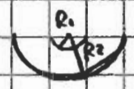
Esercizio sul tubo ad U

Un liquido incomprimibile in un tubo a U

S = sezione del tubo

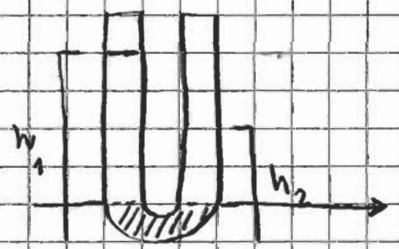
V = volume del liquido

ρ = densità del fluido



$$V_0 = \pi(R_2 - R_1)\cdot S$$

1) posizione di equilibrio



$$h_1 + h_2 = 2H = \text{cost}$$

$$E_{pot} = m_1 \frac{h_1}{2} g + m_2 \frac{h_2}{2} g$$

$\frac{h_1}{2}$: pos. c.di m. del liquido 1/2 ; $\frac{h_2}{2}$: pos. c.di m. del liquido 2/2

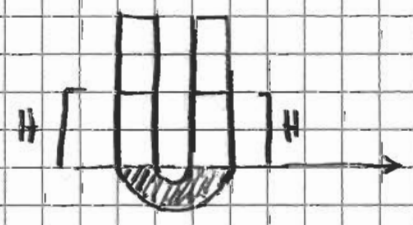
$$= \rho S h_1 \frac{h_1}{2} g + \rho S h_2 \frac{h_2}{2} g$$

$$= \rho S \frac{g}{2} \left(h_1^2 + h_2^2\right) = \rho S \frac{g}{2} \left(h_1^2 + 4H^2 + h_1^2 - 4H h_1\right)$$

$$= \rho S \frac{g}{2} \left(2h_1^2 + 4H^2 - 4H h_1\right)$$

$$\frac{dE_{pot}}{dh_1} = \rho S \frac{g}{2} \left(4h_1 - 4H\right) = 0 \Rightarrow$$

$$h_1 = H \; ; \; h_2 = H \Rightarrow$$

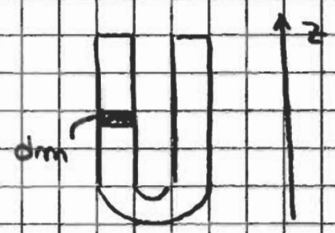


$$V = 2HS + V_0$$

$$H = \frac{V - V_0}{2S} = \frac{V - \pi(R_2 - R_1)S}{2S}$$

2) frequenza piccole oscillazioni e moto della superficie sapendo che $h_1(t=0) = \frac{3}{2}H$ e $v(t=0) = 0$

$$dE_{cin} = \frac{1}{2}\, dm\, \dot{z}^2$$



$$\Downarrow$$

$$E_{cin} = \frac{1}{2}\rho V \dot{z}^2$$

$$\Rightarrow \quad \frac{1}{2}\rho V \dot{z}^2 + \rho S g \frac{\alpha}{2}\left(2z^2 + 4H^2 - 4Hz\right) = E$$

$$\frac{dE}{dt} = 0 = \frac{1}{2}\rho V \cdot 2 \dot{z}\ddot{z} + \rho S g \frac{\alpha}{2}\left(4z\dot{z} - 4H\dot{z}\right)$$

$$V\ddot{z} + 2Sgz - 2SgH = 0$$

$$z(t) = A\cos(\omega t + \varphi) + z_{part} \qquad \omega = \sqrt{\frac{2Sg}{V}}$$

$$z_{part} \text{ t.c. } 2Sg z_{part} - 2Sg H = 0 \qquad z_{part} = H$$

$$z(t) = A\cos\left(\sqrt{\frac{2Sg}{V}}\, t + \varphi\right) + H$$

$$\dot{z}(t) = -A\sqrt{\frac{2Sg}{V}}\, \sin\left(\sqrt{\frac{2Sg}{V}}\, t + \varphi\right)_{t=0} = 0 \Rightarrow \quad \varphi = 0$$

$$z(0) = A + H = \frac{3}{2}H \quad \Rightarrow \quad A = \frac{1}{2}H$$

$$z(t) = H\left[\frac{1}{2}\cos\left(\sqrt{\frac{2Sg}{V}}\, t\right) + 1\right]$$