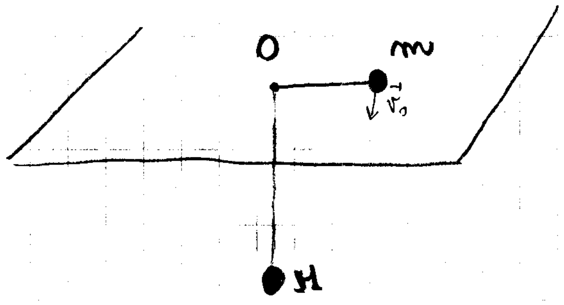
# Esercizio delle due masse attaccate ad un filo



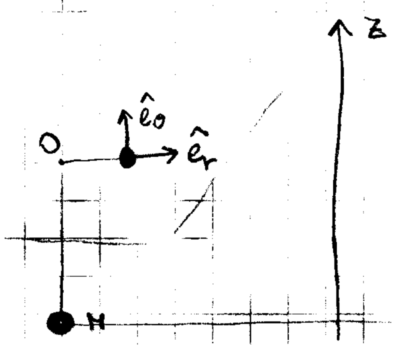

A $t=0$ $\quad d =$ distanza di $m$ da $O$

$\vec{v}_0 =$ velocità di $m$, diretta $\perp$ a $\overline{Om}$

## Equazioni del moto:

Per $m$ coord polari

Per $M$, asse $z$




$$\Rightarrow \begin{cases} M\ddot{z} = -Mg + T \\ m(\ddot{r} - r\dot{\theta}^2) = -T \\ m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \end{cases}$$

Filo inestensibile $\Rightarrow$ $\dot{z} = \dot{r}$ ; $\ddot{z} = \ddot{r}$

$$\Rightarrow \begin{cases} M\ddot{r} = -Mg + T \\ m\ddot{r} - mr\dot{\theta}^2 = -T \\ m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0 \end{cases} \Rightarrow (m+M)\ddot{r} - mr\dot{\theta}^2 = -Mg \qquad (1)$$

Stesse equazioni da principi di conservazione:

Si conserva (i) energia

(ii) mom. angolare rispetto a O

$$\Rightarrow E = \frac{1}{2}m\left[\dot{r}^2 + (r\dot{\theta})^2\right] + \frac{1}{2}M\dot{z}^2 + Mgz$$

$$\vec{L}_0 = mr^2\dot{\theta}\,\hat{z}$$

$$\Rightarrow \quad \frac{dE}{dt} = m\ddot r \dot r + m r \dot r \dot\theta^2 + m r^2 \dot\theta \ddot\theta + M \dot z \ddot z + M g \dot z = 0$$

$$\begin{array}{l} \ddot z = \ddot r \\ \dot z = \dot r \end{array} \Rightarrow \quad (m+M)\ddot r \dot r + m r^2 \dot\theta \ddot\theta + m r \dot r \dot\theta^2 + M g \dot r = 0$$

$$\frac{d|\vec L_0|}{dt} = 2m r \dot r \dot\theta + m r^2 \ddot\theta$$

$$= m\, r\left(r\ddot\theta + 2\dot r \dot\theta\right) = 0 \Rightarrow \text{II eqn. di (1)}$$

$$\Rightarrow \quad r\ddot\theta = -2\dot r\dot\theta$$

$$\Rightarrow \quad (m+M)\ddot r \cancel{\dot r} + m\dot\theta r(-2\cancel{\dot r}\dot\theta) + m r \cancel{\dot r}\dot\theta^2 + M g \cancel{\dot r} = 0$$

$$(m+M)\ddot r - m r \dot\theta^2 = -Mg \Rightarrow \text{I eqn. di (1)}.$$

$|\vec v_0|$ t.c. m fa un moto circ. uniforme

moto circ. unif. $\Rightarrow$ $\dot r = \ddot r = 0$ $\quad r = d$

$$\dot\theta = \omega_0$$

$$\Rightarrow \quad -m d\,\omega_0^2 = -Mg \Rightarrow \omega_0^2 = \frac{Mg}{md} \Rightarrow v_0^2 = \omega_0^2 d^2$$

$$= \frac{Mgd}{m} = {v^*}^2$$

Supponiamo $v_0 \neq v^*$. Sia $v_0 < v^*$: può lui cadere in O?

No!! Infatti $\vec L_0$ si conserva $\Rightarrow \forall t$

$$m\dot\theta r^2 = m\,\omega_0 d^2$$

$$\Rightarrow \quad E = \frac{1}{2}(m+M)\dot r^2 + Mgz + \frac{1}{2} m r^2\dot\theta^2 = \frac{1}{2}(m+M)\dot r^2 + Mgz + \frac{L_0^2}{2mr^2}$$

$$r = 0 \Rightarrow E = \infty \quad !!$$

$\Rightarrow$ troviamo $r_{min}$ :

$$E(t=0) = \frac{1}{2} m v_0^2 = E(t) = \frac{1}{2}(m+M)\dot{r}^2 + \frac{L_0^2}{2mr^2} + Mgz$$

$$r_{min} \Rightarrow \dot{r} = 0 \Rightarrow$$

$$\frac{1}{2} m v_0^2 = \frac{L^2}{2mr_{min}^2} + Mg z_{min}$$

$$z_{min} \equiv z \text{ per } r_{min} \Rightarrow z = -(d - r_{min})$$

$$\frac{1}{2} m v_0^2 = \frac{L^2}{2mr^2} - Mgd + Mgr$$

$$\frac{1}{2} m v_0^2 + Mgd = \frac{L_0^2}{2mr^2} + Mgr$$

$$\frac{1}{2} m v_0^2 + Mg(d-r) = \frac{1}{2}\frac{m^{\cancel{2}} v_0^2 d^2}{\cancel{m}\, r^2}$$

$$\frac{1}{2} m v_0^2 \frac{r^2 - d^2}{r^2} = Mg(r-d) \Rightarrow r = d \text{ è una soluzione. Se } r \neq d$$

$$\frac{1}{2} m v_0^2 (r+d) = Mgr^2 \longrightarrow \text{soluzione}$$

$$r = \frac{m v_0^2}{4Mg}\left[1 \pm \sqrt{1 + \frac{8Mgd}{m v_0^2}}\right]$$

da escludere, perchè $r < 0$ !

$$\Rightarrow \begin{cases} r_{max} = d \\ r_{min} = \dfrac{m v_0^2}{4Mg}\left[1 + \sqrt{1 + \dfrac{8Mgd}{m v_0^2}}\right] \end{cases}$$