquad w_0(r) = \frac{d}{dr} \ln \phi_0^l(e_0^l, r)$$

$$\hat{a}_0^- \left|\phi_0^l(e_0^l)\right\rangle = 0$$

Let's define $\hat{h}_1^l = \hat{a}_0^- \, \hat{a}_0^+ + e_0^l$



$$\left|\phi_1^l(e)\right\rangle = \frac{\hat{a}_0^-}{\sqrt{e - e_0^l}} \left|\phi_0^l(e)\right\rangle$$

$$\left|\phi_0^l(e)\right\rangle = \frac{\hat{a}_0^+}{\sqrt{e - e_0^l}} \left|\phi_1^l(e)\right\rangle$$

Operators $\hat{u}_0^\pm$:

$$\left|\phi_1(e)\right\rangle = \hat{u}_0^- \left|\phi_0(e)\right\rangle$$

$$\hat{u}_0^- = \frac{1}{\sqrt{\hat{h}_1 - e_0^l}} \hat{a}_0^-$$

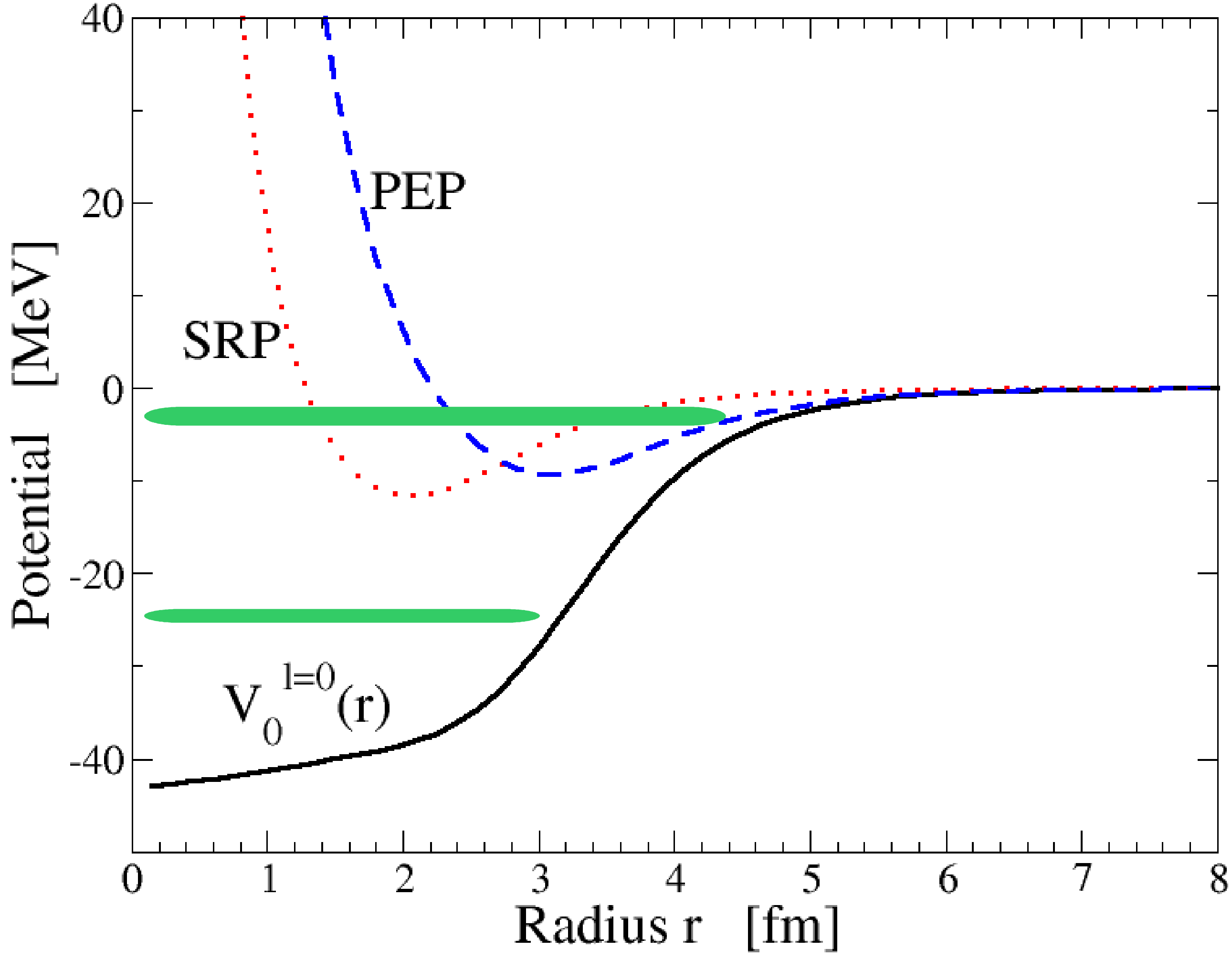
$$\hat{h}_1 = \hat{u}_0^- \, \hat{h}_0 \, \hat{u}_0^+$$

J. Margueron & P. Chomaz, PRC 2005

Quantum Mechanics Supersymmetry

SRP: State Removal Potential

PEP: Phase Equivalent Potential



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Quantum Mechanics Supersymmetry

$$\hat{h}_0^l = \hat{a}^+ \hat{a}^- + e_0^l$$

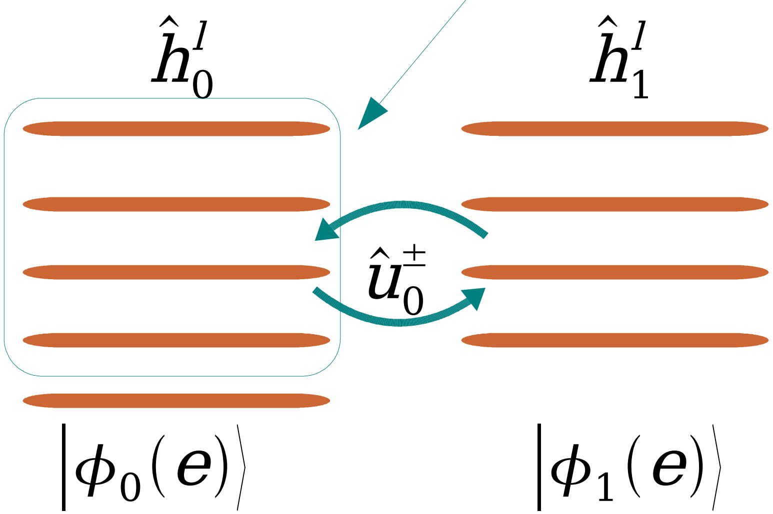
$$\hat{h}_1^l = \hat{a}^- \hat{a}^+ + e_0^l$$

Operators $\hat{u}_0^{\pm}$:

$$|\phi_1(e)\rangle = \hat{u}_0^- |\phi_0(e)\rangle$$

$$\hat{u}_0^- = \frac{1}{\sqrt{\hat{h}_1 - e_0^0}} \hat{a}_0^-$$

$$\hat{h}_1 = \hat{u}_0^- \hat{h}_0 \hat{u}_0^+$$



projector: $\hat{p} = \hat{1} - |\phi_0(e_0)\rangle\langle\phi_0(e_0)|$

$\hat{u}_0^{\pm}$ are pseudo-unitary :

$$\hat{u}_0^- \hat{u}_0^+ = \hat{1}$$

SuSy $\hat{u}_0^+ \hat{u}_0^- = \hat{p}$ **PROJECTION**

SuSy ⇔ PROJECTION

⊥ to the frozen core

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Quantum Mechanics Supersymmetry

$$\hat{h}_0^l = \hat{a}^+ \hat{a}^- + e_0^l$$

$$\hat{h}_1^l = \hat{a}^- \hat{a}^+ + e_0^l$$

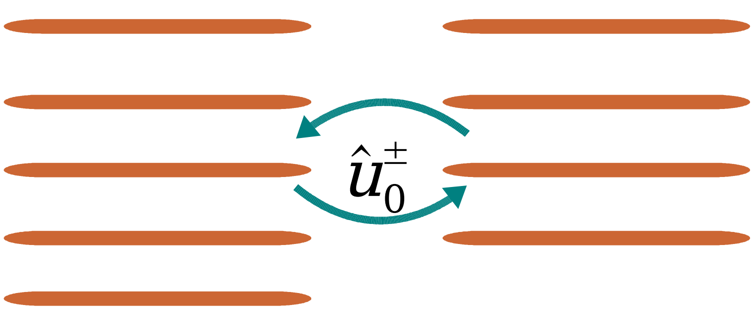
Operators $\hat{u}_0^\pm$:

$$|\phi_1(e)\rangle = \hat{u}_0^- |\phi_0(e)\rangle$$

$$\hat{u}_0^- = \frac{1}{\sqrt{\hat{h}_1 - e_0^0}} \hat{a}_0^-$$

$$\hat{h}_1 = \hat{u}_0^- \hat{h}_0 \hat{u}_0^+$$

$\hat{h}_0^l$ $\hat{h}_1^l$



$|\phi_0(e)\rangle$ $|\phi_1(e)\rangle$

Time dependant processes :

Excitation operator: $\hat{f}_0$

Perturbed Hamiltonian : $\hat{h} = \hat{h}_0 + \hat{f}_0$

Valence space Hamiltonian : $\hat{h}_v = \hat{p}\hat{h}\hat{p}$

$$\hat{h}_v |\phi_0(t)\rangle = i\frac{d}{dt}|\phi_0(t)\rangle$$

$$\hat{u}_0^+ \hat{u}_0^- = \hat{p}$$

$$\hat{u}^+{}_0 \hat{u}_0^- \hat{h} \hat{u}_0^+ \hat{u}_0^- |\phi(t)\rangle = i\frac{d}{dt}|\phi(t)\rangle$$

$$\hat{h}_1 |\phi_1(t)\rangle = i\frac{d}{dt}|\phi_1(t)\rangle$$

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Quantum Mechanics Supersymmetry

$$\hat{h}_0^l = \hat{a}^+ \hat{a}^- + e_0^l$$

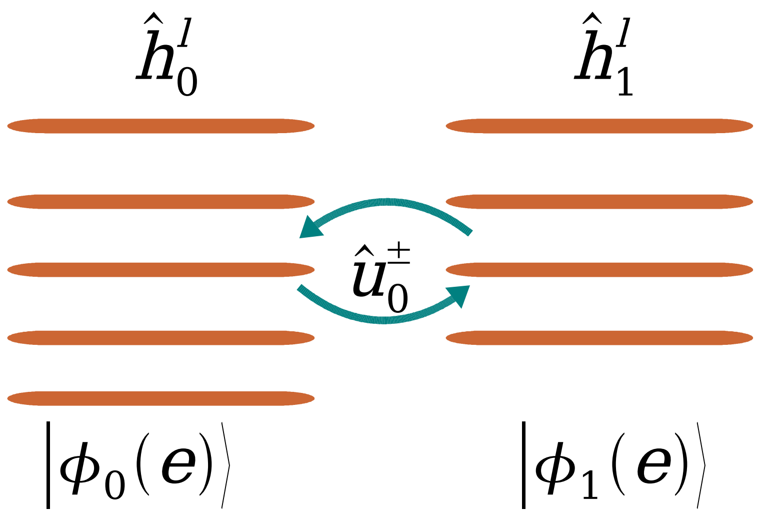
$$\hat{h}_1^l = \hat{a}^- \hat{a}^+ + e_0^l$$

Operators $\hat{u}_0^{\pm}$:

$$|\phi_1(e)\rangle = \hat{u}_0^- |\phi_0(e)\rangle$$

$$\hat{u}_0^- = \frac{1}{\sqrt{\hat{h}_1 - e_0^0}} \hat{a}_0^-$$

$$\hat{h}_1 = \hat{u}_0^- \hat{h}_0 \hat{u}_0^+$$



Excitation processes :

Excitation operator: $\hat{f}_0$

Perturbed Hamiltonian : $\hat{h} = \hat{h}_0 + \hat{f}_0$

Valence space Hamiltonian : $\hat{h}_v = \hat{p}\hat{h}\hat{p}$

SuSy transformation :

1. Remove the Ac orbitals of the core :

$$\hat{u}_k^{\pm} = \prod_{i=1}^{A_c} \hat{u}_i^{\pm}$$

2. Transformation of the Hamiltonian:

$$\hat{h}_{v,k} = \hat{u}_k^- \hat{h} \hat{u}_k^+ = \hat{h}_k + \hat{f}_k$$

$$\hat{f}_k = \hat{u}_k^- \hat{f}_0 \hat{u}_k^+$$

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Quantum Mechanics Supersymmetry

$$\hat{h}_0^l = \hat{a}^+ \ \hat{a}^- + e_0^l$$

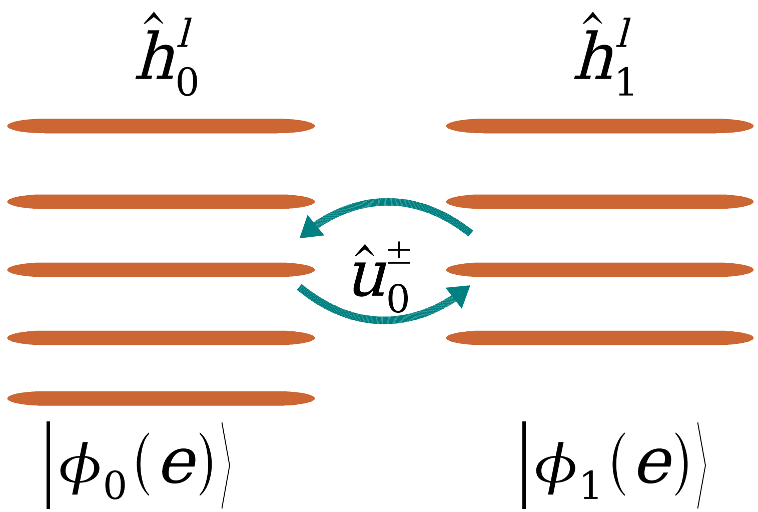
$$\hat{h}_1^l = \hat{a}^- \ \hat{a}^+ + e_0^l$$

Operators $\hat{u}_0^{\pm}$:

$$|\phi_1(e)\rangle = \hat{u}_0^- \, |\phi_0(e)\rangle$$

$$\hat{u}_0^- = \frac{1}{\sqrt{\hat{h}_1 - e_0^0}} \hat{a}_0^-$$

$$\hat{h}_1 = \hat{u}_0^- \ \hat{h}_0 \ \hat{u}_0^+$$



Excitation processes :

Excitation operator: $\hat{f}_0$

Perturbed Hamiltonian : $\hat{h} = \hat{h}_0 + \hat{f}_0$

$$\hat{h}_{v,k} = \hat{u}_k^- \ \hat{h} \ \hat{u}_k^+ = \hat{h}_k + \hat{f}_k \qquad \hat{f}_k = \hat{u}_k^- \ \hat{f}_0 \ \hat{u}_k^+$$

Internal approximation :

$$\hat{h}_{v,k} \approx \hat{h}_k + \hat{f}_0$$

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Quantum Mechanics Supersymmetry

Electric excitation: $\hat{f}_0(\lambda, L, M) = \hat{r}^{\lambda}\, \hat{Y}_{L,M}$

$$\langle l'm' | \hat{f}_k(\lambda, L, M) | l m \rangle = \hat{f}_k^{l'l}(\lambda) \langle l'm' | \hat{Y}_{L,M} | l m \rangle$$

$$\hat{f}_k^{l'l}(\lambda) = \hat{u}_{k-1}^{l'-}\, \hat{r}^{\lambda}\, \hat{u}_{k-1}^{l+}$$ non diagonal in the r-space

Doorway state: ——

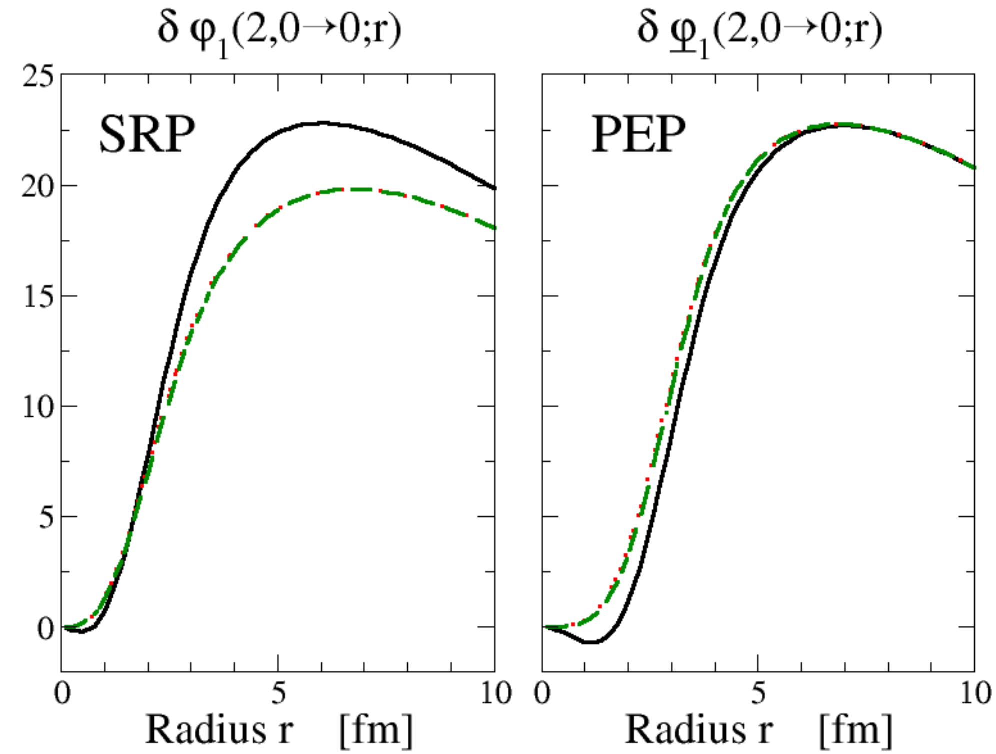
$$|\delta\phi_k(\lambda, l \to l')\rangle = \hat{f}_k^{l'l}(\lambda) |\phi_k^l\rangle$$

Internal approximation: - - - - -

$$|\delta\phi_k(\lambda, l \to l')\rangle = \hat{f}_0^{l'l}(\lambda) |\phi_k^l\rangle$$

Diagonal approximation: - - - - -

$$|\delta\phi_k(\lambda, l \to l')\rangle = diag(\hat{f}_k^{l'l}) |\phi_k^l\rangle$$



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Quantum Mechanics Supersymmetry

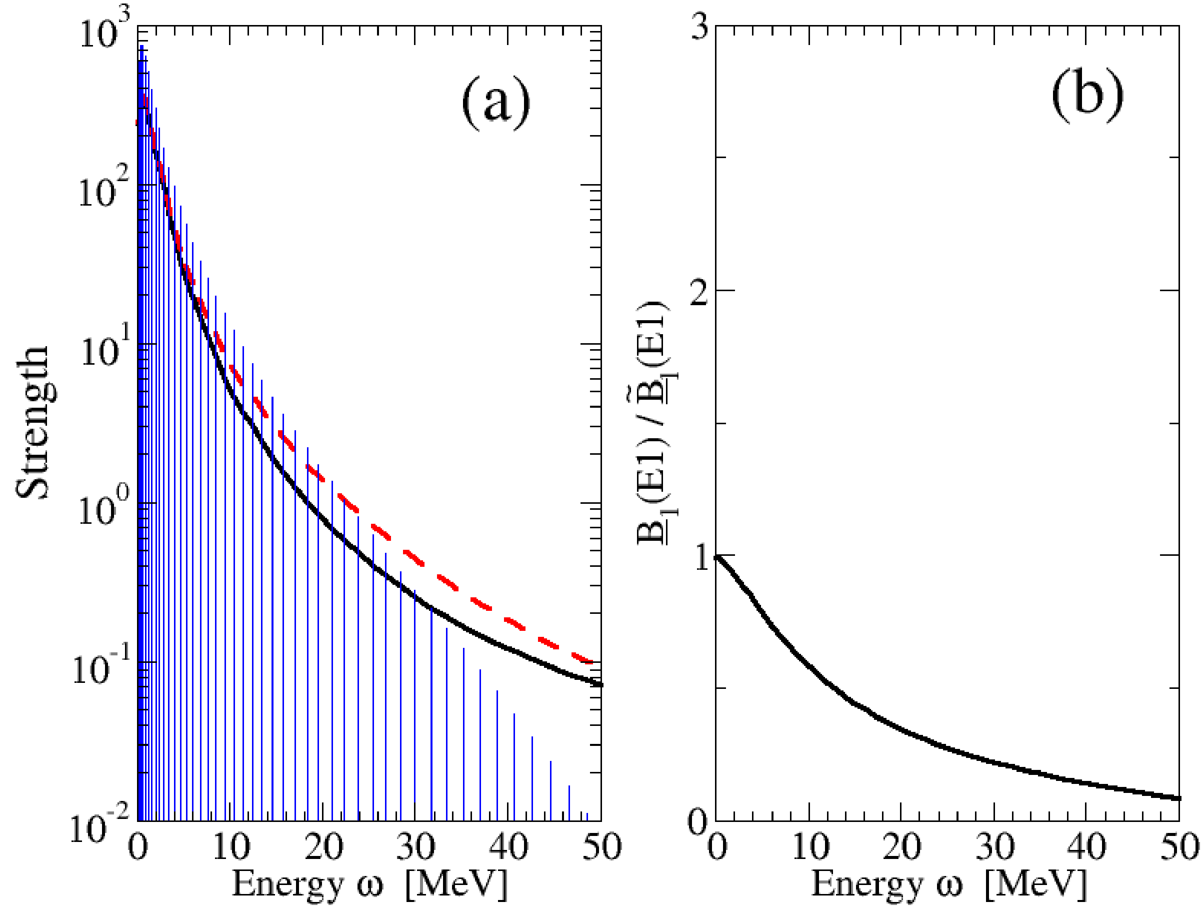
Electric excitation: $\hat{f}_k^{l'l}(\lambda) = \hat{u}_{k-1}^{l'-}\,\hat{r}^{\lambda}\,\hat{u}_{k-1}^{l+}$

Transition probability :

$$B_k(E\lambda, i \to f) = \langle \phi_k^f | \hat{f}_k^{l_f l_i}(\lambda) | \phi_k^i \rangle$$

Energy weigthed sum rule :

$$m_t(E\lambda) = \int d\omega\, \omega^t S(E\lambda, \omega)$$



	$(m_0 - \tilde{m}_0)/m_0$	$(m_1 - \tilde{m}_1)/m_1$	$(m_2 - \tilde{m}_2)/m_2$
$E0$	1.4%	3.9%	17.4%
$E1$	-6.8%	-33.3%	-93.1%

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Quantum Mechanics Supersymmetry

Gaussian excitation: $\hat{f}_0 = \exp - r^2/\mu^2$

Doorway state:

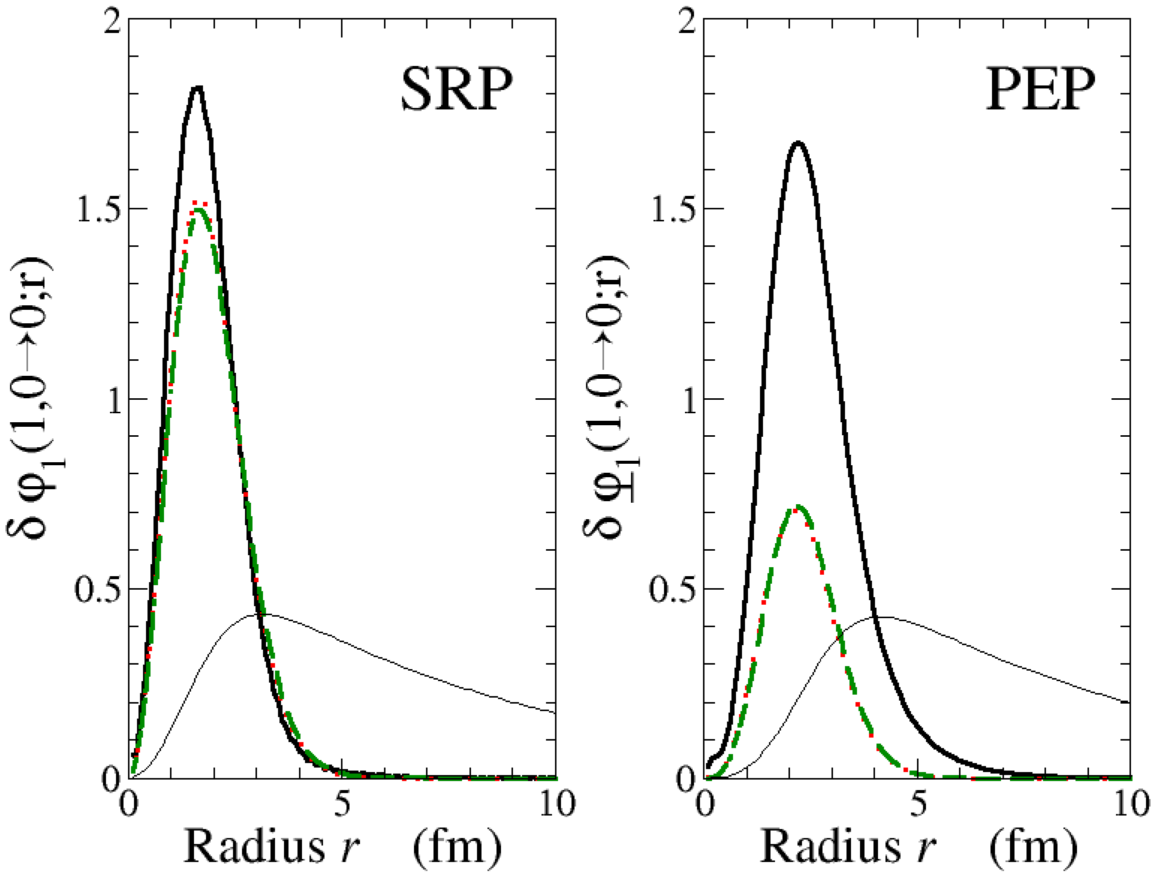
$$\left|\delta\phi_k(\lambda, l \to l')\right\rangle = \hat{f}_k^{l'l}(\lambda)\left|\phi_k^l\right\rangle$$

Internal approximation:

$$\left|\delta\phi_k(\lambda, l \to l')\right\rangle = \hat{f}_0^{l'l}(\lambda)\left|\phi_k^l\right\rangle$$

Diagonal approximation:

$$\left|\delta\phi_k(\lambda, l \to l')\right\rangle = diag(\hat{f}_k^{l'l})\left|\phi_k^l\right\rangle$$



➪ **Strong effects if the excitation operator overlap the core potential**

Quantum Mechanics Supersymmetry

- ✔ We have introduced new operators : $\hat{u}_0^{\pm}$
- ✔ We propose a new framework for SuSy transformations :
  - ◆ It is equivalent to the Projection method
  - ◆ It contains the usual internal approximation

Results :

- ✔ The internal approximation is no longer accurate if the excitation operator overlap the core potential
- ✔ Or in the case of 2 particles in the valence space
  - ⇨ 2body interaction has to be transformed !

$$\hat{u}^{-}(1)\,\hat{u}^{-}(2)\,V(\hat{r}_1-\hat{r}_2)\,\hat{u}^{+}(1)\,\hat{u}^{+}(2)$$

Outlooks :

Apply SuSy to 2-body n-n interaction (^{6}He, ^{11}Li, ...) :

- ◆ Structure
- ◆ Reaction mechanism