ANALYTICITY IN θ AND INFINITE VOLUME LIMIT OF THE TOPOLOGICAL SUSCEPTIBILITY IN SU(3) THEORY

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Bari, September 2004

$$\mathcal{L}_{QCD} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} + \sum_f \bar{\Psi}_f (i\not{D} - m_f)\Psi_f$$

- It is invariant under $\mathcal{P}$.

- Does QCD break $\mathcal{P}$ spontaneously?

- Vafa-Witten ('84) advanced arguments against such a possibility.

- Let us consider a $\mathcal{P}$-odd operator X,

$$\mathcal{P} X \mathcal{P}^{-1} = -X.$$

- To study whether X drives a $\mathcal{P}$-breaking, VW wrote the partition function in the Euclidean,

$$Z(\theta) \equiv \int \mathcal{D}\bar{\Psi}\mathcal{D}\Psi\mathcal{D}A^a_\mu \; e^{-S_E + i\theta\int d^4x_E X_E}$$

$$(\theta \in \mathbb{R})$$

and showed that always $Z(\theta) \leqslant Z(0)$ meaning that the free energy $E(\theta) = -\ln Z(\theta)$ develops a minimum at $\theta = 0$.

- This result triggered a debate about its implications.

  - The main objection being that $E(\theta)$ must be nonsingular at $\theta=0$ and in VW this question is left unanswered.

  - Actually there are arguments that show that in general $E(\theta)$ can be singular at $\theta=0$ (Azcoiti–Galante '99).

  - If the P-odd probe is the topological charge density

$$X = Q(x)$$

then it is possible to show (Asorey-Aguado '02, '04) that $Z(\theta)$ is finite throughout the whole $\mathbb{C}$ plane and arguments can be given against the appearance of dangerous Lee-Yang zeroes.

  - Our scope is to give an upper bound to the P-breaking order parameter $\langle Q \rangle$ by doing lattice simulations.

$$\left(Q \equiv \int d^4x \; Q(x) \right)$$

- Let us assume that single measurements of Q yield

$$Q = \alpha a^4 V + \eta$$

→ Q is an integer (in the $\overline{MS}$)

→ We shall work by putting $a=1$

→ $V = L^4$ (dimensionless)

→ η is some distribution that satisfies

$$\langle \eta \rangle = 0 \qquad \langle \eta^2 \rangle \neq 0$$

- The topological susceptibility

$$\chi = \frac{\langle Q^2 \rangle}{V} = |\alpha|^2 V + \frac{\langle \eta^2 \rangle}{V} \equiv |\alpha|^2 V + \chi_0$$

contains a term linear in V and this is what we want to study on the lattice.

- We made the study for pure Yang-Mills SU(3) theory.

- To calculate χ we make use of the well-tested "Pisa-technology"

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Maggiore
Panogopoulos
Vicari...
'90, '92, '93, '96, '97, ...

$$\chi_L \equiv \frac{\langle (Q_L^{(1)})^2 \rangle}{V}$$

$$\chi_L = a^4 \chi \cdot Z^2 + M$$

- Z relates the $\overline{MS}$ definition of Q with its lattice-regularized determination.

  We calculate it by imposing $Q = 1$ on a heated (heat-bath) 1-instanton configuration.

- M shows up as a result of the contact terms in χ_L. It contains mixings with opportune operators (perturbative tail, etc.)

  We evaluate it by demanding $\chi = \frac{n^2}{V}$ on a n-instanton background configuration.

- In order to make reliable our calculation we have to reduce the statistical errors as much as possible.

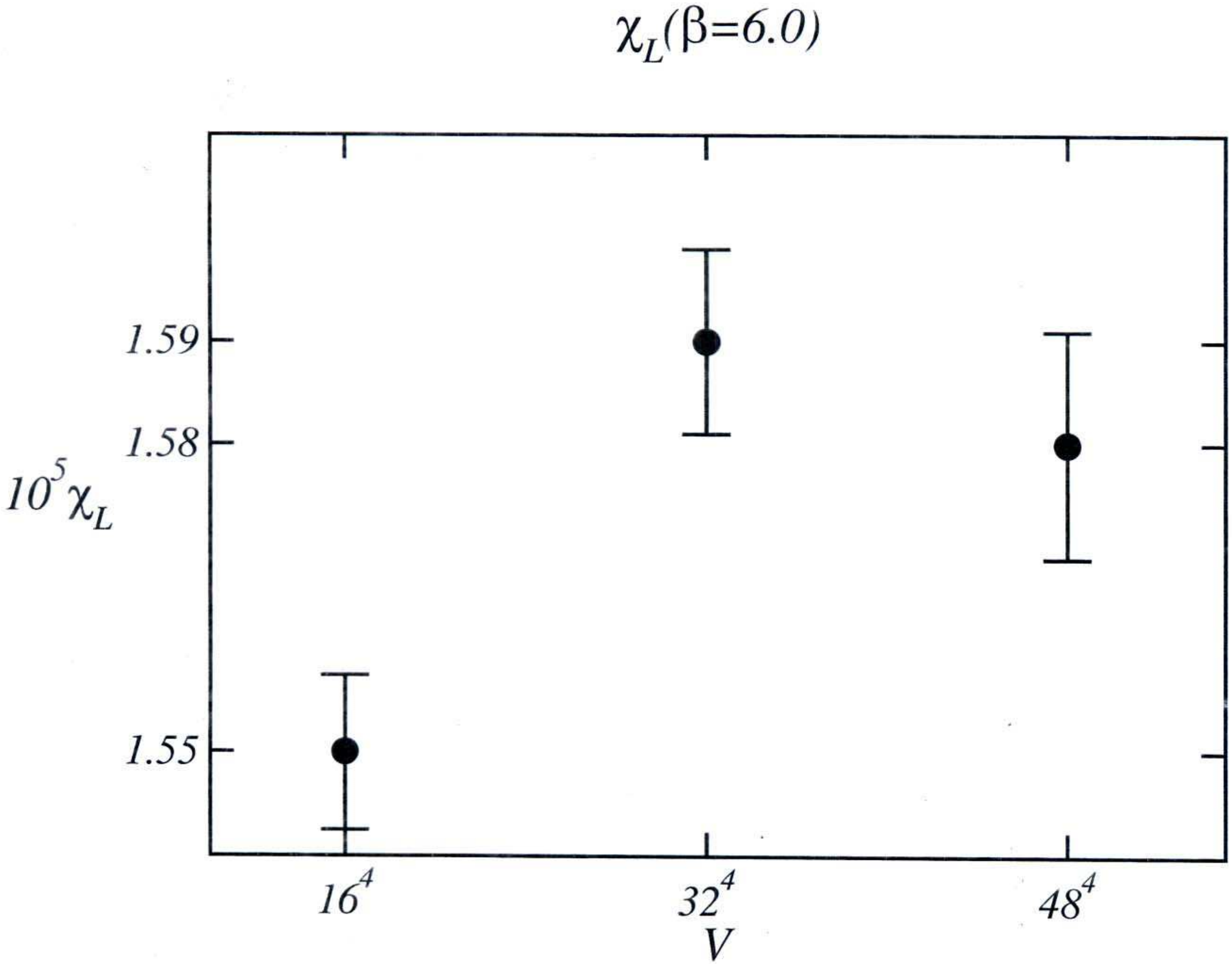
$L = 16^4$ stat = 100 000

$L = 32^4$ stat = 60 000

$L = 48^4$ stat = 50 000

- $\beta = 6.0$

- Data statistically independent (several HB steps among successive measurements).


$\chi_L(\beta=6.0)$
$10^5\chi_L$
1.59
1.58
1.55
16^4
32^4
48^4
V

- Data look stable (mainly for larger lattices).

- Before drawing any conclusion we also calculate Z and M as a function of L

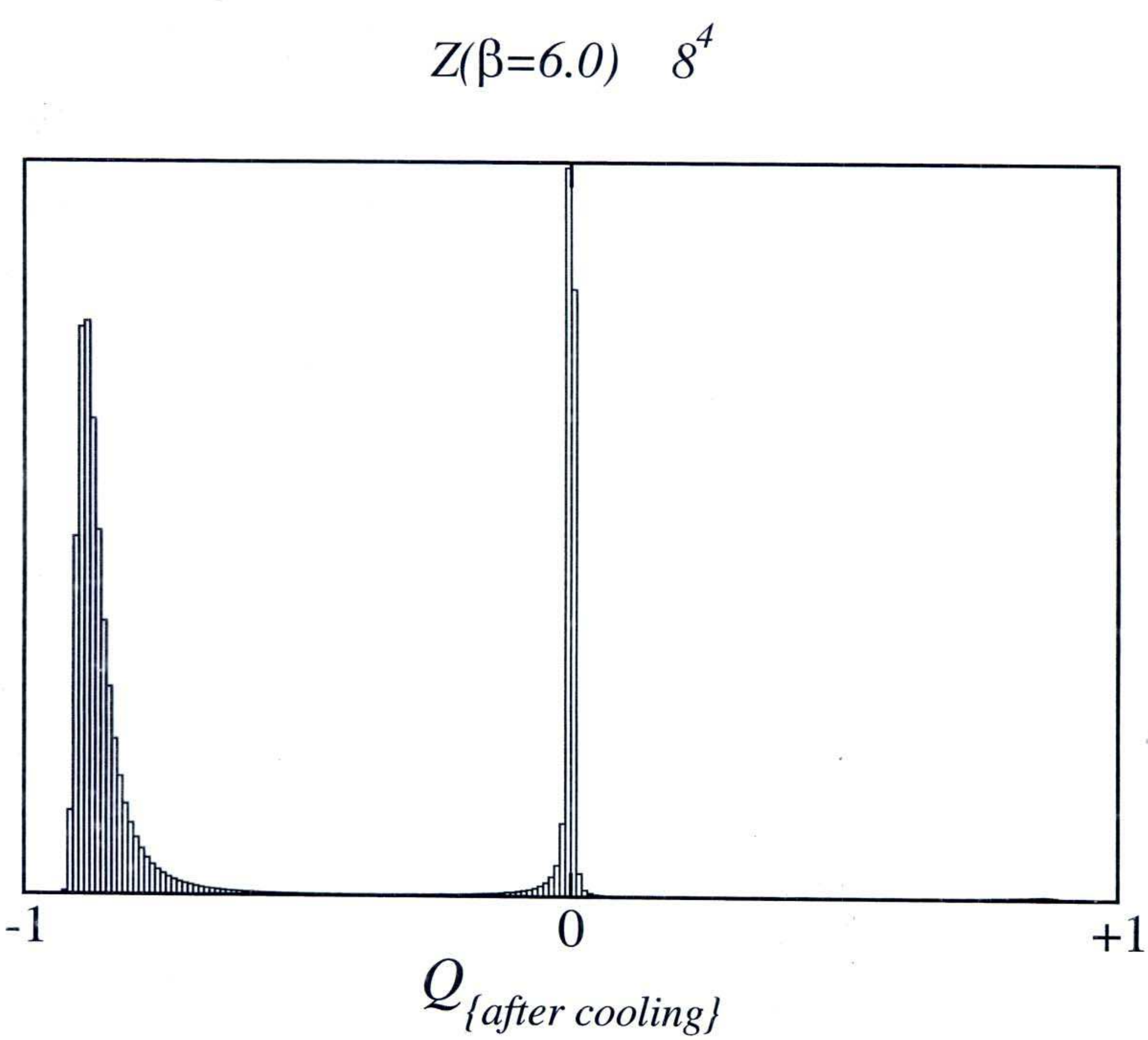
  (At least in P.T. we know that a mild L-dependence shows up)

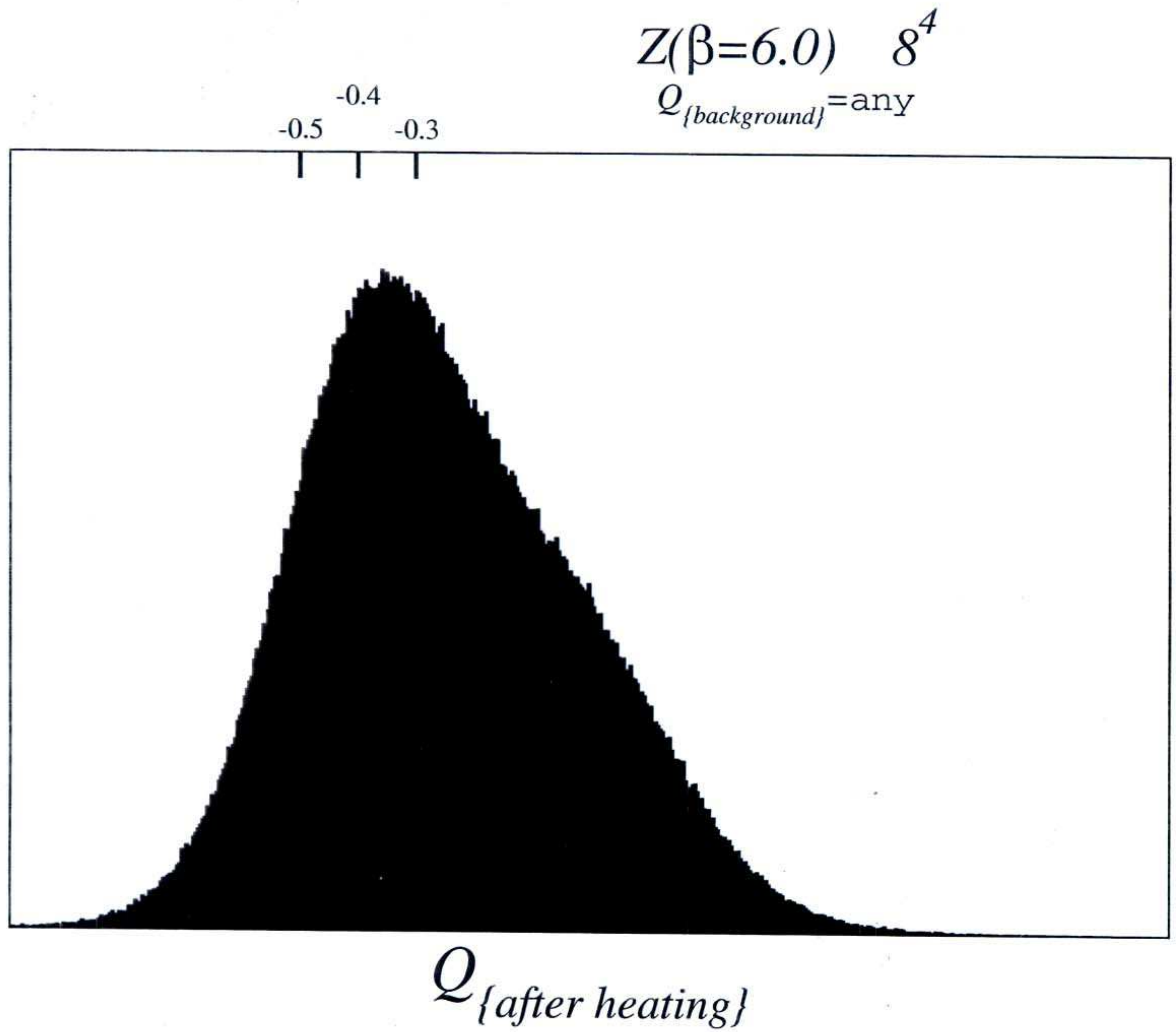
We calculate Z and M on 8^4 and 16^4 and extrapolate by the functional forms

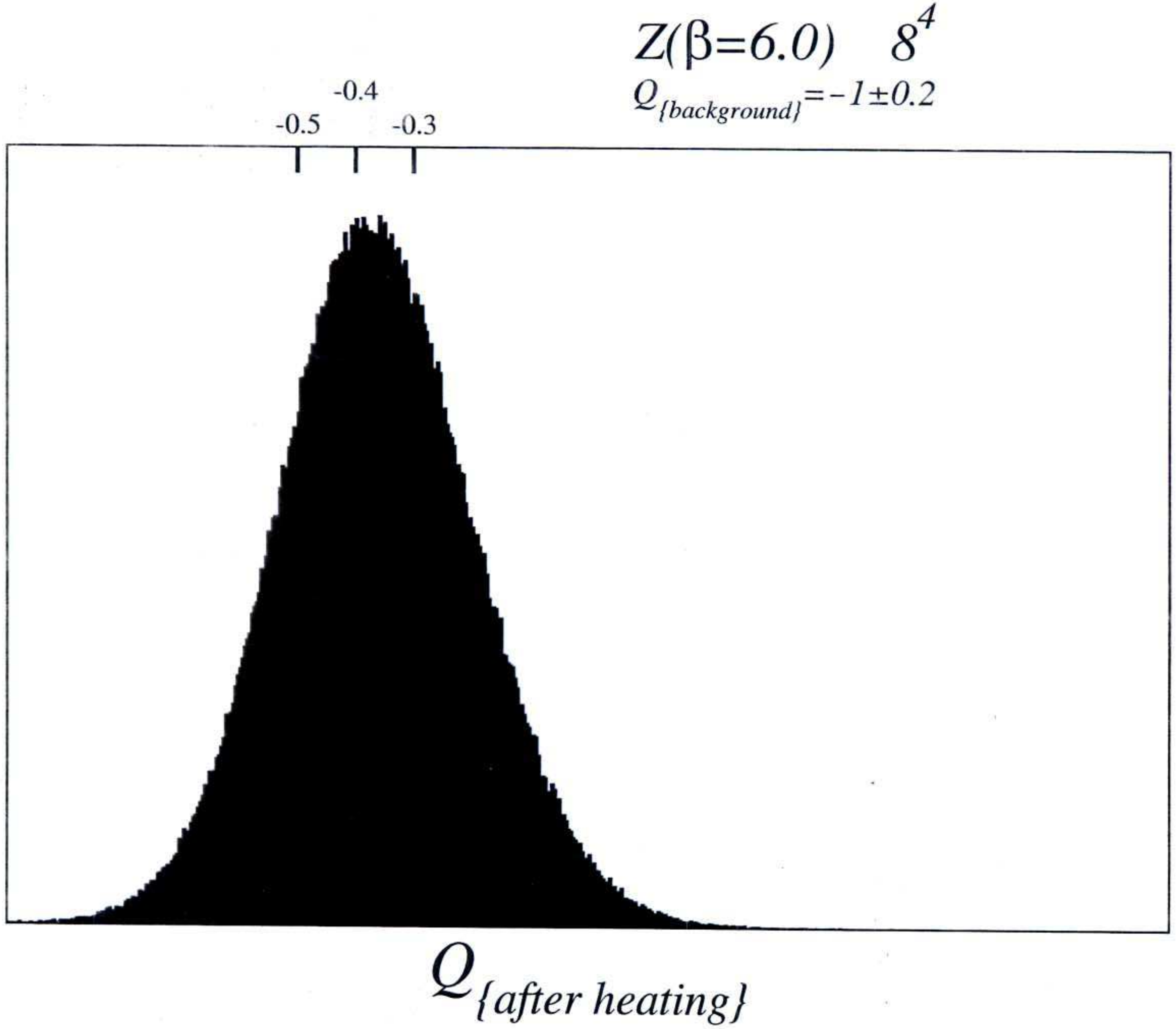
$$Z(L) = Z_0 + \frac{Z_1}{L}$$

$$M(L) = M_0 + \frac{M_1}{L}$$

- Because both calculations are based on the determination of χ or Q on a fixed-charge background, we have to be sure that the heat-bath updatings do not modify that background.


Z(β=6.0) 8^4
-1
0
+1
Q{after cooling}


Z(β=6.0) 8^4
Q_{background}=any
-0.5
-0.4
-0.3
Q_{after heating}


Z(β=6.0) 8^4
Q_{background}=-1±0.2
-0.5
-0.4
-0.3
Q_{after heating}

The results are

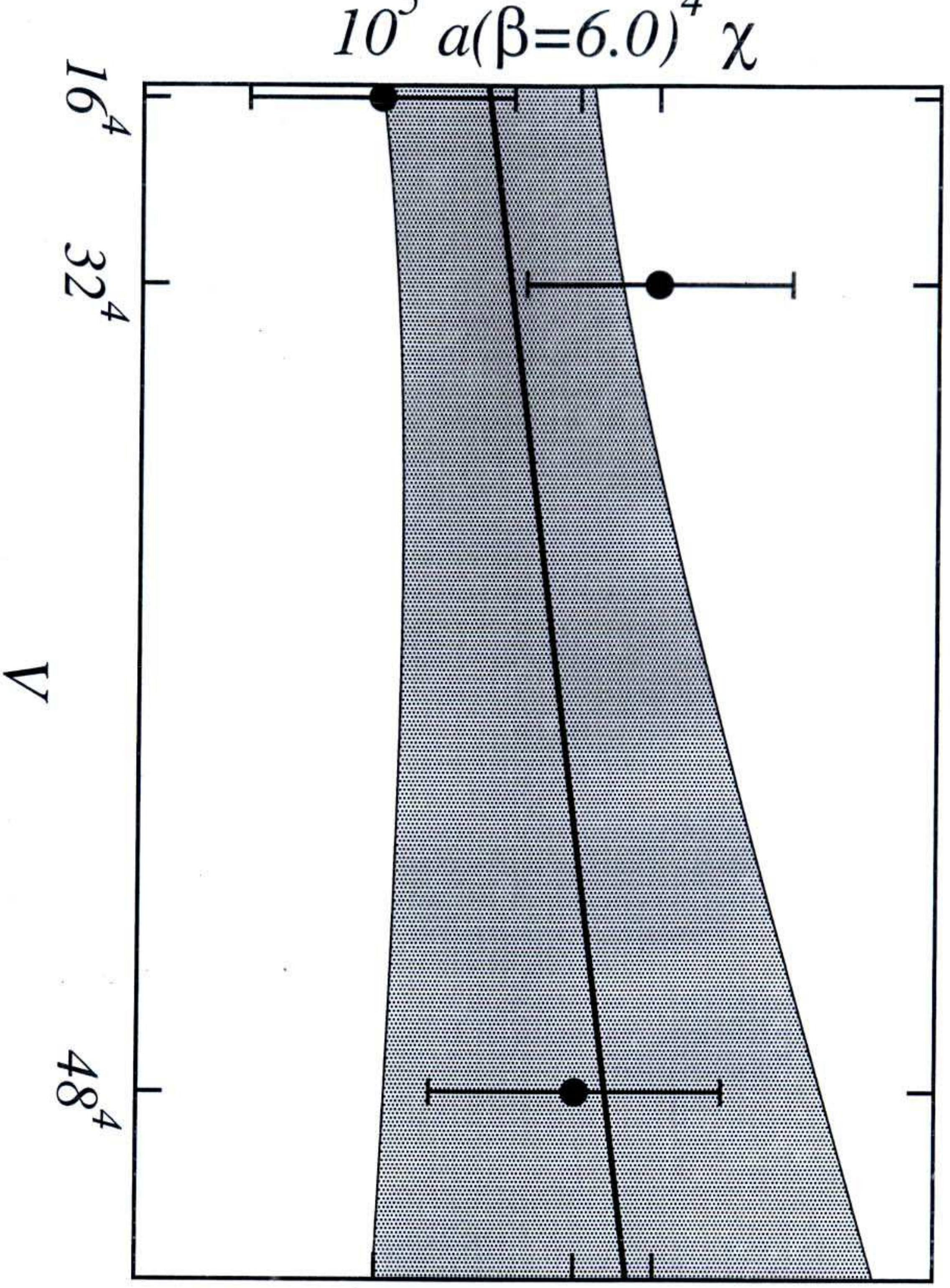
$$Z \begin{cases} 8^4 & 0.407(5) \\ 16^4 & 0.409\,(3) \end{cases}$$

$$M \begin{cases} 8^4 & 0.5830(3)\;\; 10^{-5} \\ 16^4 & 0.583\,(2)\;\; 10^{-5} \end{cases}$$

$$Z = 0.411\,(8) - \frac{0.03\,(9)}{L}$$

$$10^5 \cdot M = 0.583\,(4) + \frac{0.00\,(3)}{L}$$

Z & M are very steady



$10^5\, a(\beta=6.0)^4\, \chi$
16^4
32^4
48^4
V
5.78
5.93
5.99

- A best fit yields

$$|\alpha|^2 a^8 = 1.7\ (2.7) \quad 10^{-13}$$

Hence

$$|\alpha|\, a^4 < 6 \cdot 10^{-7}$$

Using data for a from Karsch et al. '96 we conclude

$$|\alpha| < \left(\frac{0.2}{fm}\right)^4$$

- As a check for our best fit, notice that at $\beta = 6$ $a = 0.1$ fm, therefore our limit means that there are not P-breaking generated instantons in volumes "as small as" 50^4 (the max size we utilized!)

CONCLUSIONS

- There is no trace of $\mathcal{P}$-breaking generated instantons in our simulations.

- An upper bound is

$$\left|\frac{\langle Q\rangle}{V}\right| < \left(\frac{1}{5\,\mathrm{fm}}\right)^4 \qquad \left(\frac{|\alpha|}{m_n^4} \lesssim 10^{-6}\right)$$

- As a by-product we give the number

$$\chi = \left(170(1)\ \mathrm{MeV}\right)^4$$