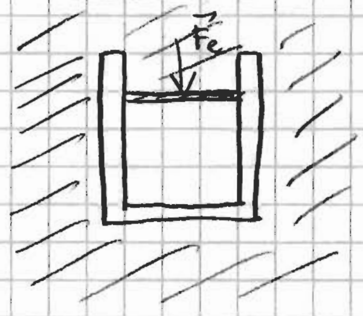
Esercizio sulle politropiche



contenitore con massa m e cal. spec. c
n moli. Pistone e resto adiabatici
Variamo di dV il volume reversibilmente
Trovare l'eq. della trasformazione.

1° principio $\qquad dU = \delta Q_g - \delta L$

$$nC_V\, dT = \delta Q_g - p\, dV$$

$$\delta Q_{gas} + \delta Q_{cont} = 0 \implies \delta Q_{gas} = -\delta Q_{cont} = -mc\, dT$$

$$\implies (nC_V + mc)\, dT = -p\, dV$$

$pV = nRT$
$\qquad \implies$
$p\,dV + V\,dp = nR\,dT$

$$\frac{(nC_V + mc)}{nR}(p\,dV + V\,dp) + p\,dV = 0$$

$$\frac{\overbrace{nC_V + nR}^{nC_p} + mc}{nR}\, p\,dV = -\frac{nC_V + mc}{nR}\, V\,dp$$

$$\underbrace{\left(\frac{nC_p + mc}{nC_V + mc}\right)}_{\alpha} p\,dV = -V\,dp$$

$$\alpha\, p\,dV = -V\,dp \longrightarrow \boxed{pV^{\alpha} = \text{cost}} \quad \text{politropica}$$

Nota: $\quad m$ piccolo $\implies mc \ll nC_V \qquad \alpha = \gamma$ adiab.

$\qquad m$ grande $\implies mc \gg nC_{p,V} \qquad \alpha = 1$ isoterma

Capacità termica molare per la trasformazione:

$$nC = \frac{\delta Q_g}{dT} \qquad \delta Q_g \underset{1^\circ \text{princ.}}{=} dU + p\,dV = nC_V\,dT + p\,dV$$

$$\Downarrow$$

$$\boxed{p\,dV = n(C - C_V)\,dT}$$

$$pV = nRT \qquad p\,dV + V\,dp = nR\,dT$$

$$pV^{\alpha} = p_0V_0^{\alpha} \Rightarrow \alpha p \cancel{V^{\alpha-1}}\,dV + V^{\alpha}\,dp = 0$$

$$\Downarrow$$

$$-\alpha p\,dV = V\,dp$$

$$p\,dV\,(1-\alpha) = nR\,dT$$

$$\boxed{p\,dV = \frac{nR}{1-\alpha}\,dT}$$

$$\cancel{n}(C - C_V)\cancel{dT} = \cancel{n}\frac{R}{1-\alpha}\cancel{dT}$$

$$\boxed{C = C_V + \frac{R}{1-\alpha}}$$

Per $\alpha = \frac{C_p}{C_V}\ (\equiv \gamma)$ $\qquad C = C_V + \frac{R}{1 - \frac{C_p}{C_V}} = C_V + \cancel{R}\frac{C_V}{\frac{\cancel{C_V - C_p}}{-1}} = 0$

Torna, poiché $\alpha = \gamma$ è l'adiabatica

Riscriviamo $\quad C = C_V + \frac{R}{1 - \frac{nC_p + mc}{nC_V + mc}} = C_V + \frac{R\,(nC_V + mc)}{n(-R)}$

$$= -\frac{mc}{n}$$

$\Rightarrow$ C può essere neg.: $\Delta T < 0$ anche se assorbo calore.

Basta che $\delta L > \delta Q$.

Infine