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Topology, Chiral and Polyakov Loop Transitions at Finite Density Two-colour QCD

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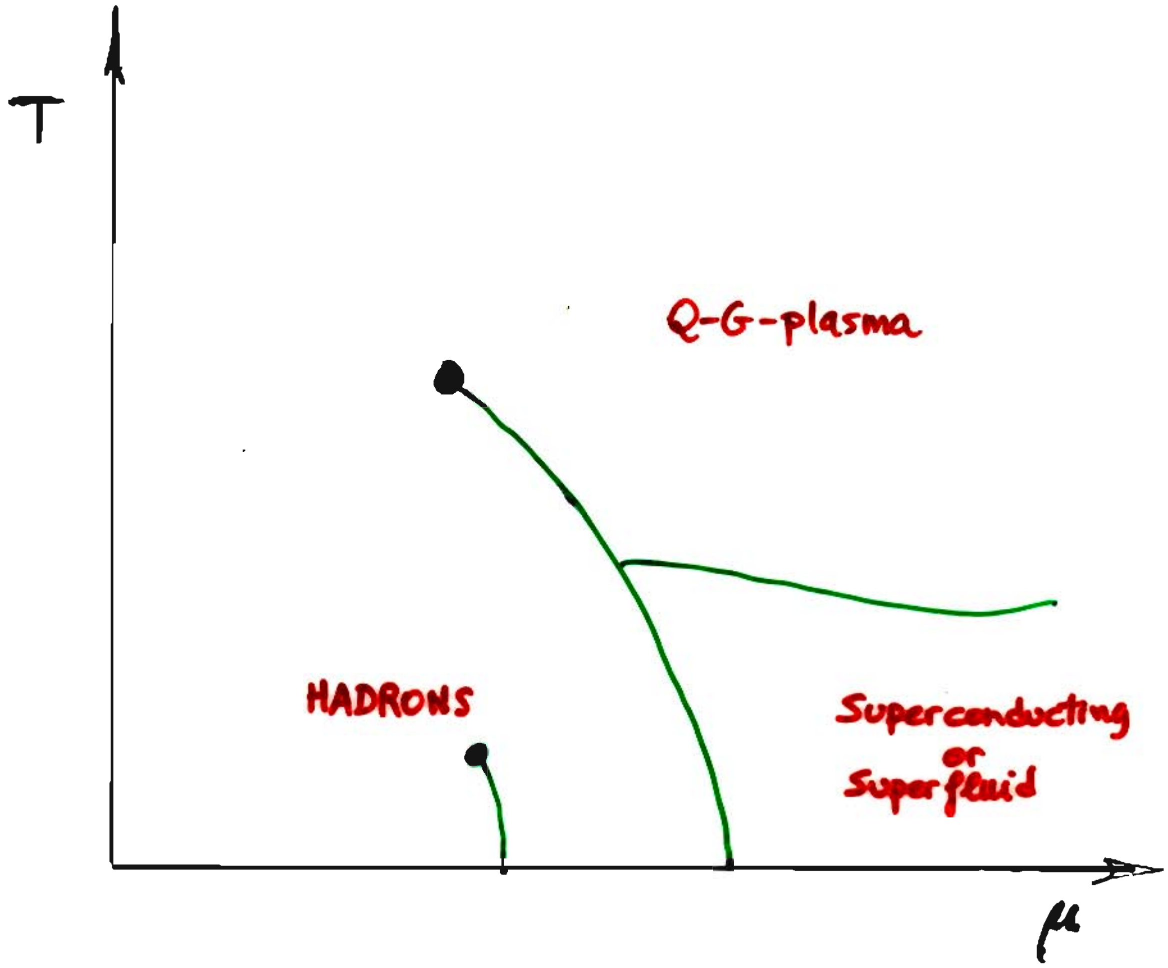
Tucson, July 2006



- We want to study
  a) the fate of the topological susceptibility
  b) the concomitancy of all transitions.

- We shall simulate SU(2) to avoid the "sign-problem".

- The study of QCD at high T and ρ_B opens the scenario of new states of matter.

- They are characterized by several symmetries and order parameters:

  - $\langle \bar{\psi}\psi \rangle$ signals breaking of chiral symmetry in massless theory.

  - Polyakov loop (or Wilson line) indicates the confinement-deconfinement transition in pure gauge theory.

  - Several observables (like particle susceptibilities or the topological susceptibility) can be used to detect an effective restoration of the $U_A(1)$ symmetry.

  - $\langle \psi^T C \psi \rangle$ condensates triggers color superconductivity.

- Suscettività Topologica χ

$$\chi = \int d^4x \quad \langle 0 | T\{Q(x)\, Q(0)\} | 0 \rangle$$

$$Q(x) = \frac{g^2}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$

$$\partial_\mu J^5_\mu = -2N_f\, Q(x)$$

$$J^5_\mu = \sum_{i=1}^{N_f} \bar{\psi}_i \gamma_\mu \gamma_5 \psi_i$$

- In teoria pura di gauge $\chi_{quenched}$

$$m^2_{\eta'} \approx \frac{2N_f}{F_\pi^2} \chi_{quenched}$$

$$\chi_{quenched} \approx (180\ \mathrm{MeV})^4$$

- In QCD $\left(\frac{m}{\Lambda_{QCD}} \longrightarrow 0\right)$

$$\chi \approx \frac{1}{N_f}\, m \langle \bar{\psi}\psi \rangle$$

- Not all conclusions derived from SU(2) are immediately applicable to SU(3).

A survey of analogies & differences can be found in

Hands, Kogut, Lombardo, Morrison hep-9902034
Kogut, Stephanov, Toublan, Verbaarschot, Zhitnitsky hep-ph/0001171

The main issues are:

- In SU(2) baryons and mesons belong to the same representation due to the equivalence between 2 and $\bar{2}$
- In SU(2) a $\langle\psi\psi\rangle$ condensate can be colour-blind and a superfluid phase can appear.

  In SU(3) such a condensate carries colour and the corresponding phase becomes superconducting, much as it happens in the BCS-model.
- Gluon observables keep similarities among SU(2) and SU(3). Therefore instanton physics (and topology) are very similar in both theories.

- It is well-known that χ undergoes an abrupt drop at the critical temperature.

It has been shown to be true for

B.A, D'Elia, Di Giacomo $\left\{ \begin{array}{ll} \text{pure SU(3) theory} & \text{hep-lat/9605013} \\ \text{QCD } N_f = 4, 2 & \text{hep-lat/0004020} \end{array} \right.$

↪ see Figure

- Transitions are concomitant (Polyakov, $\langle\bar{\psi}\psi\rangle$, χ)

We make use of two graphical representations of the concomitancy of transitions:

- $\frac{dO}{dT}$ versus T (or a parameter β related to T)

↪ see Figure

- $\bar{O}_1$ vs $\bar{O}_2$ where

$$\bar{O} \equiv \frac{O - O_{min}}{O_{max} - O_{min}} \in [0,1] \quad \text{if} \nearrow$$

$$\bar{O} \equiv \frac{O_{max} - O}{O_{max} - O_{min}} \in [0,1] \quad \text{if} \searrow$$

↪ See Figure

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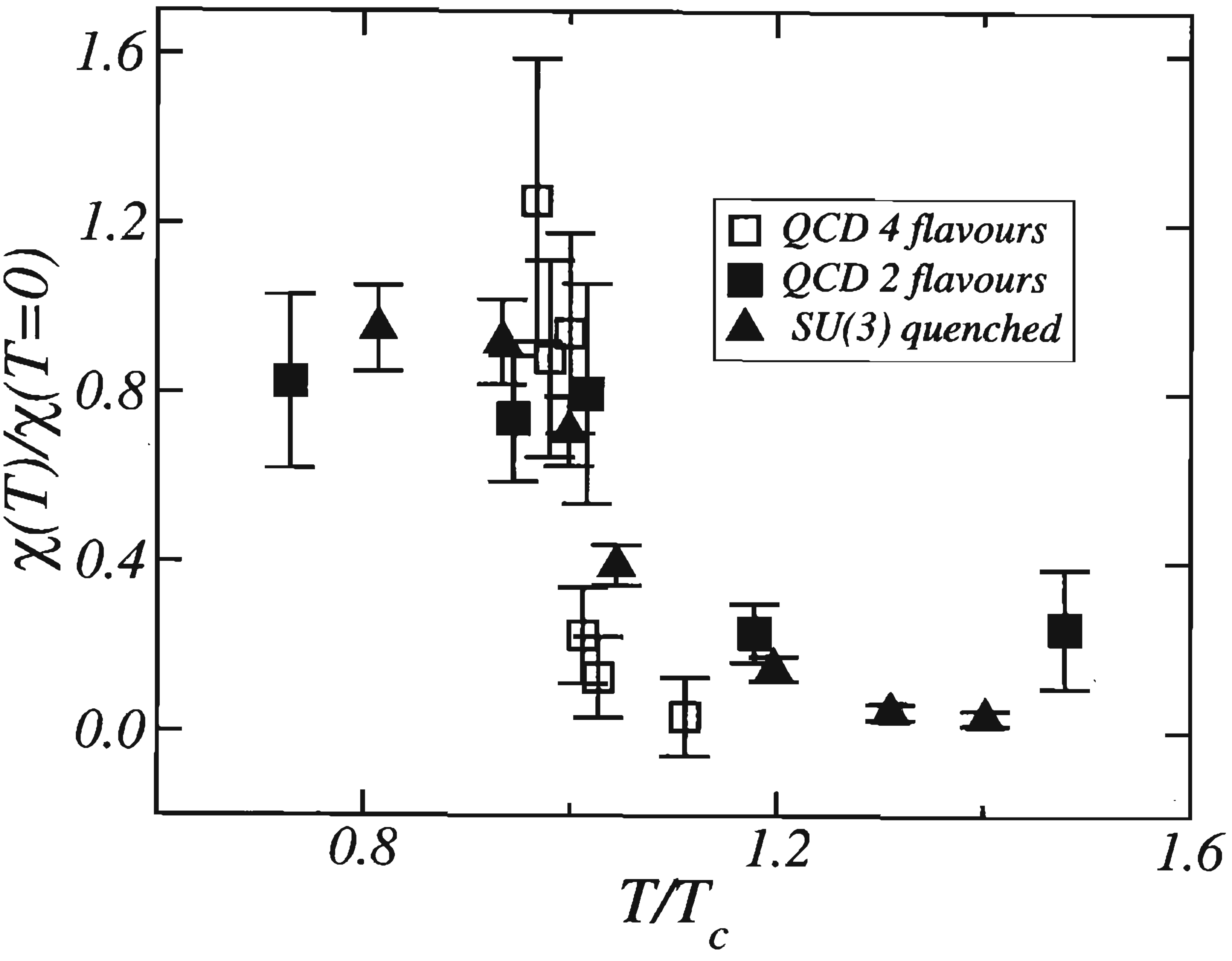
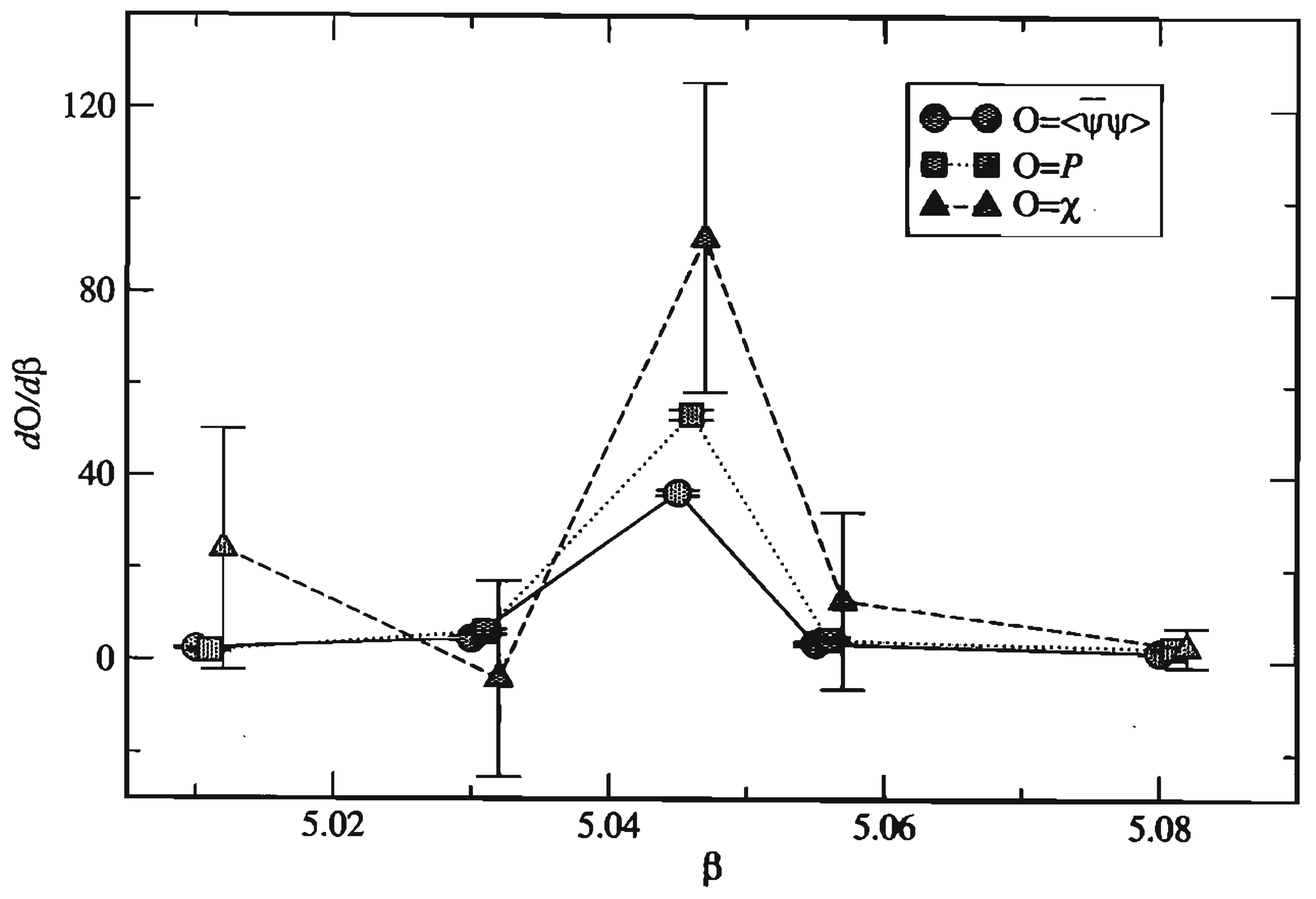


QCD 4 flavours
QCD 2 flavours
SU(3) quenched
χ(T)/χ(T=0)
T/T_c
0.0
0.4
0.8
1.2
1.6
0.8
1.2
1.6

8/8

(8)

SU(3) $N_f=4$



- Let us describe the work done in the case of SU(2) unquenched at finite μ_B and T.

  - 8 flavours of staggered quarks (Hybrid Molecular Dynamics).
  - $am = 0.07$, $\beta = \frac{4}{g^2_{latt}} = 1.5$
  - $L_s^3 \times L_t = 14^3 \times 6$
  - We measured
    - Polyakov loop
    - $\langle \bar{\psi}\psi \rangle$
    - ρ_B
    - $\mathrm{Tr}\,\Box$
    - Q_L

unquenched & quenched

- To measure the topology

[Pisa group: B.A. Di Giacomo D'Elia et. al.]

- 1^{st}: introducing $Q_L(x)$ such that

$$Q_L(x) \xrightarrow[a \to 0]{} a^4 \frac{g^2_{latt}}{32\pi^2} \epsilon_{\xi\nu\gamma\sigma} F^a_{\xi\nu}(x) F^a_{\gamma\sigma}(x)$$

- 2^{nd}: Calculate the total topological charge on the lattice

$$Q_L \equiv \sum_x Q_L(x)$$

- 3^{rd}: Extract

$$\chi_L \equiv \frac{\langle Q_L^2 \rangle}{V}$$

$$V = L_s^3 \times L_t$$

- The problem now is that χ_L is related to the physical value χ in a complicated manner:

$$\chi_L = a^4 Z^2 \chi + M$$

B.A. Campostrini, Di Giacomo, Panagopoulos, Vicari,
Nucl. Phys. B329 683
Phys. Lett. B212 206
Phys. Lett. B268 241

- There are definitions of Q_L which yield

$$Z = 1 , \qquad M = 0$$

Neuberger hep-lat/9707022

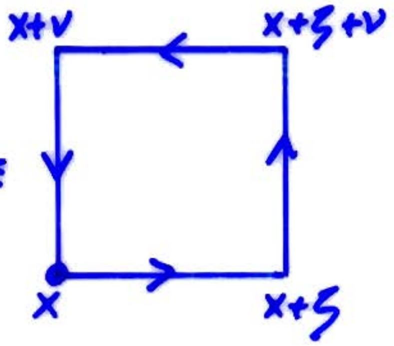
Lüscher hep-lat/9802011

Giusti, Rossi, Testa hep-lat/0402027

- We have used

$$Q_L(x) = -\frac{1}{2^9 \pi^2} \sum_{\xi,\nu,\rho,\sigma=\pm 1}^{\pm 4} \tilde{\epsilon}_{\xi\nu\rho\sigma} \, \mathrm{Tr}\left\{ \Box_{\xi\nu}(x) \, \Box_{\rho\sigma}(x) \right\}$$

Di Vecchia, Fabricius, Rossi, Veneziano
Nucl. Phys. B192 392
Phys. Lett. B108 323

$\Box_{\xi\nu}(x) \equiv$ 

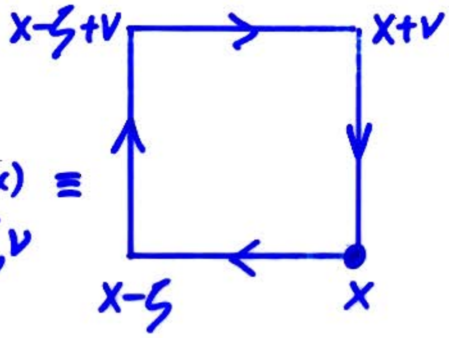
$$\begin{cases} \tilde{\epsilon}_{1234} = +1 \\ \tilde{\epsilon}_{\tau(\xi\nu\rho\sigma)} = \mathrm{sign}(\tau) \, \tilde{\epsilon}_{\xi\nu\rho\sigma} \\ \tilde{\epsilon}_{-\xi,\nu\rho\sigma} = -\tilde{\epsilon}_{\xi\nu\rho\sigma} \end{cases}$$

$\sigma \rightarrow$

$$U_{-\nu}(x) \equiv U_{\nu}^{\dagger}(x-\hat{\nu})$$

in such a way that:

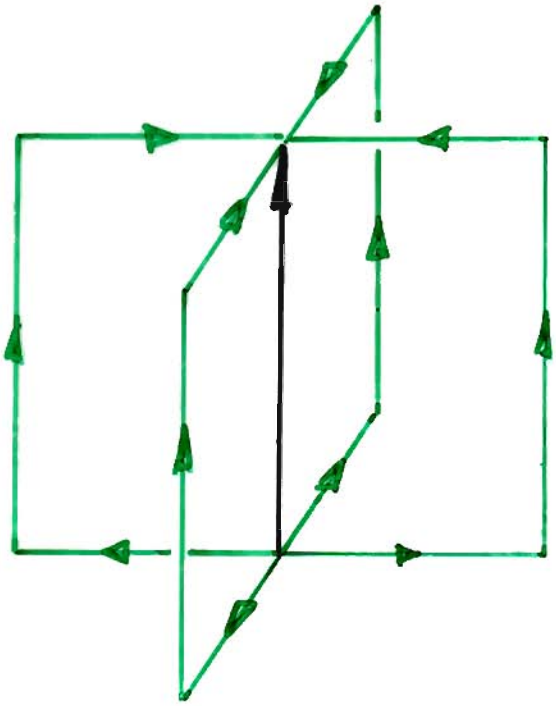
$\square_{-\xi,\nu}(x) \equiv$



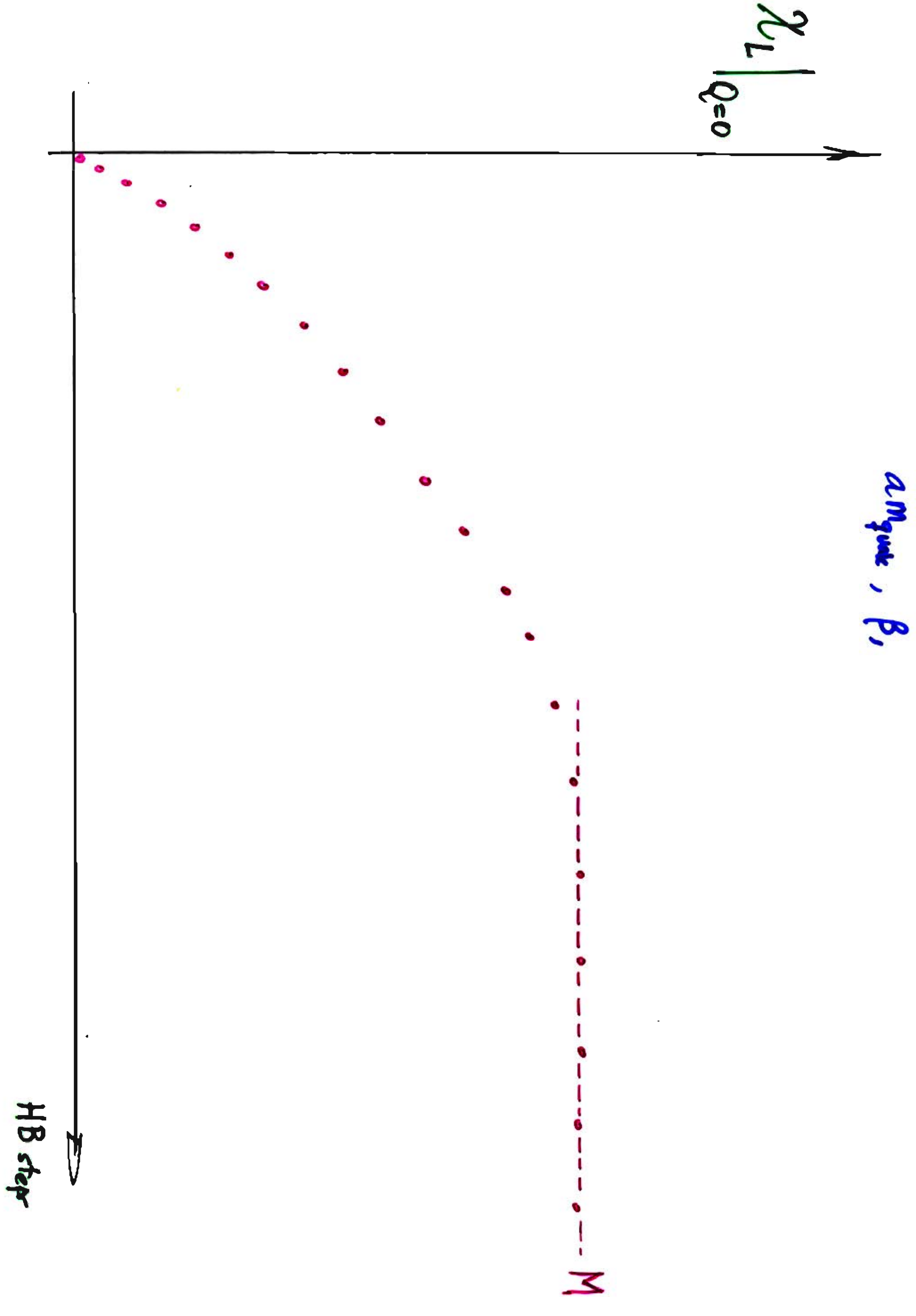
- This definition is particularly symmetric. This fact reduces the statistical noise...

- ...A further reduction of noise is obtained by using smeared links

Christou, Di Giacomo, Panagopoulos, Vicari, hep-lat/9510023



- Small statistical (or quantum) noise implies M lower and Z closer to 1, thus leading to a clearer signal for χ.


$\chi_L|_{Q=0}$
$a\,m_{quark}$, β,
HB step
M

- We actually only need M because both Z and M cannot depend on μ_B.

We shall plot $\dfrac{\chi(\mu)}{\chi(\mu=0)} = \dfrac{\chi_L(\mu) - M}{\chi_L(\mu=0) - M}$

Notice that this ratio is free from Z and a^4.

See 3 Figs

- Fermion saturation initiates at $a\mu_B \sim 0.4$ and becomes complete at $a\mu_B \simeq 1.2$

We have checked that at the same saturation μ_B, also P_B becomes 1 tr $\square$ becomes that of quenched, Polyakov vanishes again ...

See Figs.

9-F

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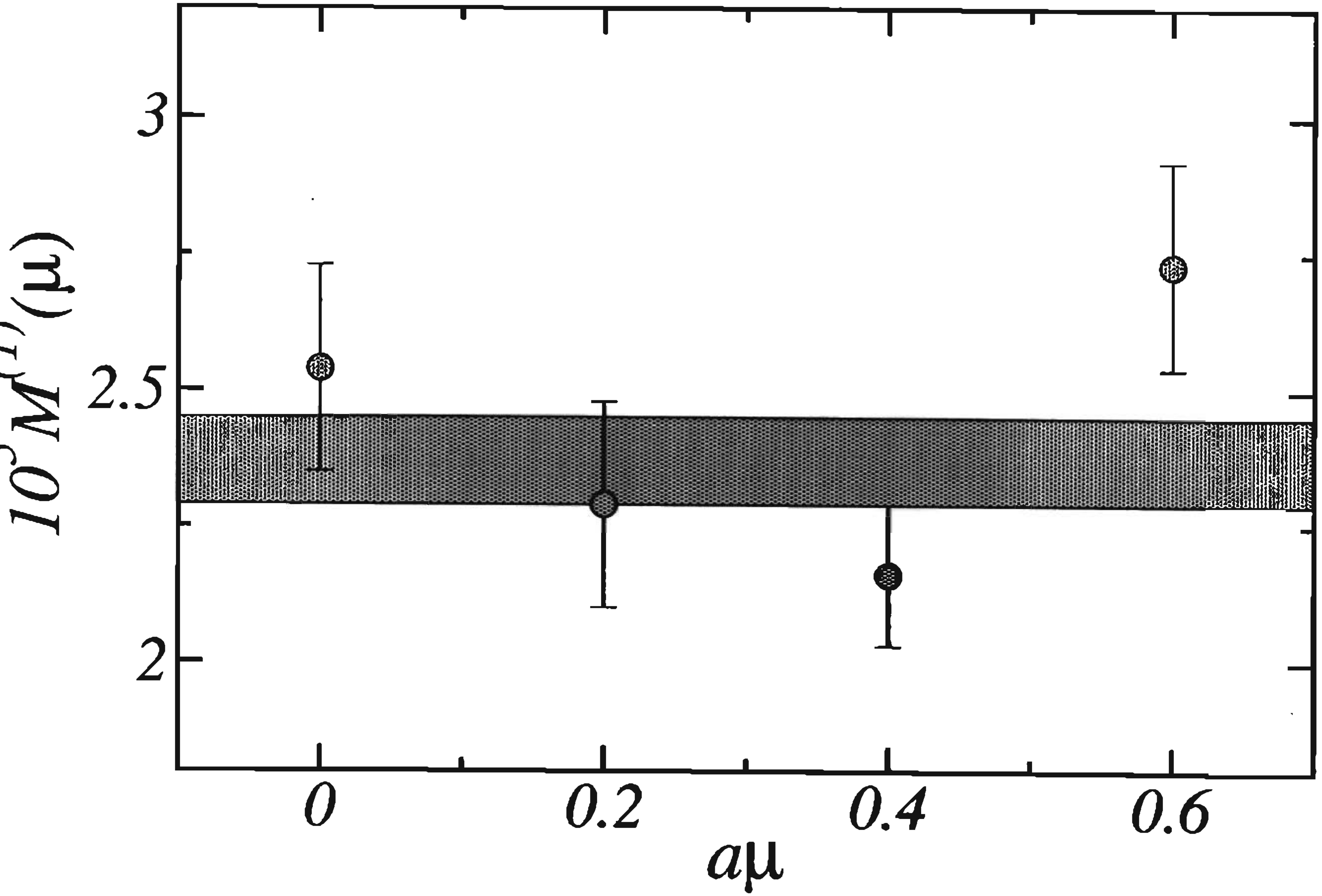

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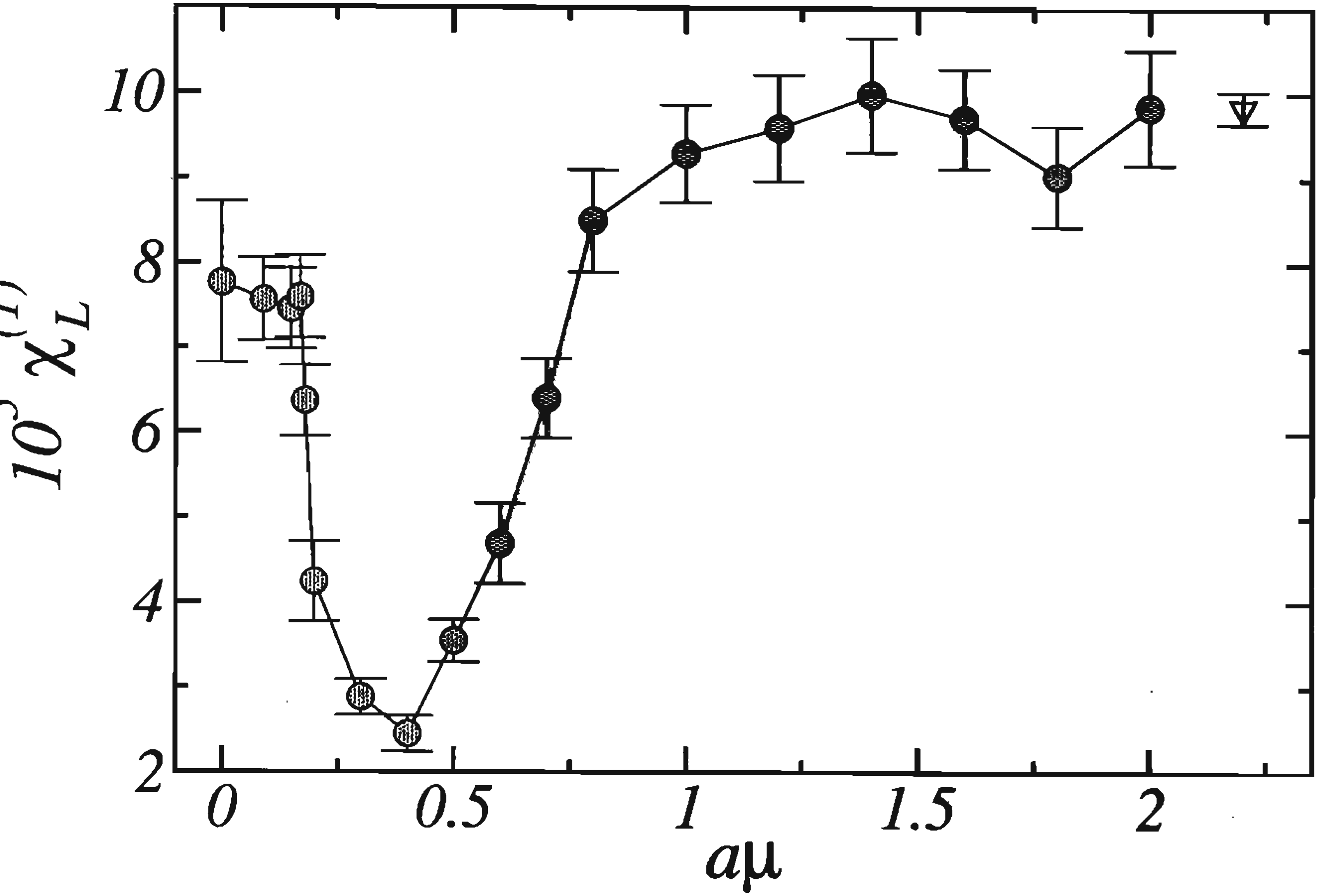

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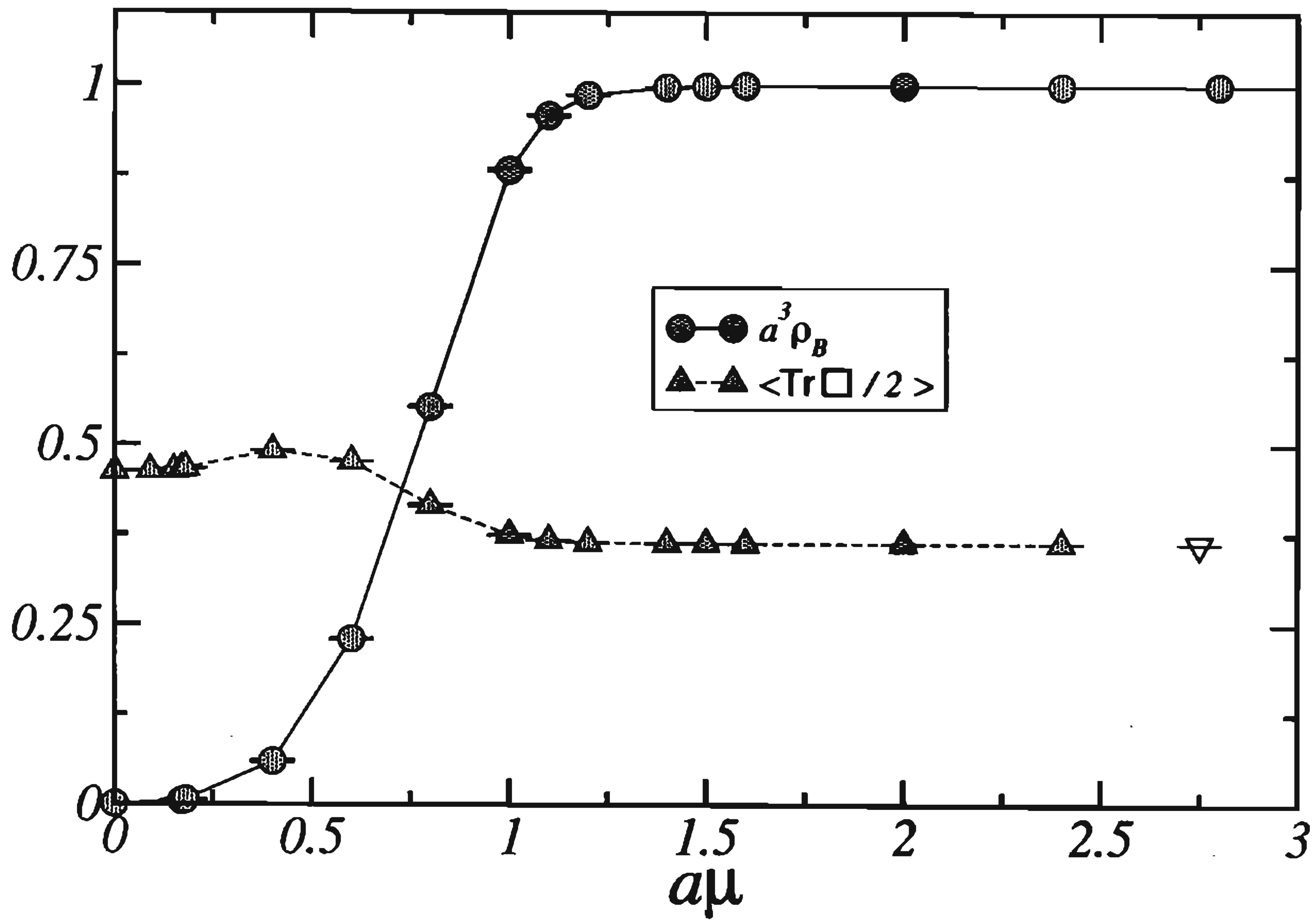
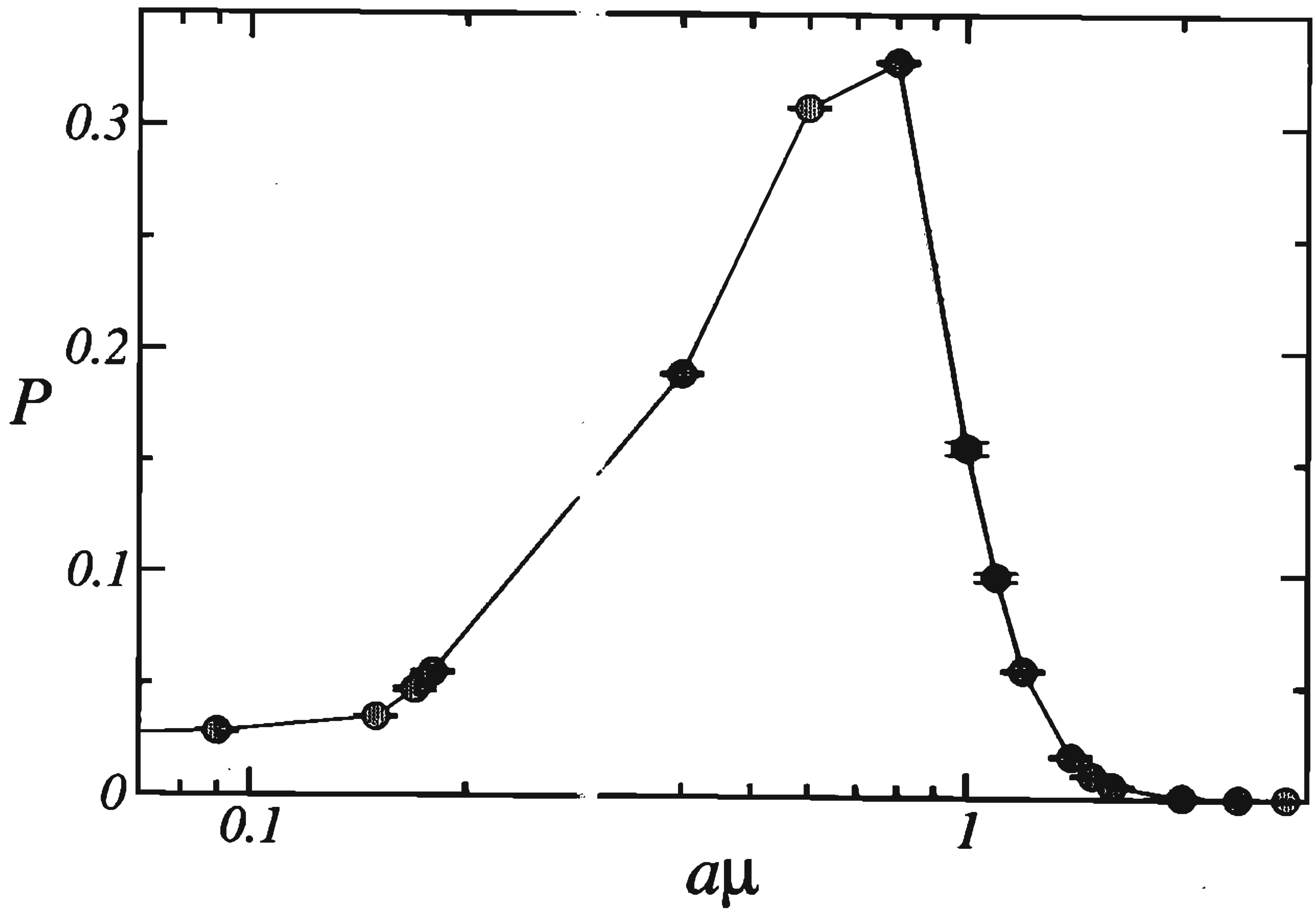


$a^3\rho_B$
$\langle \mathrm{Tr}\Box/2 \rangle$
0
0.25
0.5
0.75
1
0
0.5
1
1.5
2
2.5
3
$a\mu$

26

I-6



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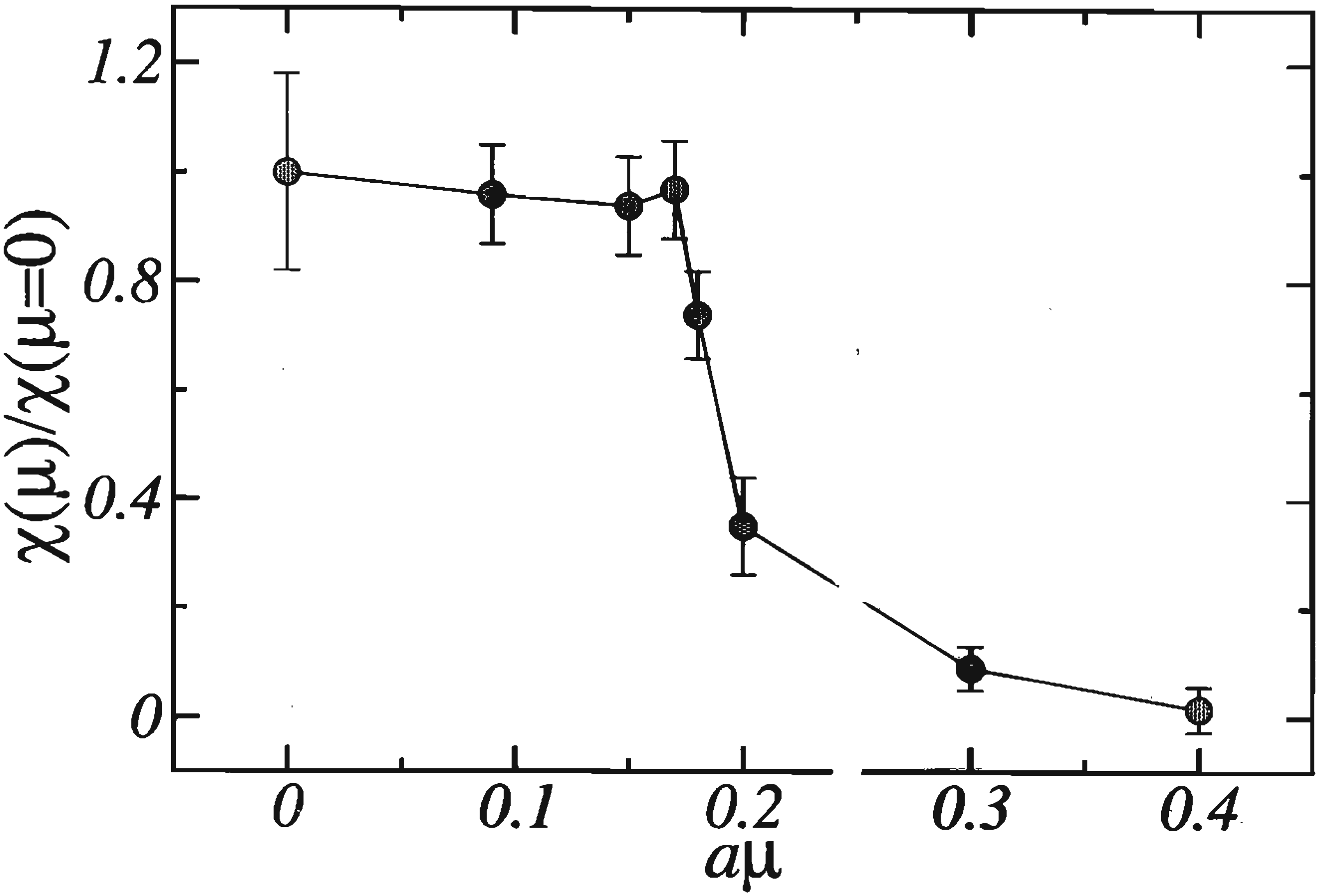

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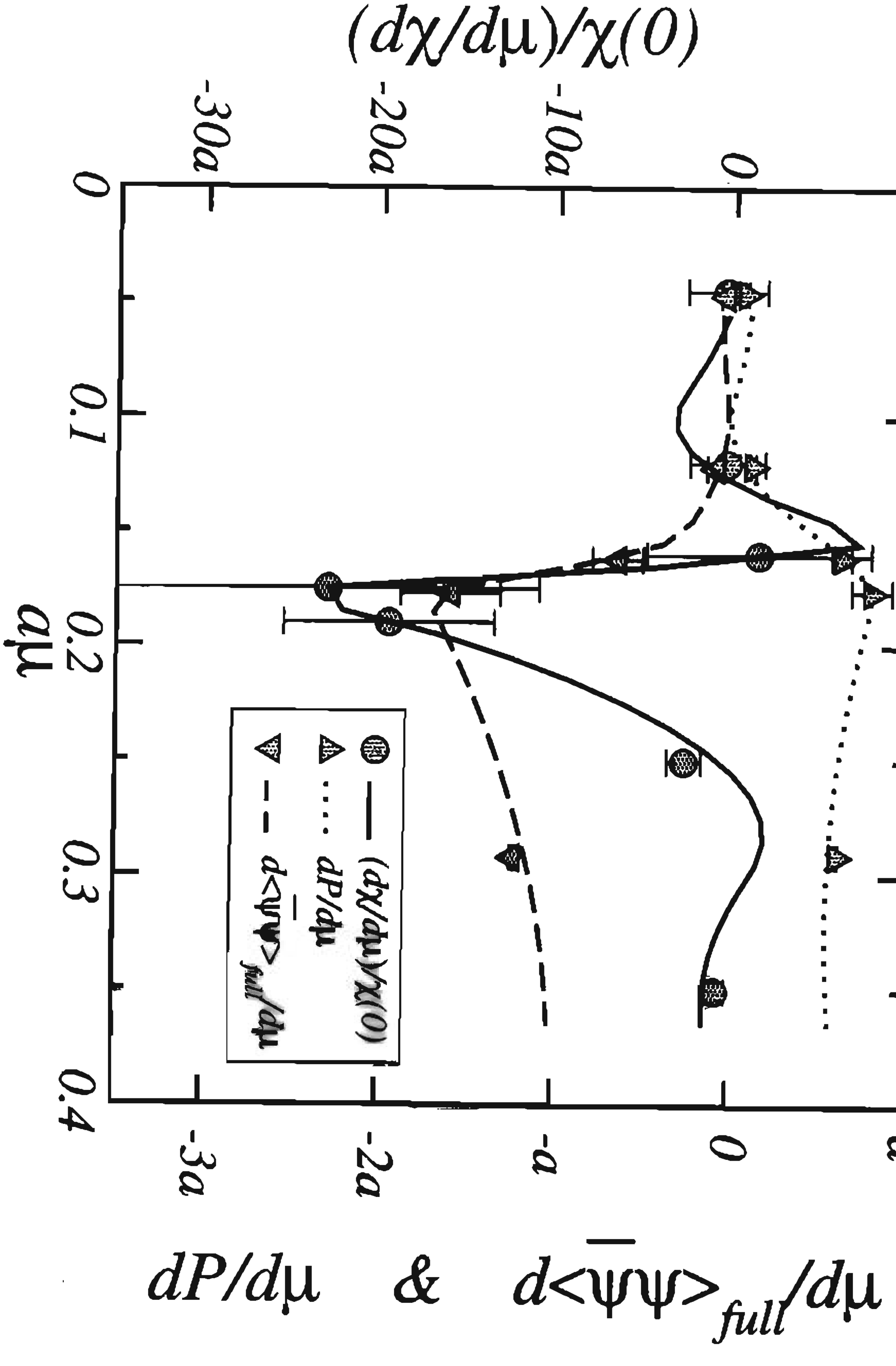


(dχ/dμ)/χ(0)
10a
0
-10a
-20a
-30a
aμ
0
0.1
0.2
0.3
0.4
a
0
-a
-2a
-3a
dP/dμ & d<ψ̄ψ>full/dμ
(dχ/dμ)/χ(0)
dP/dμ
d<ψ̄ψ>full/dμ

Numerics

- $a\mu_c = 0.175(5)$

- Assuming $T_c \sim 100-200$ MeV
  - Many quarks make $T_c \downarrow$
  - Heavy quarks make $T_c \uparrow$
  - T_c (quenched) ~ 250 MeV
    T_c (real QCD) ~ 170 MeV

  we have calculated $\beta_c\ (\mu=0) \simeq 1.594(6)$

  → See Figure

- From T_c we obtain $a(\beta_c) = \dfrac{1}{6\,T_c}$

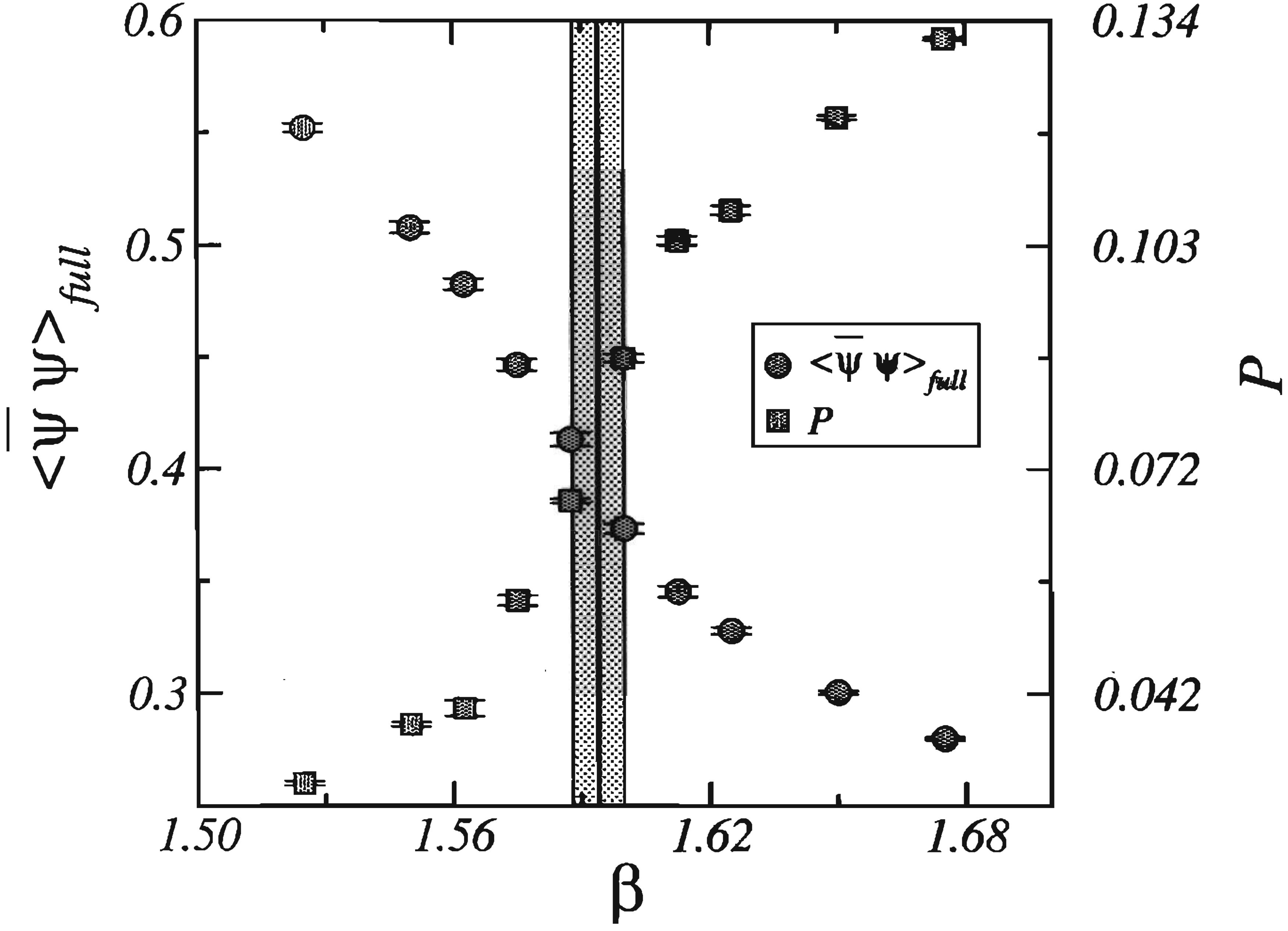
- From β_c, $a(\beta_c)$ we obtain $a(\beta=1.5)$ by using the 2-loop beta function

$$\Lambda_L\, a(\beta) = e^{-\pi^2\beta} \left(\frac{1}{2\pi^2\beta}\right)^{5/2} (1+\dots)$$

- $a(\beta=1.5) = 0.64\ \underbrace{(\pm 4)}_{\beta_c}\ \underbrace{\left({}^{+33}_{-16}\right)}_{T_c}$ fm

13

29



0.6
0.5
0.4
0.3
$\langle\bar{\psi}\,\psi\rangle_{full}$
0.134
0.103
0.072
0.042
P
1.50
1.56
1.62
1.68
β
$\langle\bar{\psi}\,\psi\rangle_{full}$
P

- The requirement of $T < T_c$ and the small lattice size yielded a coarse lattice.

- $$\mu_c \simeq 54 \underset{a\mu_c}{(\pm 2)} \underset{\beta_c}{(\pm 4)} \underset{T_c}{(\pm 18)} \text{ MeV}$$

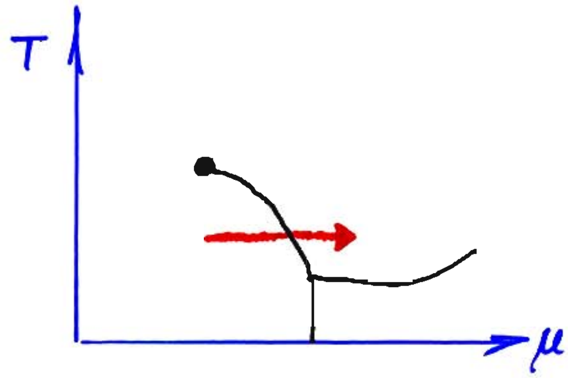
$$T \simeq 51 \underset{\beta_c}{(\pm 4)} \underset{T_c}{(\pm 17)} \text{ MeV}$$

$$T/T_c \simeq 0{,}34 \underset{\beta_c}{(\pm 3)}$$

$$\rho_{B,c}/T_c^3 = 0.047 \underset{a^3\rho_{B,c}}{(13)} \underset{\beta_c}{(12)}$$

FINAL REMARKS & OUTLOOK

- It is thought that the superfluid phase still confines. Hence we consider that we have varied μ like this ?



- χ drops much as it happens at finite T transitions
- All transitions look concomitant.
- Fermion saturation hinders simulation at very high μ.
- Reliable results as for χ, $\langle\bar{\Psi}\Psi\rangle$ and P and their coincidence. Also reliable the values of μ_c, T, ... ($\mu_c(T=0) \simeq m_\pi/2$). However it would be desirable:
  - ▲ to work on larger volumes and finer lattices,
  - ▲ to have less and lighter quarks
  - ▲ to explore lower T in order to cross the superfluid phase.