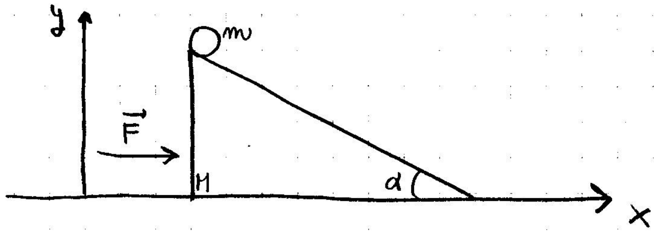
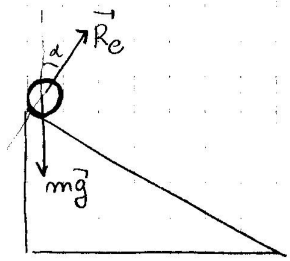
Esercizio del corpo sul cuneo libero di muoversi



$\vec{F}$ t.c. il cuneo non si muova.

1) F ?

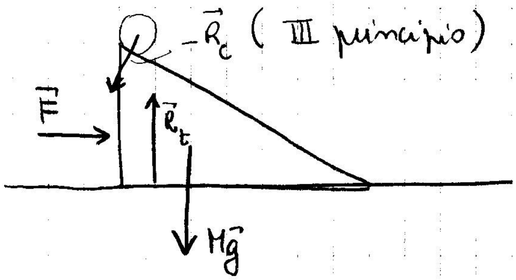
- Forze su m



$m\vec{g}$, $\vec{R}_e$

$$m\,\ddot{\vec{r}}_m = m\vec{g} + \vec{R}_e \qquad (1)$$

- Forze su M



$\vec{F}$, $-\vec{R}_e$, $M\vec{g}$, $\vec{R}_t$

$$M\,\ddot{\vec{r}}_M = M\vec{g} + \vec{R}_t + \vec{F} - \vec{R}_e \qquad (2)$$

Scompongo (1) e (2) su x e y:

$$\begin{cases} m\,\ddot{x}_m = R_e \sin\alpha \\ m\,\ddot{y}_m = -mg + R_e \cos\alpha \end{cases}$$

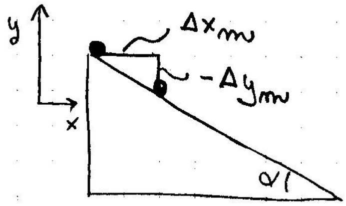
$$\begin{cases} M\,\ddot{x}_M = F - R_e \sin\alpha \\ M\,\ddot{y}_M = -Mg + R_t - R_e \cos\alpha \end{cases} \qquad (3)$$

Il cuneo non si muove $\Rightarrow$ $\ddot{x}_M = \ddot{y}_M = 0$

$$\Rightarrow \begin{cases} F = R_e \sin\alpha \\ R_t = Mg + R_e \cos\alpha \end{cases} \qquad (4)$$

Il blocco si muove lungo il piano inclinato $\Rightarrow$

$$\frac{(-\Delta y_m)}{\Delta x_m} = tg\,\alpha \quad \Rightarrow \quad \ddot{y}_m = -tg\,\alpha\; \ddot{x}_m$$



$$\Rightarrow \quad a \equiv \ddot{x}_m \qquad \begin{cases} m\,a = R_c \sin\alpha \\ -m\,a\,tg\,\alpha = -mg + R_c \cos\alpha \end{cases} \qquad (5)$$

$$m a \sin\alpha = mg\cos\alpha - R_c \cos^2\alpha = mg\cos\alpha - R_c + R_c \sin^2\alpha$$

$$\cancel{m a \sin\alpha} = mg\cos\alpha - R_c + \cancel{m a \sin\alpha}$$

$$R_c = mg\cos\alpha$$

$$a = g\cos\alpha\sin\alpha$$

$$\Rightarrow \begin{cases} \ddot{x}_m = g\cos\alpha\sin\alpha \\ \ddot{y}_m = -g\sin^2\alpha \end{cases}$$

$$\begin{cases} F = mg\cos\alpha\sin\alpha \\ R_t = Mg + mg\cos^2\alpha \end{cases} \qquad (6)$$

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Risolviamo in quest'altra maniera:

$$\text{sistema } \underbrace{(M+m)}_{m_t} \longrightarrow \vec{F},\ M\vec{g},\ m\vec{g},\ \vec{R}_t$$

$$\begin{cases} m_t\,\ddot{x}_{CM} = F \\ m_t\,\ddot{y}_{CM} = -(M+m)g + R_t \end{cases} \qquad (7)$$

Inoltre so che

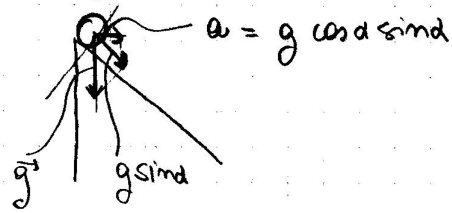
$$\ddot{x}_M = \ddot{y}_M = 0 \quad \Rightarrow \quad \begin{cases} m_t \, \ddot{x}_{CM} = m \, \ddot{x}_m \\ m_t \, \ddot{y}_{CM} = m \, \ddot{y}_m \end{cases}$$

Uso inoltre $\quad \ddot{y}_m = - \operatorname{tg}\alpha \, \ddot{x}_m \quad \Rightarrow$

$$\ddot{x}_m = a$$

$$\begin{cases} ma = F \\ - ma \operatorname{tg}\alpha = -(M+m)g + R_t \end{cases}$$

infine so che
(perchè il blocco
corre lungo il piano)



$$\Rightarrow \quad a = g \cos\alpha \sin\alpha \quad \Rightarrow \quad \begin{cases} F = mg \cos\alpha \sin\alpha \\ R_t = Mg + mg \cos^2\alpha \end{cases}$$

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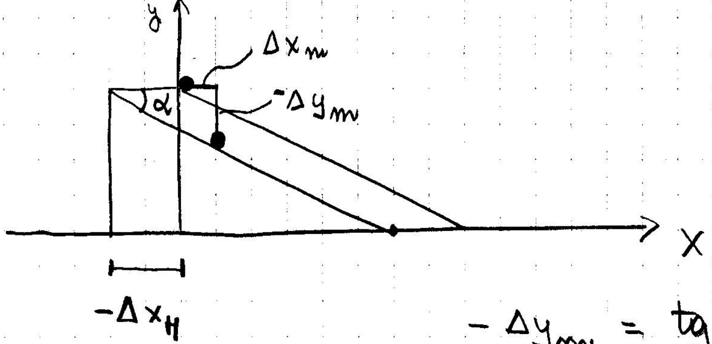
2) Tolgo F : come si muove il sistema?

Uso Eqns. (3) con $F = 0$

$$\begin{cases} m \, \ddot{x}_m = R_c \sin\alpha \\ m \, \ddot{y}_m = - mg + R_c \cos\alpha \\ M \, \ddot{x}_M = - R_c \sin\alpha \\ M \, \ddot{y}_M = - Mg + R_t - R_c \cos\alpha \end{cases} \qquad (8)$$

$\ddot{y}_M = 0$ (il cuneo può solo scivolare sul piano orizzontale)

Relazione tra $\ddot{x}_M$, $\ddot{x}_m$ e $\ddot{y}_m$:



$$-\Delta y_m = tg\,\alpha\,(\Delta x_m - \Delta x_M) \qquad (9)$$

$\Rightarrow \quad -\ddot{y}_m = tg\,\alpha\,(\ddot{x}_m - \ddot{x}_M)$

$\Rightarrow$ (8) diventa : $\quad \Big[$ uso $\quad a \equiv \ddot{x}_m$, $\quad A = \ddot{x}_M$

$$\begin{cases} m\,a = R_e \sin\alpha \\ -m\,tg\alpha\,(a - A) = -mg + R_e\cos\alpha \\ M\,A = -R_e\sin\alpha \\ 0 = -Mg + R_t - R_e\cos\alpha \end{cases} \qquad (10)$$

4 eqns e 4 incognite (a, A, R_e, R_t)

Risolviamo :

$m\,(a - A)\sin\alpha = mg\cos\alpha - R_e\cos^2\alpha$

$R_e\sin^2\alpha - \frac{m}{M}(-R_e\sin\alpha)\sin\alpha = mg\cos\alpha - R_e\cos^2\alpha$

$\Rightarrow \quad R_e\left(1 + \frac{m}{M}\sin^2\alpha\right) = mg\cos\alpha$

$$R_e = \frac{m\,M}{M + m\sin^2\alpha}\,g\cos\alpha$$

$$a = \frac{M g\cos\alpha\sin\alpha}{M + m\sin^2\alpha}$$

$$A = - \frac{mg \cos\alpha \sin\alpha}{M + m \sin^2\alpha}$$

$$a_y = \operatorname{tg}\alpha \, A - \operatorname{tg}\alpha \, a = - \frac{(m+M) g \sin^2\alpha}{M + m \sin^2\alpha}$$

$$R_t = Mg + \frac{M m}{M + m \sin^2\alpha} g \cos^2\alpha$$

$$= \frac{M^2 g + Mmg}{M + m \sin^2\alpha} = Mg \frac{m+M}{M + m \sin^2\alpha}$$

<u>Traiettorie</u>:

M : moto rettilineo unif. accelerato lungo $-\hat{x}$,

m : $\quad x_m(t) = + \frac{1}{2} a t^2$

$\quad y_m(t) = l_0 \sin\alpha - \frac{1}{2} a_y t^2$

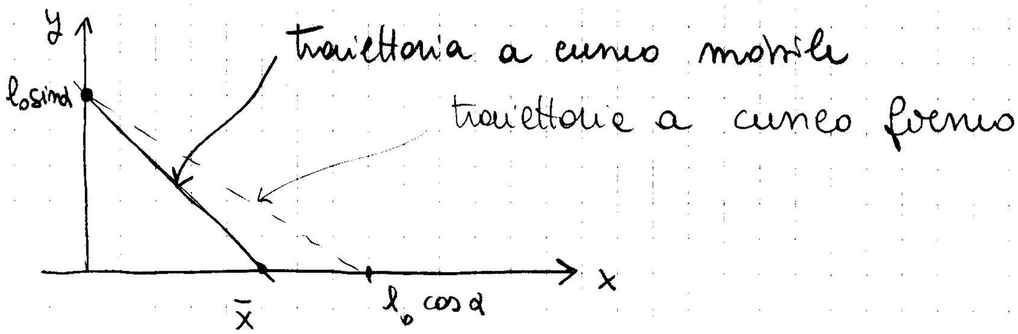
$$x_m(t) = \frac{1}{2} a t^2$$

$$(y_m - l_0 \sin\alpha) = - \frac{1}{2} a_y t^2$$

$$y_m - l_0 \sin\alpha = - \frac{a_y}{a} x_m$$

$$y_m = l_0 \sin\alpha - \frac{(m+M) \cancel{g} \sin^2\alpha}{M \cancel{g} \cos\alpha \cancel{\sin\alpha}} x_m$$

$$y = l_0 \sin\alpha - \frac{m+M}{M} \operatorname{tg}\alpha \, x$$



$$\bar{x} = l_0 \sin\alpha \; \frac{M}{m+M} \; \frac{\cos\alpha}{\sin\alpha} = l_0 \cos\alpha \; \frac{M}{m+M}$$

Altro modo per risolvere il problema:

Costanti del moto: Energia

$v_{CM,x}$ $\quad a_{CM,x}$ ← non ci sono forze est. lungo x

$$E = \frac{1}{2} M (\dot{x}_M^2 + \dot{y}_M^2) + \frac{1}{2} m (\dot{x}_m^2 + \dot{y}_m^2)$$
$$+ M g y_M + m g y_m$$

$$v_{CM_x} = m \dot{x}_m + M \dot{x}_M = 0$$
$$a_{CM_x} = m \ddot{x}_m + M \ddot{x}_M = 0$$

← inizialmente il sistema è fermo

E costante $\Rightarrow$ $\frac{dE}{dt} = 0$

$$\frac{dE}{dt} = M \dot{x}_M \ddot{x}_M + M \dot{y}_M \ddot{y}_M + m \dot{x}_m \ddot{x}_m + m \dot{y}_m \ddot{y}_m$$
$$+ M g \dot{y}_M + m g \dot{y}_m = 0$$

informazioni sul moto: vedi Eq. (9)
$$\begin{cases} \dot{y}_M = \ddot{y}_M = 0 \\ \dot{y}_m = \operatorname{tg}\alpha\,(\dot{x}_M - \dot{x}_m) \\ \ddot{y}_m = \operatorname{tg}\alpha\,(\ddot{x}_M - \ddot{x}_m) \end{cases}$$