Superluminal speeds, an experiment and some theoretical considerations

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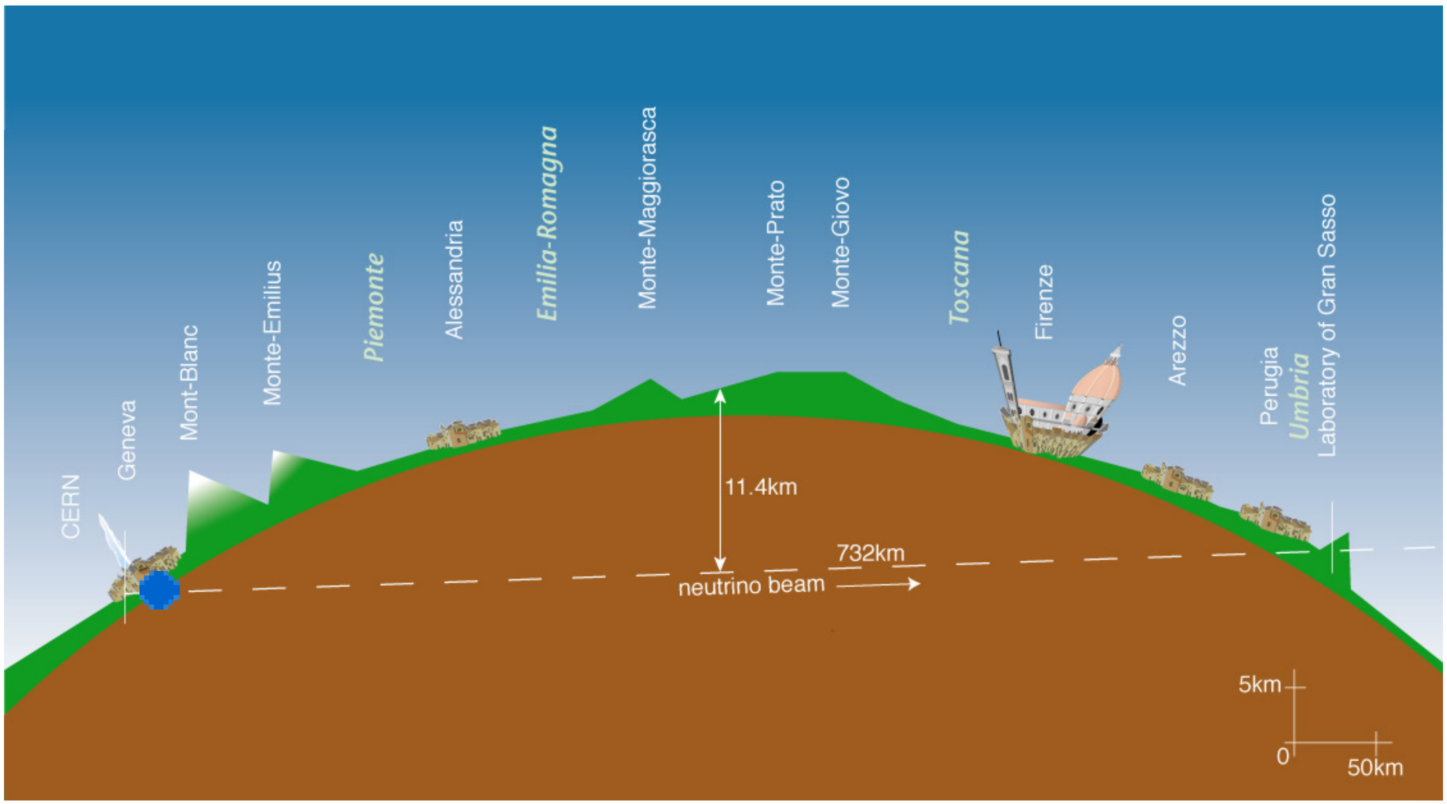
[Phys. Rev. D85 047501 (2012) or arXiv:1111.0805]

Málaga, June 2012

CNGS Experiment



CNGS Experiment

Gran Sasso setup

Cern production

p

↓

graphite

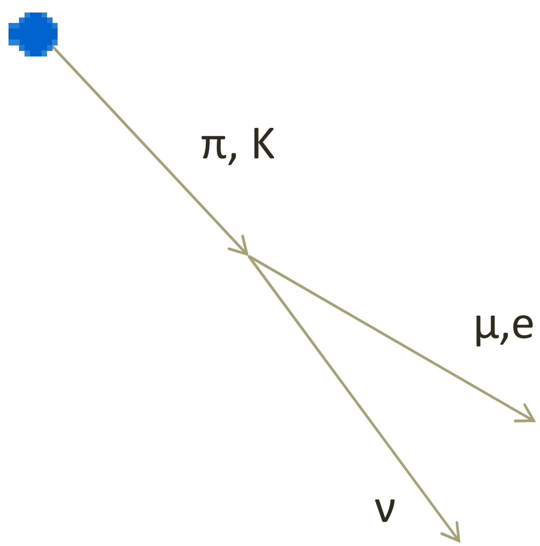
↓

π, K

↓

μ, ν

CNGS Experiment



- m(π)=140 MeV
- m(K)=494 MeV
- m(μ)=106 MeV
- m(e)=0.5 MeV
- m(ν)≈0

small transverse momentum in $\pi \rightarrow \mu\nu$

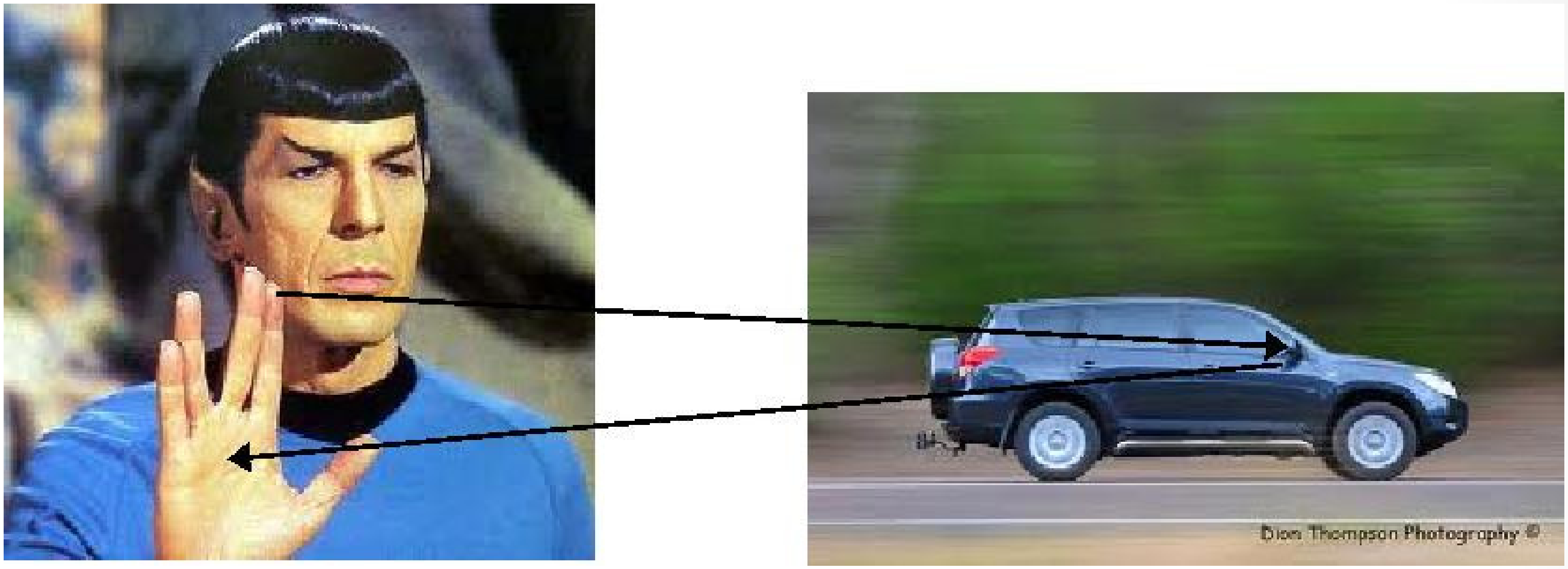
maximizes the number of collimated ν.

CNGS Experiment

- From 10^{19} initial protons at Cern, 10^{4} neutrinos events were recorded at Gran Sasso and only 10^{3} true candidates.
- Since an analogous experiment (MINOS at Fermilab in 2007) found an indication of possible superluminal neutrino speed, it was interesting to determine this speed also at CNGS.
- Dividing the total distance Cern-SPS to Gran Sasso facility by the time of flight of neutrinos, it came the astonishing result:

$$(v-c)/c=2.37 \pm 0.32 {}^{+0.34}_{-0.24}\ 10^{-5}$$

This result clashes with the basic principles of special relativity.



If a faster-than-light message is sent by an observer at rest towards another observer in motion and the latter replies back the message, the answer might arrive before the message was sent!!

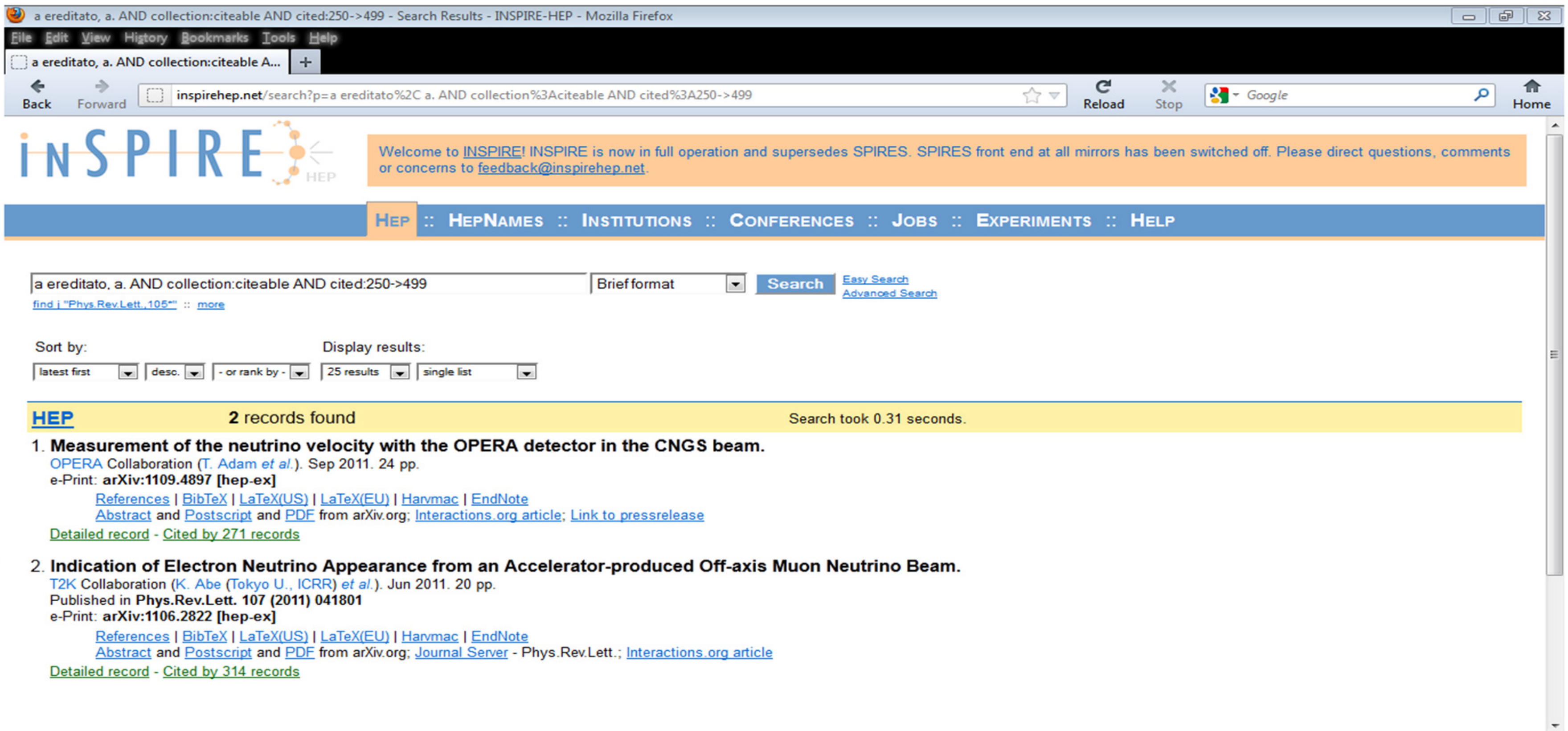
CNGS Experiment (end)

Ereditato making statements before discovering a misconnection among GPS cables.

By March 2012 ICARUS collaboration found a result compatible with c.

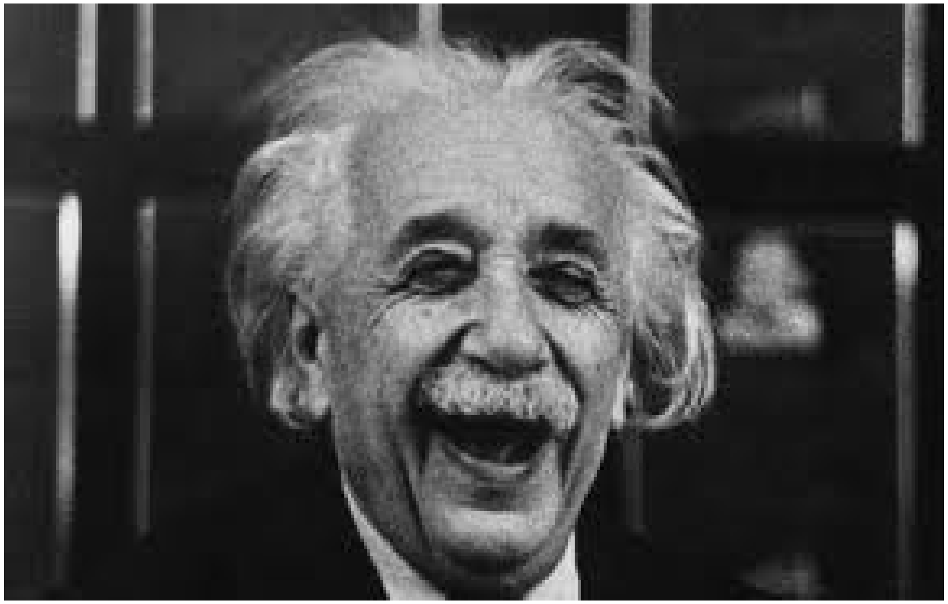
…but not all has been a waste of time…

- The surprising findings of OPERA team spurred a great deal of research in order to understand what was going on about neutrinos and relativity.



- We are going to see how the inclusion of gravity changes drastically all conclusions about superluminal speeds.
- The above argument demonstrating that superluminal signals would generate logical contradictions applies rigorously only in special relativity: the theory of Einstein without gravity.
- Moreover, what really can attain superluminal values are the mean velocities, not the instantaneous ones…
- …but most of the velocities usually measured are precisely mean (in particular the one described by the OPERA team).
- An exception are the velocities of celestial objects when they are determined by studying the Doppler effect on the spectrum of light emitted by the object.

A tour around relativity



Don't be afraid, just smile!!

A tour around relativity

The pivotal paradigm in relativity is the metric

$$g_{\mu\nu}$$

Greek indices (μ,ν,α,β,…) stand indistinctly for time or spatial coordinates.

Nought index (0) stands for the time coordinate.

Latin indices (i, j, k,…) indicate spatial coordinates.

ijk … run from 1 to 3.

μναβ,… run from 0 to 3.

A tour around relativity

$$\begin{pmatrix} g_{00} & \cdots & g_{03} \\ \vdots & \ddots & \vdots \\ g_{30} & \cdots & g_{33} \end{pmatrix}$$

$g_{\mu\nu}=g_{\nu\mu}$ is symmetric, as every metric!

Given infinitesimal increments $dt, d\vec{x}$, the quantity

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu}\, dx^{\mu} dx^{\nu}$$

is called squared *proper length*.

A tour around relativity

It is clear that, under coordinate transformations $x^{\mu} \rightarrow x'^{\mu}$, dx^{μ} goes over to

$$dx'^{\mu} = \frac{\partial x'^{\mu}}{\partial x^{\alpha}}\, dx^{\alpha}$$

Since, mathematically, a metric is a tensor, it transforms in such a way as to leave ds invariant.

THEREFORE, UNDER ANY COORDINATE TRANSFORMATION s REMAINS UNVARIED.

A tour around relativity

Given an energy-momentum tensor describing the matter contents, the following expression

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^2} T_{\mu\nu}}$$

provides a non-linear partial differential equation of second order for the metric. $R_{\mu\nu}$ is the Ricci tensor and R the scalar of curvature.

$T_{\mu\nu}$ is the matter tensor. For instance, for a certain density of matter ρ, $T_{00} = \rho$ and all the other $T_{\mu\nu}$ zero.

We shall not solve this equation, but show two physically interesting solutions…

A tour around relativity

NO MATTER: Minkowski metric

$$ds^2 = d(ct)^2 - d\vec{x}^2$$

SPHERICAL MASS DISTRIBUTION: Schwarzschild metric

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) d(ct)^2 - \frac{dr^2}{1 - \frac{2GM}{c^2 r}} - r^2 d\theta^2 - r^2 \sin\theta^2 d\varphi^2$$

A tour around relativity

A VERY IMPORTANT ADVICE

Coordinates are mere labels, like street numbers.

THUS:

Distances or time intervals cannot be calculated by just coordinate differences (like in usual Euclidean space with Cartesian coordinates).

A tour around relativity

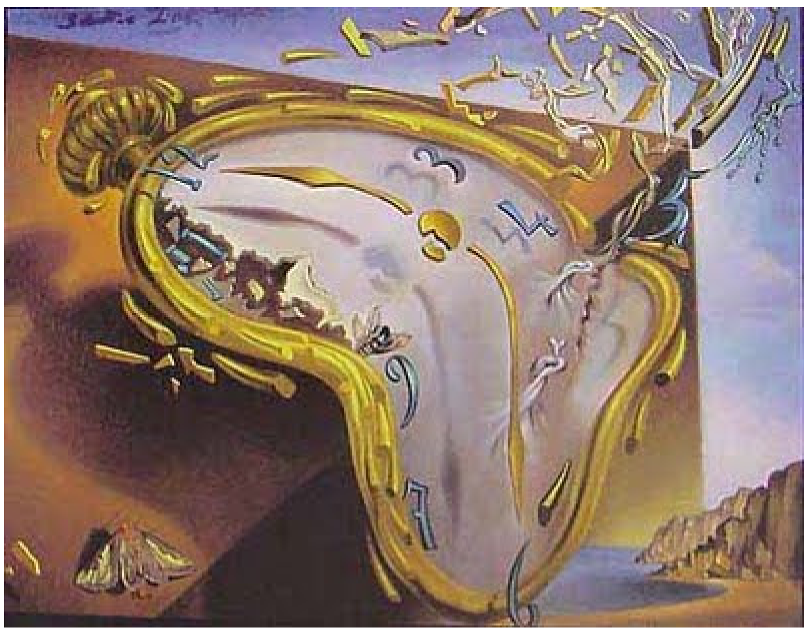
What is ds (or s)?

It depends!

Much as the spatial Euclidean metric serves for calculating distances in R^3, s (or ds) allows to determine both time intervals and spatial distances.

A tour around relativity

TIME INTERVALS



The time displayed by a clock at rest at the spatial position P (coordinates $\vec{x}_P$) is s/c.

It is presumed that the 0-coordinate (which is $x^0 \equiv ct$) changes but the spatial coordinates remain fixed and equal to $\vec{x}_P$.

A tour around relativity

The case of the Minkowski metric is very simple and also well-known: the ticking of the clock time equals the successive values taken by the time coordinate ct.

However, the case of the Schwarzschild metric suggests that clocks at different heights tick differently. Indeed, given a unique coordinate time interval, Δt, clocks at radial coordinates r_P and $r_Q$indicate

CLOCK TIME AT $\boldsymbol{r_P} = \frac{s}{c} = \sqrt{1 - \frac{2GM}{c^2 r_P}}\, t$

CLOCK TIME AT $\boldsymbol{r_Q} = \frac{s}{c} = \sqrt{1 - \frac{2GM}{c^2 r_Q}}\, t$

Therefore the ratio of the two times is not 1…

A tour around relativity

Robert Pound (picture) and
Glen Rebka demonstrated
experimentally the correctness
of the above statement.

In 1959 they compared atomic clocks
separated by a height of 22.5 meters,
(resorting to the recently discovered
Mösbauer effect). The agreement
was excellent.

A tour around relativity

SPATIAL DISTANCES

Spatial distances
are the value of $\sqrt{-s^2}$
at fixed time coordinate.

A tour around relativity

Therefore the true distance between r_P and r_Q in a Schwarzschild metric (what one would measure by counting how many rulers can be laid in order to exactly cover the separation from r_P to r_Q) is given by

TRUE DISTANCE = $\int \sqrt{-ds^2} = \int_{r_P}^{r_Q} \frac{dr}{\sqrt{1-\frac{2GM}{c^2 r}}}$

NOTE: The true distance is **NOT** $r_Q - r_P$ because coordinates are just labels!

A tour around relativity

$$\textbf{TRUE DISTANCE} \approx r_Q - r_P + \frac{GM}{c^2} \log \frac{r_Q}{r_P}$$

For the Earth, this correction is extremely small:

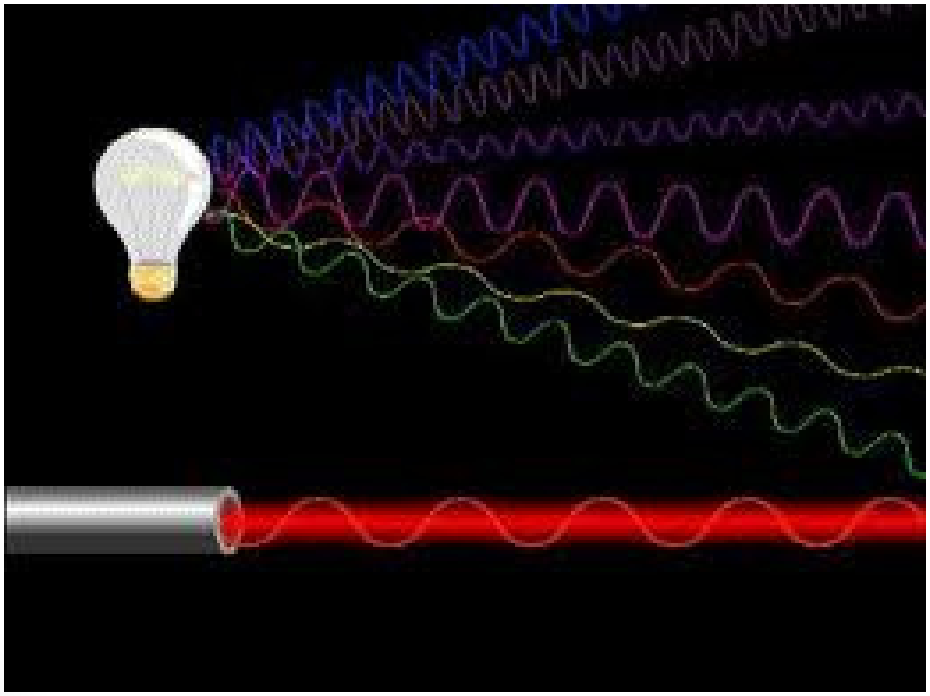
$$\frac{GM}{c^2} \approx 4\ \text{mm}.$$

For r_P = Earth radius and $r_Q = r_P + \delta$ $(\delta \ll r_P)$,

$$\log \frac{r_Q}{r_P} \approx \frac{\delta}{r_P} \ll 1.$$

A tour around relativity

If both spatial and temporal coordinates are left to vary, we can follow the trajectory of moving particles.



An interesting case is that of light. In special relativity

$$\left\|\frac{d\vec{x}}{dt}\right\| = c$$

Hence, for light rays $ds = 0$ always.

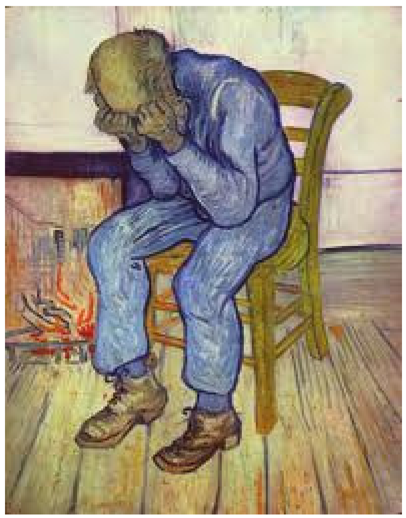
A tour around relativity

This is true for special relativity, that is, in absence of gravity.

WHAT ABOUT GRAVITY?

Does light travel at **299,792,458** meters/second also in gravity?

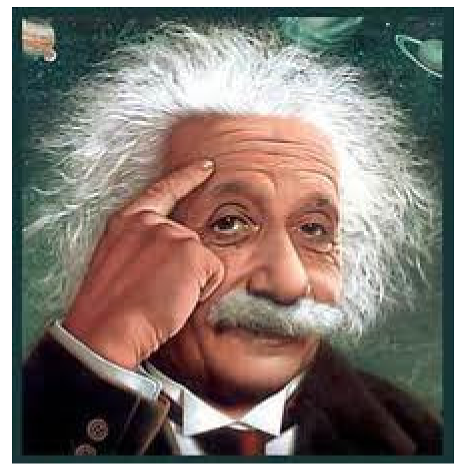
A tour around relativity



WHAT TO DO?

A tour around relativity

PRINCIPLE OF EQUIVALENCE



A tour around relativity

The **PRINCIPLE OF EQUIVALENCE** states:

If you are small enough and you are falling down, you cannot feel gravity.

A tour around relativity

A tour around relativity

More mathematically: at any point in spacetime a coordinate transformation can be found such that in a close neighbourhood of this point the metric in the new coordinates is Minkowski.

Since in these coordinates (and close to the chosen point) everything behaves as in special relativity, the speed of light is surely c and $ds = 0$.

Ergo, being s coordinate invariant, it remains constant in the original (not motivated by the principle of equivalence) coordinates.

Hence light rays go at c LOCALLY and $ds = 0$.

CONGRATULATIONS!!

WE HAVE NOW GOT ALL NECESSARY INGREDIENTS.

Let us consider a light ray following the spatial trajectory

$$x^1(\lambda), x^2(\lambda), x^3(\lambda)$$

from point P to point Q where parameter λ varies from λ_P at P to λ_Q at Q.

Our goal is to calculate the MEAN VELOCITY of the light ray.

The total travelled distance is

$$\Delta L = \int_{\lambda_P}^{\lambda_Q} d\lambda \sqrt{-\sum_{i,j} g_{ij} \dot{x}^i \dot{x}^j}$$

where overdots denote λ derivatives.

As clocks run differently in different spatial positions, we have to clearly specify WHERE the observer lies.

We choose to place him at the end of the trajectory, Q, although this detail is largely immaterial.

So, according to what we have just learnt,
FIRSTLY it should be obtained the coordinate time interval Δt needed by the light ray to travel from P to Q and
SECONDLY the time really displayed by the observer's clock at Q will be determined.

To find Δt we resort to the vanishing property of s in light.

Specifically:

$$\Delta t = \frac{1}{c}\int_{\lambda_P}^{\lambda_Q} d\lambda \sqrt{-\frac{1}{g_{00}}\sum_{i,j} g_{ij}\dot{x}^i\,\dot{x}^j}$$

Consequently,

$$\Delta \tau = \frac{\sqrt{g_{00}(Q)}}{c}\int_{\lambda_P}^{\lambda_Q} d\lambda \sqrt{-\frac{1}{g_{00}}\sum_{i,j} g_{ij}\dot{x}^i\,\dot{x}^j}$$

The ratio $\frac{\Delta L}{\Delta\tau}$ gives the desired MEAN VELOCITY.

The mean value theorem enables us to simplify the expression.

There is indeed an intermediate $\lambda_\xi \in [\lambda_P, \lambda_Q]$ for which

$$\int_{\lambda_P}^{\lambda_Q} d\lambda \sqrt{-\frac{1}{g_{00}}\sum_{i,j} g_{ij}\dot{x}^i\,\dot{x}^j} = \frac{1}{\sqrt{g_{00}(\lambda_\xi)}}\int_{\lambda_P}^{\lambda_Q} d\lambda \sqrt{-\sum_{i,j} g_{ij}\dot{x}^i\,\dot{x}^j}$$

Finally,

$$\boxed{\frac{\Delta L}{\Delta \tau} = \sqrt{\frac{g_{00}(\lambda_\xi)}{g_{00}(\lambda_Q)}}\,c}$$

The surprise is that this ratio is barely equal to c.

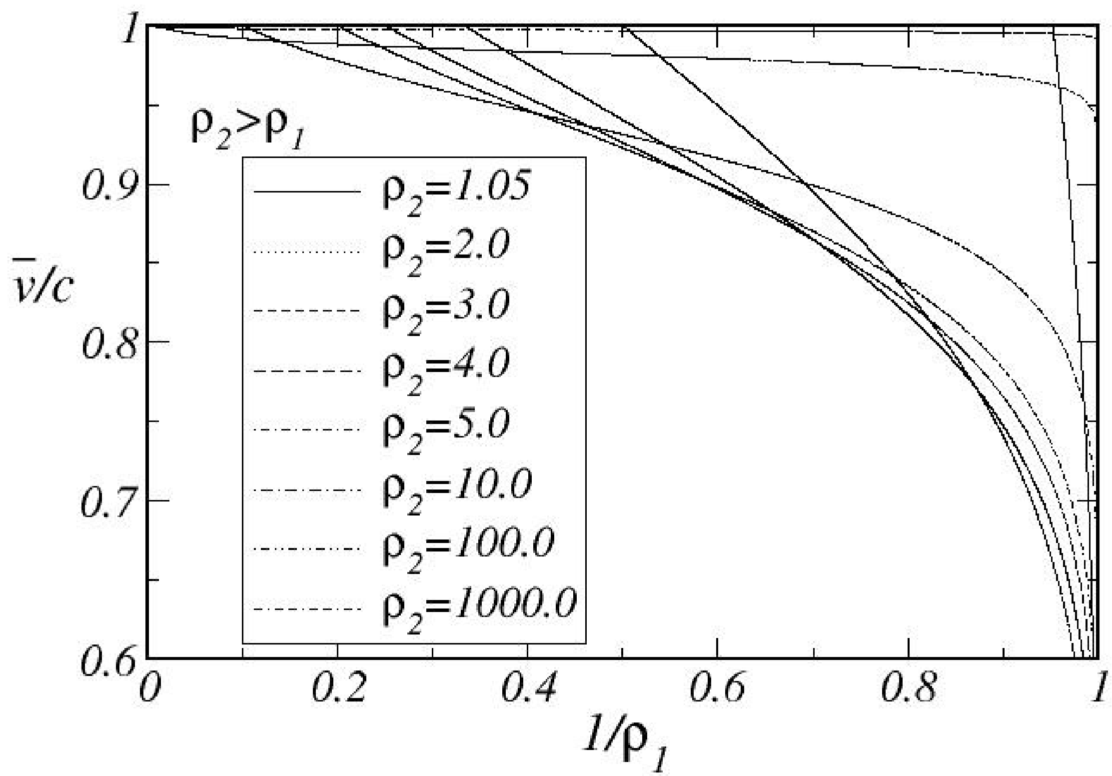
In summary, mean velocities need not be c.

But instantaneous velocities ARE always c,
(as prescribed by the PRINCIPLE OF EQUIVALENCE).

Indeed, if we make points P and Q collapse, then also the intermediate point labelled with λ_ξ tends to Q and so the above ratio of radicands tends to 1.

$$\lim_{P \to Q} \frac{\Delta L}{\Delta \tau} = \lim_{\lambda_\xi \to \lambda_Q} \sqrt{\frac{g_{00}(\lambda_\xi)}{g_{00}(\lambda_Q)}}\, c = c$$

The Schwarzschild metric offers an analytically calculable instance of the above effect:



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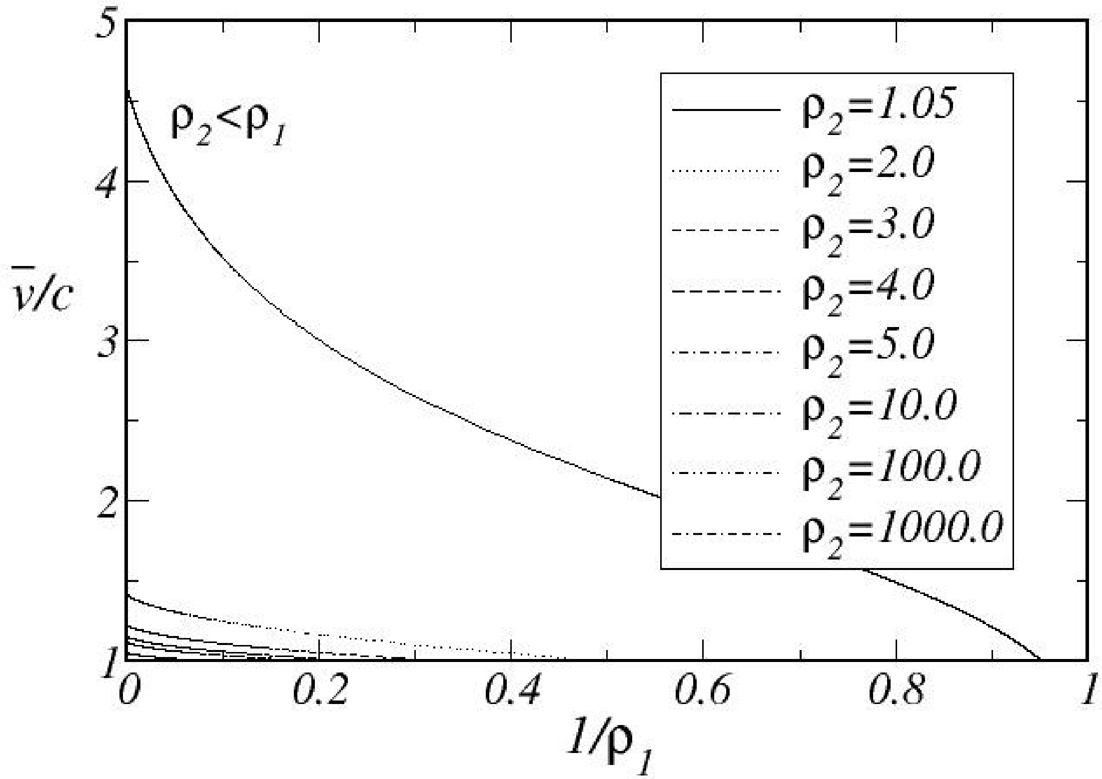


The terrestrial gravity is too weak.
Effects like the one described in this seminar are completely negligible.

In particular, the excess velocity claimed by the OPERA team is almost three orders of magnitude larger than our prediction.

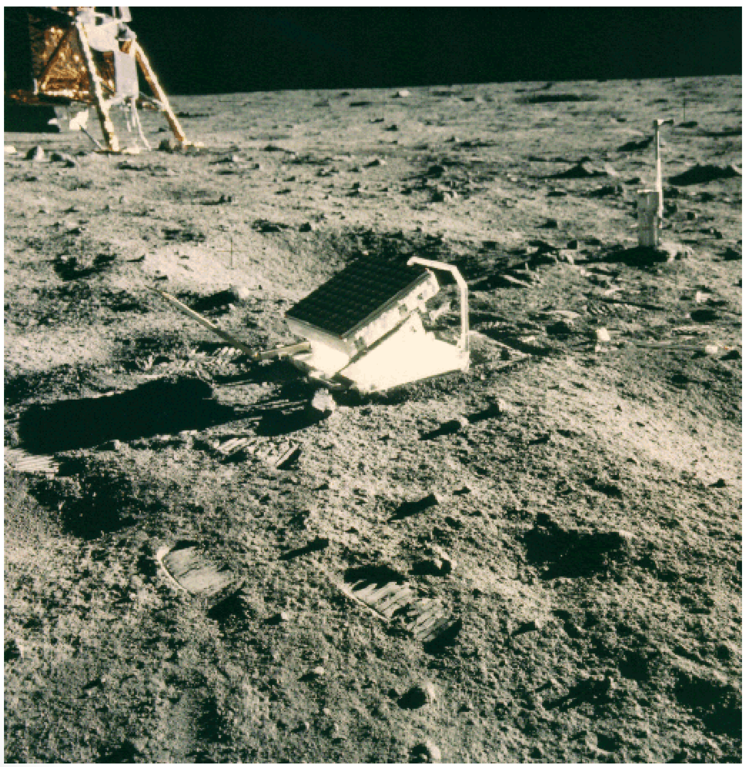
There is, though, an old experiment that necessarily has detected such an effect: the LUNAR LASER RANGING

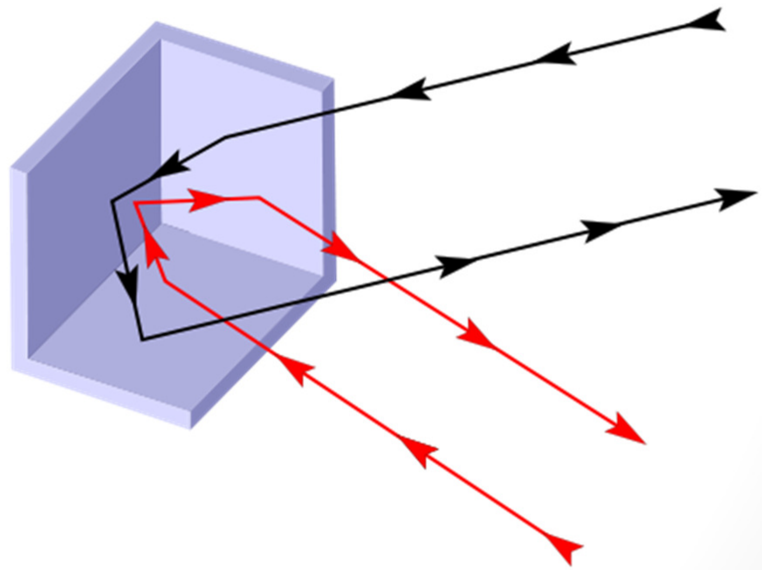
It consists in sending a laser pulse to the Moon and observing the reflected light. The (round trip time)/2 multiplied by c yields the Earth-Moon distance…

BUT THE SPEED OF THE SIGNAL IS NOT c !

Lunar Laser Ranging

Retroreflectors:





Lunar Laser Ranging

If the effect described here is taken into account, the correction turns out to be about 53 cm., well beyond the stipulated precision of the experiment (about 1-2 cm.).

Nevertheless, the LLR collaboration recorded the coordinates of the Moon and other planets and satellites (in a certain solar system coordinate set). In this way, they implicitly included the effects studied here.

Conclusions

- Care must be taken in defining a velocity in the context of general relativity.
- Local definitions comply with the basic tenets of the theory: they yield exactly c but…
- non-local definitions (like the above-analysed MEAN VELOCITY) do not necessarily observe this rule.
- Relativity may still teach us many surprising aspects of Nature…