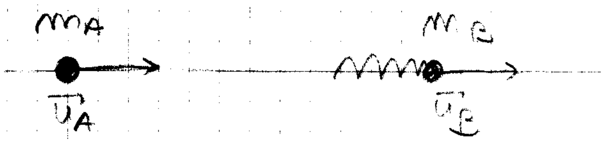


massa della molla ~0

$|\vec{u}_A| > |\vec{u}_B|$

Calcolare $\vec{v}_A$ e $\vec{v}_B$ dopo l'urto:

(i) Risolviamo con le eqn. del moto

$$\begin{cases} M \ddot{x}_{CM} = 0 \\ \mu \ddot{x} = -k(x - l_0) \end{cases}$$

per A in contatto con la molla

$x \equiv x_B - x_A$

$$x_{CM}(t) = \underbrace{x_{CM}(0)}_{m_B l_0 / M} + v_{CM}\, t \qquad v_{CM} = \frac{m_A u_A + m_B u_B}{M}$$

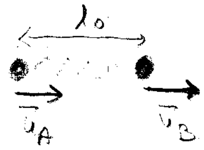
$$x(t) = l_0 + A \sin(\omega t + \varphi) \qquad \omega = \sqrt{\frac{k}{\mu}} \qquad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{\mu}{k}}$$

$$\begin{cases} x(0) = l_0 \\ \dot{x}(0) = u_B - u_A = u_{rel} \end{cases} \Rightarrow \begin{cases} A \sin\varphi = 0 \\ A\omega \cos\varphi = v_{rel} \end{cases} \Rightarrow$$

$$x(t) = l_0 + \frac{u_{rel}}{\omega} \sin \omega t$$

Seguiamo come evolve la situazione:

1) $t = 0$



2) $t = \frac{T}{4} = \frac{\pi}{2\omega}$ $\qquad \dot{x}(t) = \frac{v_{rel}}{\cancel{\omega}} \cancel{\omega} \cos \omega t = u_{rel} \cos \frac{\omega \pi}{2\omega} = 0$

$$\Rightarrow x_{min} = l_0 - \frac{u_{rel}}{\omega}$$

$$\begin{aligned} v_A &= v_{CM} - \frac{m_B}{M} \dot{x}(t) \\ v_B &= v_{CM} + \frac{m_A}{M} \dot{x}(t) \end{aligned} \Rightarrow v_A = v_B$$

3) $t = \frac{T}{2} = \frac{\pi}{\omega}$ $\qquad x(t) = l_0$ $\Rightarrow$ la pallina A si distacca

$$\dot{x}\left(\frac{T}{2}\right) = \frac{u_{rel}}{\omega} \omega \cos \pi = -u_{rel}$$

fine dell' urto!

$$\Rightarrow \quad v_B - v_A = u_A - u_B \quad !$$

$$v_A = v_{CM} + \frac{m_B}{M} u_{rel} = \frac{m_A u_A + m_B u_B + m_B u_B - m_B u_A}{M}$$

$$v_B = v_{CM} - \frac{m_A}{M} u_{rel} = \frac{m_A u_A + m_B u_B - m_A u_B + m_A u_A}{M}$$

$$\begin{cases} v_A = \dfrac{(m_A - m_B)\, u_A + 2 m_B u_B}{M} \\[2ex] v_B = \dfrac{2 m_A u_A + (m_B - m_A)\, u_B}{M} \end{cases}$$

Nota: se $K \to \infty$ $\quad \frac{T}{2} \to 0 \Rightarrow$ urto istantaneo

$$I_B = \int_0^{T/2} F_{el} \cdot dt = -k\,(x - \ell_0)\, dt$$

$$= -k \frac{u_{rel}}{\omega} \int_0^{T/2} \sin \omega t \, dt = +k \frac{u_{rel}}{\omega^2} \cos \omega t \Big|_0^{T/2}$$

$$= k \frac{u_{rel}}{\omega^2} (-1-1) = -2k \frac{u_{rel}}{\omega^2}$$

$$= -2 \cancel{k}\, u_{rel} \frac{\mu}{\cancel{k}}$$

$$= -2\, u_{rel}\, \mu$$

$I_B = -2 \dfrac{m_A m_B}{M} (u_B - u_A)$. Ovviamente $I_A = -I_B$ e $I_A + I_B = 0$

Del resto:

$$\Delta Q = m_A u_A + m_B u_B - m_A v_A - m_B v_B =$$

$$= \frac{1}{M} \Big(\cancel{m_A^2 u_A} + m_A m_B u_A + \cancel{m_B^2 u_B} + \cancel{m_A m_B u_B}$$

$$- \cancel{m_A^2 u_A} + \cancel{m_A m_B u_A} - \cancel{2 m_A m_B u_B}$$

$$- \cancel{2 m_A m_B u_A} - \cancel{m_B^2 u_B} + \cancel{m_A m_B u_B} \Big) = 0$$

ii) Con le conservazioni: non ci sono $\vec{F}_{ext} \Rightarrow \vec{Q}_{tot} = \text{cost}$

Forze conservative $\Rightarrow E = \text{cost}$

Nota $E_{pot}(t=0) = E_{pot}(T/2) = 0 \Rightarrow$ vicolo come sempre.