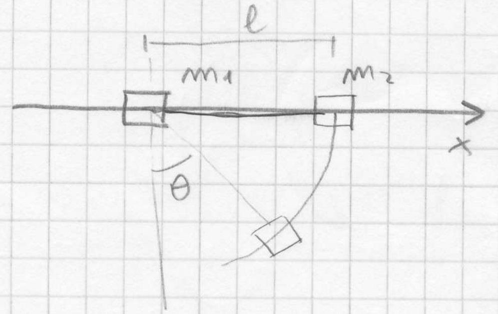
Esercizio del pendolo doppio

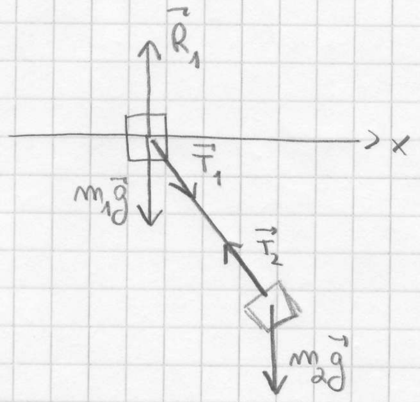


m_1 libero di scorrere lungo x senza attrito.

Dato m_1, m_2 e l (lunghezza del filo), trovare

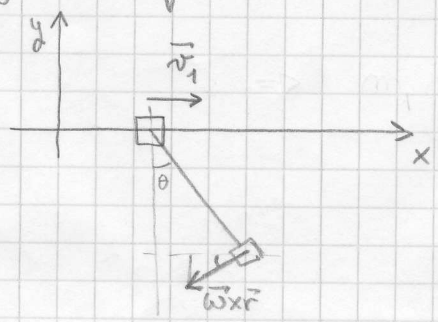
$v_1(\theta)$ e $v_2(\theta)$

Forze



Forze esterne // $\bar{y}$ $\Rightarrow$ $m_1 v_1 + m_2 v_{2x} = 0$ (1)

In generale posso scrivere $\vec{v}_2 = \vec{v}_1 + \vec{\omega}\times\vec{r}$



$$\Rightarrow \quad v_{2,x} = v_1 - \omega l \cos\theta$$

$$v_{2y} = - \omega l \sin\theta$$

$$\Rightarrow \quad m_1 v_1 + m_2 v_1 - m_2 \omega l \cos\theta = 0$$

$$\Rightarrow \quad v_1 = \frac{m_2}{m_{12}} \omega l \cos\theta$$

Adesso devo trovare ω:

$$\mathcal{L}_{Forze} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \leftarrow \text{teorema delle forze vive}$$

$$\mathcal{L}_{R_1} = 0 \quad (\vec{R}_1 \perp d\vec{r}_1)$$

$$\mathcal{L}_{m_1 g} = 0 \quad (m_1\vec{g} \perp d\vec{r}_1)$$

$$\mathcal{L}_{T_1} = \int \vec{T}_1 \cdot d\vec{r}_1 = T_1 \cdot \Delta x_1 \sin\theta$$

$$\mathcal{L}_{T_2} = \int \vec{T}_2 \cdot d\vec{r}_2 = \int \vec{T}_2 \cdot \left(d\vec{r}_1 + dt(\vec{\omega}\times\vec{r}_2)\right) = \int \vec{T}_2 \cdot d\vec{r}_1 = -\mathcal{L}_{T_1}$$

$dt(\vec{\omega}\times\vec{r}_2) \perp$ filo

$$\Rightarrow \mathcal{L}_{Forze} = \mathcal{L}_{m_2 g} = m_2\, g\, l \cos\theta$$

$$m_2 g l \cos\theta = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(v_1^2 + \omega^2 l^2 - 2 v_1 \omega l \cos\theta\right)$$

$$= \cancel{\frac{1}{2} m_{12} \frac{m_2^2}{m_{12}^2} \omega^2 l^2 \cos^2\theta} + \frac{1}{2} m_2 \omega^2 l^2$$

$$- m_2 \frac{m_2}{m_{12}} \omega^2 l^2 \cos^2\theta \quad \frac{1}{2}$$

$$= \frac{1}{2} m_2 \omega^2 l^2 \left(1 - \frac{m_2}{m_{12}} \cos^2\theta\right)$$

$$= \frac{1}{2} m_2 \omega^2 l^2 \left(\frac{m_1 + m_2 \sin^2\theta}{m_{12}}\right)$$

$$\Rightarrow \omega = \sqrt{2\, \frac{g}{l} \cos\theta \, \frac{m_1 + m_2}{m_1 + m_2 \sin^2\theta}}\ .$$

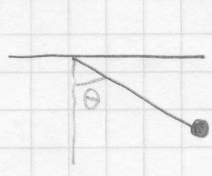
Consideriamo $m_1 \to \infty$:

$$\omega \to \sqrt{2\frac{g}{l} \cos\theta}$$

$$|\vec{v}_1| \to 0$$

$$|\vec{v}_2| \to \sqrt{2 g\, l \cos\theta}$$

Questo caso corrisponde al pendolo semplice!



$$\mathcal{L} = m g l \cos\theta = \frac{1}{2} m v^2$$

$$v = \sqrt{2 g\, l \cos\theta} = |\vec{v}_2| \quad !$$