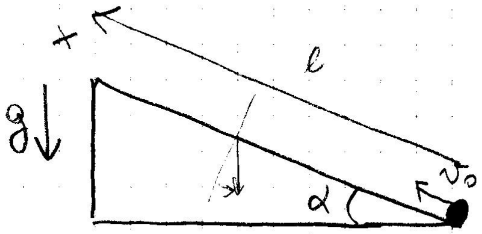
Esercizio della pallina sul piano inclinato



v_0 t.c. cada di sotto?
Data ℓ

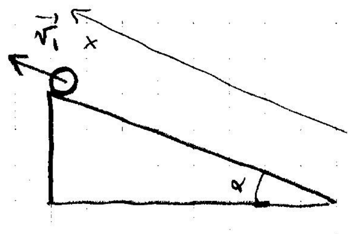
$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{g}\,t^2 \quad ; \quad \vec{v} = \vec{v}_0 + \vec{g}\,t$$

lungo x :
$$x = x_0 + v_0 t - \frac{1}{2} g \,\mathrm{sen}\alpha\, t^2$$
$$v_x = v_0 - g\,\mathrm{sen}\alpha\, t$$

t^* t.c. $v_x = 0 \Rightarrow \dfrac{v_0}{g\,\mathrm{sen}\alpha} = t^*$

$$\ell = \frac{v_0^2}{2g\,\mathrm{sen}\alpha} \Rightarrow v_0 \geq \sqrt{2g\ell\,\mathrm{sen}\alpha}$$

Supponiamo $v_0 > \sqrt{2g\ell\,\mathrm{sen}\alpha}$, dove cade la pallina?



Trovare v_1:

$x = v_0 t - \frac{1}{2} g\,\mathrm{sen}\alpha\, t^2$ $\qquad$ $\ell = v_0 t - \frac{1}{2} g\,\mathrm{sen}\alpha\, t^2 \Rightarrow$

$v_x = v_0 - g\,\mathrm{sen}\alpha\, t$

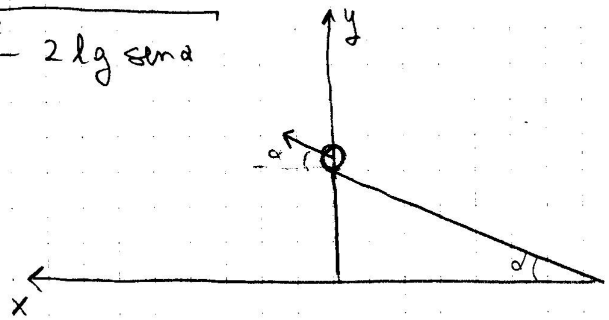
$$g\,\mathrm{sen}\alpha\, t^2 - 2v_0 t + 2\ell = 0$$

$$v_1 = v_0 - v_0 \mp \sqrt{v_0^2 - 2\ell g\,\mathrm{sen}\alpha} \quad \leftarrow \quad t = \frac{v_0 \pm \sqrt{v_0^2 - 2\ell g\,\mathrm{sen}\alpha}}{g\,\mathrm{sen}\alpha}$$
$$= \mp \sqrt{v_0^2 - 2\ell g\,\mathrm{sen}\alpha}$$

$\Rightarrow$ il segno giusto è "$-$" in t $\Rightarrow$

$$v_1 = \sqrt{v_0^2 - 2 l g \operatorname{sen}\alpha}$$

Adesso



$$\begin{cases} x(t) = v_1 \cos\alpha \, t \\ y(t) = l \operatorname{sen}\alpha + v_1 \operatorname{sen}\alpha \, t - \frac{1}{2} g t^2 \end{cases}$$

$$y(t) = 0 \implies l \operatorname{sen}\alpha + v_1 \operatorname{sen}\alpha \, t - \frac{1}{2} g t^2 = 0$$

$$g t^2 - 2 v_1 \operatorname{sen}\alpha \, t - 2l = 0$$

unico segno possibile, perchè $t > 0$!

$$t = \frac{v_1 \operatorname{sen}\alpha + \sqrt{v_1^2 \operatorname{sen}^2\alpha + 2lg}}{g}$$

$$\implies x = v_1 \cos\alpha \, \frac{v_1 \operatorname{sen}\alpha}{g} \left(1 + \sqrt{1 + \frac{2lg}{v_1^2 \operatorname{sen}^2\alpha}} \right)$$

$$x = v_1^2 \, \frac{\cos\alpha \operatorname{sen}\alpha}{g} \left(1 + \sqrt{1 + \frac{2lg}{v_1^2 \operatorname{sen}^2\alpha}} \right).$$