

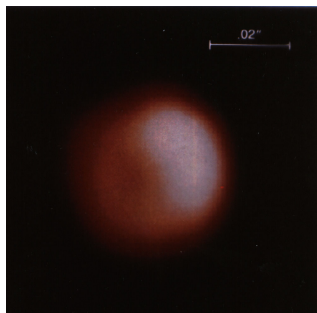
Outline

- 1 Introduction
 - Imaging outside of Nuclear Physics
 - Heavy-Ion Collisions
 - Observed Asymmetries
- 2 Correlation Analysis
 - Multipole Decomposition & Imaging
 - Cartesian Harmonics
- 3 Illustration
 - Relative Source
 - Classical Coulomb Correlations
- 4 Summary



Astronomy

Intensity/phase interferometry first used to assess sizes of astronomical objects. Astronomers have since moved to details:



red giant Betelgeuse

Can we do comparably well?

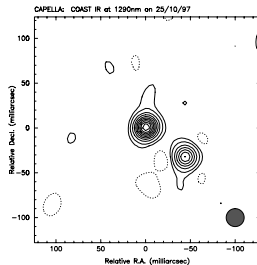


Figure 5.7: Reconstructed image of Capella, from data taken on 25 October 1997 at a wavelength of $1.3\ \mu\text{m}$. The contours are at $-4, 4, 10, 20, 30, \dots, 90\%$ of the peak ux. The map has been restored with a circular beam for clarity.

binary star Capella, Monnier
Rep Prog Phys 66(03)789



Imaging

Geometric information from imaging. General task:

$$C(q) = \int dr K(q, r) S(r)$$

From data w/ errors, $C(q)$, determine the source $S(r)$.

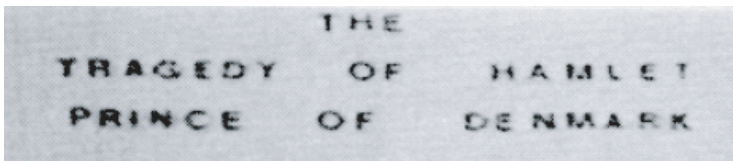
Requires inversion of the kernel K .

Optical recognition: K - blurring function, max entropy method

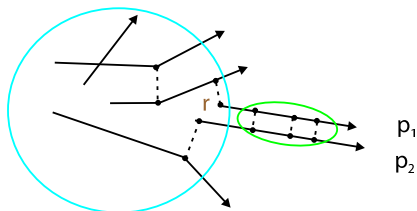
C :



S :



Factorization of Final-State Amplitude in Reactions



coarse

pronounced structure
calculable

2-ptcle inclusive cross section
at low $|\mathbf{p}_1 - \mathbf{p}_2|$

$$\frac{d\sigma}{d\mathbf{p}_1 d\mathbf{p}_2} = \int d\mathbf{r} S'_P(\mathbf{r}) |\Phi_{\mathbf{p}_1 - \mathbf{p}_2}^{(-)}(\mathbf{r})|^2$$

data source 2-ptcle wf

S' : distribution of emission
points in 2-ptcle CM

Normalizing with 1-ptcle cross sections yields correlation f:

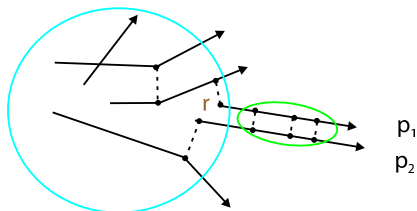
$$C(\mathbf{p}_1 - \mathbf{p}_2) = \frac{\frac{d\sigma}{d\mathbf{p}_1 d\mathbf{p}_2}}{\frac{d\sigma}{d\mathbf{p}_1} \frac{d\sigma}{d\mathbf{p}_2}} = \int d\mathbf{r} S_P(\mathbf{r}) |\Phi_{\mathbf{p}_1 - \mathbf{p}_2}^{(-)}(\mathbf{r})|^2$$

Then the relative source is normalized to unity: $\int d\mathbf{r} S_P(\mathbf{r}) = 1$.

Note: C may only give access to the density of relative emission
points in 2-ptcle CM, integrated there over time



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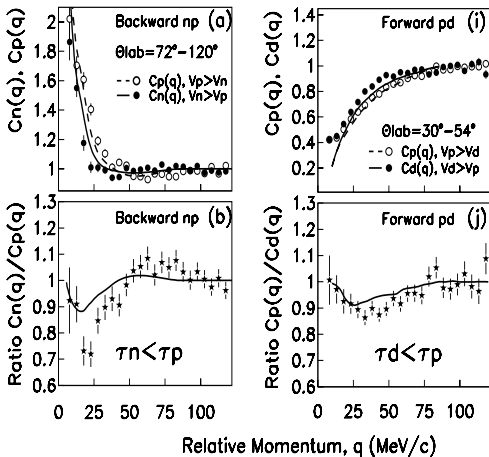
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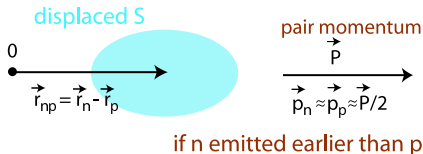
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Time Difference in Emission



Anisotropic C , dependent on orientation of \mathbf{q}
Attributable to anisotropic S :



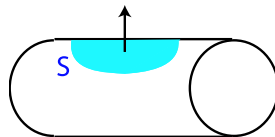
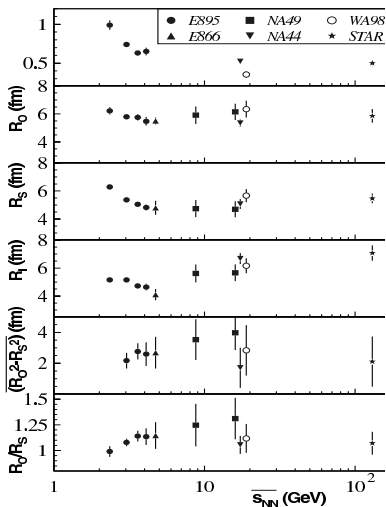
if n emitted earlier than p

Ghetti *et al.*,
PRL91(03)0927011

Model fitted to data



Geometry + Freezeout + Collective Motion



⇐ Fitted radii (longitudinal, outward & sideward) for an anisotropic Gaussian

Models fitted to data...



Integral Relation

Of interest is the deviation of correlation function from unity:

$$\mathcal{R}(\mathbf{q}) = C(\mathbf{q}) - 1 = \int d\mathbf{r} \left(|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2 - 1 \right) S(\mathbf{r}) \equiv \int d\mathbf{r} K(\mathbf{q}, \mathbf{r}) S(\mathbf{r})$$

Learning on S possible when $|\Phi_{\mathbf{q}}^{(-)}(\mathbf{r})|^2$ deviates from 1, either due to symmetrization or interaction within the pair.

The spin-averaged kernel K depends only on the relative angle between \mathbf{q} and \mathbf{r} . This facilitates the angular decomposition.

With

$$K(\mathbf{q}, \mathbf{r}) = \sum_{\ell} (2\ell + 1) K_{\ell}(q, r) P^{\ell}(\cos \theta), \quad \text{and}$$

$$\mathcal{R}(\mathbf{q}) = \sqrt{4\pi} \sum_{\ell m} \mathcal{R}^{\ell m}(q) Y^{\ell m}(\hat{\mathbf{q}}), \quad S(\mathbf{r}) = \sqrt{4\pi} \sum_{\ell m} S^{\ell m}(q) Y^{\ell m}(\hat{\mathbf{r}})$$

we reduce the 3D relation to a set of 1D:

$$\mathcal{R}^{\ell m}(q) = 4\pi \int dr r^2 K_{\ell}(q, r) S^{\ell m}(r)$$



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$$\ell = 0$$

Different multipolarities of deformation for the source and correlation functions are directly related to each other.

The $\ell = 0$ version:

$$\mathcal{R}(q) = 4\pi \int dr r^2 K_0(q, r) S(r)$$

where $\mathcal{R}(q)$, K_0 and $S(r)$ – angle-averaged correlation, kernel and source, respectively.

For pure interference, π^0 's or γ 's, $\Phi_{\mathbf{q}}^{(-)}(\mathbf{r}) = \frac{1}{\sqrt{2}} (e^{i\mathbf{q}\cdot\mathbf{r}} + e^{-i\mathbf{q}\cdot\mathbf{r}})$, the kernel $K = |\Phi|^2 - 1$ results from the interference term in $|\Phi|^2$ and the correlation-source relation is just the FT:

$$\mathcal{R}_0(q) = \frac{2\pi}{q} \int dr r \sin(2qr) S_0(r)$$



Discretization & Imaging

Source discretization w/ χ^2 fitting applies to any pair:

- 1 Discretize integral

$$\mathcal{R}_i = \sum_j 4\pi \Delta r r_j^2 K_0(q_i, r_j) S(r_j) \equiv \sum_j K_{ij} S_j$$

- 2 Vary $S(r_j)$ to minimize χ^2 :

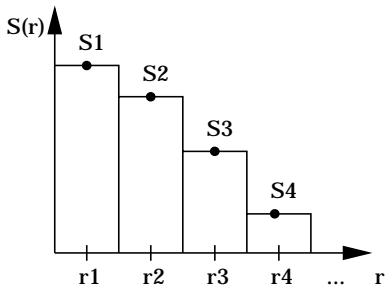
$$\chi^2 = \sum_i \frac{(\sum_j K_{ij} S_j - \mathcal{R}_i^{exp})^2}{\sigma_i^2}$$

- 3 S_j -derivative of χ^2 yields:

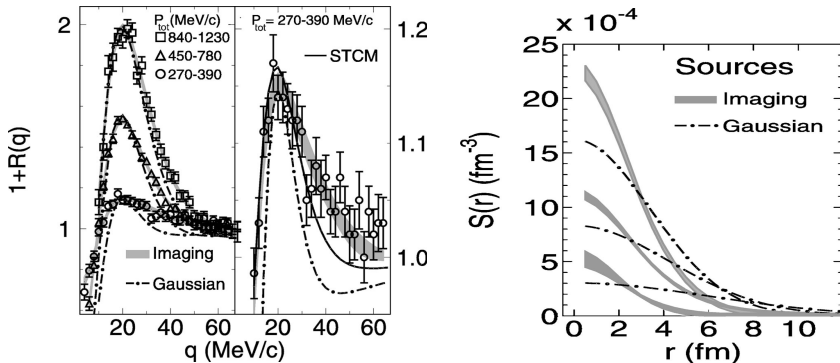
$$\sum_{ij} \frac{1}{\sigma_i^2} (K_{ij} S_j - \mathcal{R}_i^{exp}) K_{ij} = 0$$

with solution in a mtx form:

$$\mathbf{S} = (\mathbf{K}^T \mathbf{K})^{-1} \mathbf{K}^T \mathcal{R}^{exp}$$



pp Imaging

Imaging impacted interpretation of C_{pp} , Verde PRC65(02)054609

Gauss par: quickly changing radii. Imaging: quickly changing preequilibrium fraction, non-Gaussian source shapes!

$S(r \rightarrow 0)$: preequilibrium fraction, entropy, freeze-out $\rho \dots$



Anisotropies??

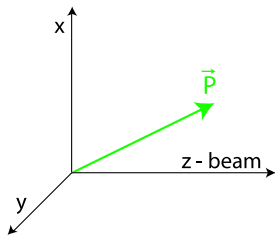
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we have

$$\mathcal{R}^{\ell m}(q) = 4\pi \int dr r^2 K_{\ell}(q, r) \mathcal{S}^{\ell m}(r)$$

A set of 1D integral relations



Problem: Why turning real quantities, R & S , into imaginary, $R^{\ell m}$ & $S^{\ell m}$? Other basis than $Y^{\ell m}$??



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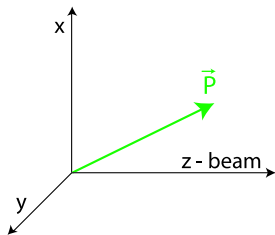
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Cartesian Basis

Take the direction vector: $\hat{n}_\alpha = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$

Rank- ℓ tensor product:

$$(\hat{n}^\ell)_{\alpha_1 \dots \alpha_\ell} \equiv \hat{n}_{\alpha_1} \hat{n}_{\alpha_1} \dots \hat{n}_{\alpha_\ell} = \sum_{\ell' \leq \ell, m} c_{\ell' m} Y^{\ell' m}$$

$\mathcal{D}^{(\ell, \ell)}$ projection operator that, within the space of rank- ℓ cartesian tensors, removes $Y^{\ell' m}$ components with $\ell' < \ell$:

$$(\mathcal{D}\hat{n}^\ell)_{\alpha_1 \dots \alpha_\ell} = \sum_m c_{\ell m} Y^{\ell m}$$

The components $\mathcal{D}\hat{n}^\ell$ are real and can be used to replace $Y^{\ell m}$.



Low- ℓ Cartesian Harmonics

$$\mathcal{D}\hat{n}^0 = 1$$

$$(\mathcal{D}\hat{n}^1)_\alpha = \hat{n}_\alpha$$

$$(\mathcal{D}\hat{n}^2)_{\alpha_1 \alpha_2} = \hat{n}_{\alpha_1} \hat{n}_{\alpha_2} - \frac{1}{3} \delta_{\alpha_1 \alpha_2}$$

$$(\mathcal{D}\hat{n}^3)_{\alpha_1 \alpha_2 \alpha_3} = \hat{n}_{\alpha_1} \hat{n}_{\alpha_2} \hat{n}_{\alpha_3} - \frac{1}{5} (\delta_{\alpha_1 \alpha_2} \hat{n}_{\alpha_3} + \delta_{\alpha_1 \alpha_3} \hat{n}_{\alpha_2} + \delta_{\alpha_2 \alpha_3} \hat{n}_{\alpha_1})$$

$$\vdots$$

\mathcal{D} can be called a detracing operator as

$$\sum_{\alpha} (\mathcal{D}\hat{n}^{\ell})_{\alpha \alpha \alpha_3 \dots \alpha_{\ell}} = 0$$



Decomposition with Cartesian Harmonics

Completeness relation ($\mathcal{D} = \mathcal{D}^\top = \mathcal{D}^2$):

$$\begin{aligned} \delta(\Omega' - \Omega) &= \frac{1}{4\pi} \sum_{\ell} \frac{(2\ell + 1)!!}{\ell!} \sum_{\alpha_1 \dots \alpha_{\ell}} (\mathcal{D}\hat{n}^{\ell})_{\alpha_1 \dots \alpha_{\ell}} (\mathcal{D}\hat{n}^{\ell})_{\alpha_1 \dots \alpha_{\ell}} \\ &= \frac{1}{4\pi} \sum_{\ell} \frac{(2\ell + 1)!!}{\ell!} \sum_{\alpha_1 \dots \alpha_{\ell}} (\mathcal{D}\hat{n}^{\ell})_{\alpha_1 \dots \alpha_{\ell}} \hat{n}_{\alpha_1} \dots \hat{n}_{\alpha_{\ell}} \end{aligned}$$

In consequence

$$\mathcal{R}(\mathbf{q}) = \int d\Omega' \delta(\Omega' - \Omega) \mathcal{R}(\mathbf{q}') = \sum_{\ell} \sum_{\alpha_1 \dots \alpha_{\ell}} \mathcal{R}_{\alpha_1 \dots \alpha_{\ell}}^{(\ell)}(q) \hat{q}_{\alpha_1} \dots \hat{q}_{\alpha_{\ell}}$$

where coefficients are angular moments

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Cartesian coefficients for \mathcal{R} & \mathcal{S} directly related to each other:

$$\mathcal{R}_{\alpha_1 \dots \alpha_\ell}^{(\ell)}(\mathbf{q}) = 4\pi \int dr r^2 K_\ell(\mathbf{q}, r) \mathcal{S}_{\alpha_1 \dots \alpha_\ell}^{(\ell)}(r)$$

For weak anisotropies, only lowest- ℓ matter:

$$\mathcal{R}(\mathbf{q}) = \mathcal{R}^{(0)}(q) + \sum_{\alpha} \mathcal{R}_{\alpha}^{(1)}(q) \hat{q}_{\alpha} + \sum_{\alpha_1 \alpha_2} \mathcal{R}_{\alpha_1 \alpha_2}^{(2)}(q) \hat{q}_{\alpha_1} \hat{q}_{\alpha_2} + \dots$$

$\mathcal{R}^{(0)}$ - angle-averaged correlation

$\mathcal{R}_{\alpha}^{(1)} \equiv R^{(1)} e_{\alpha}^{(1)}$ - dipole distortion, magnitude + direction vector

$\mathcal{R}_{\alpha\beta}^{(2)}(q) = R_1^{(2)} e_{1\alpha}^{(2)} e_{1\beta}^{(2)} + R_3^{(2)} e_{3\alpha}^{(2)} e_{3\beta}^{(2)} - (R_1^{(2)} + R_3^{(2)}) e_{2\alpha}^{(2)} e_{2\beta}^{(2)}$

- quadrupole distortion, 2 magnitude values + 3 orthogonal direction vectors



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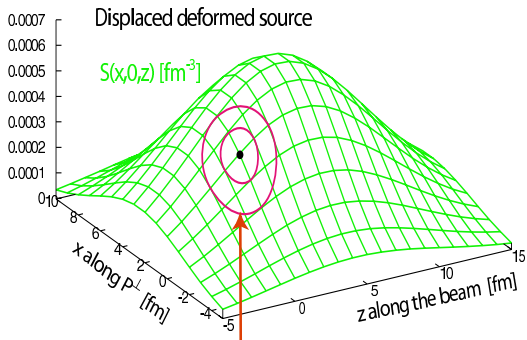
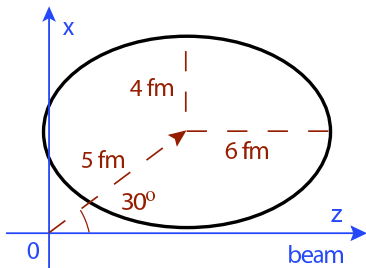
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Sample Relative Source

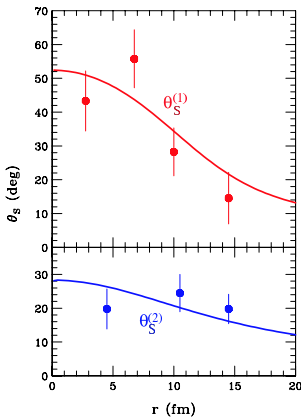
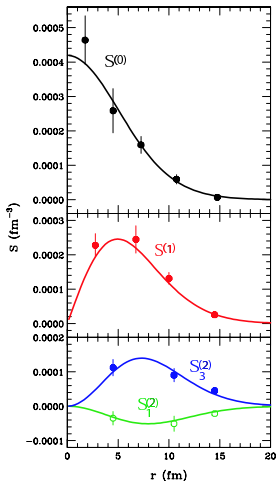
Anisotropic Gaussian, elongated along the beam axis, displaced along the pair momentum



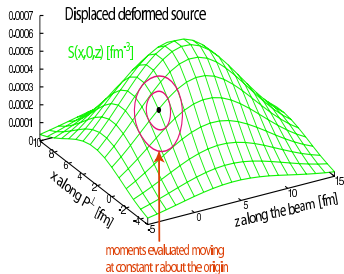
moments evaluated moving
at constant r about the origin



Low- l Characteristics



Values + Angles



$$S^{(l)} \propto r^l$$



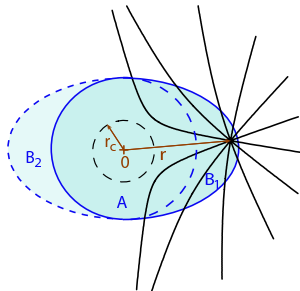
Classical Coulomb Correlations

Coulomb kernel is a function of $\theta_{\mathbf{qr}}$ and r/r_c , where r_c distance of closest approach in head-on collision, $\frac{q^2}{2m_{ab}} = \frac{Z_a Z_b e^2}{4\pi\epsilon_0 r_c}$:

$$|\phi|^2 = \frac{d^3 q_0}{d^3 q} = \frac{\Theta(1 + \cos \theta_{\mathbf{qr}} - 2r_c/r)(1 + \cos \theta_{\mathbf{qr}} - r_c/r)}{\sqrt{(1 + \cos \theta_{\mathbf{qr}})^2 - (1 + \cos \theta_{\mathbf{qr}})2r_c/r}}$$

$$K_0 = \Theta(r - r_c) \sqrt{1 - r_c/r} - 1$$

Correlation reflects the distribution of relative Coulomb trajectories emerging from an anisotropic source.



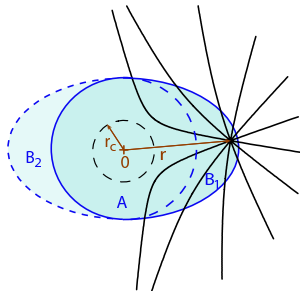
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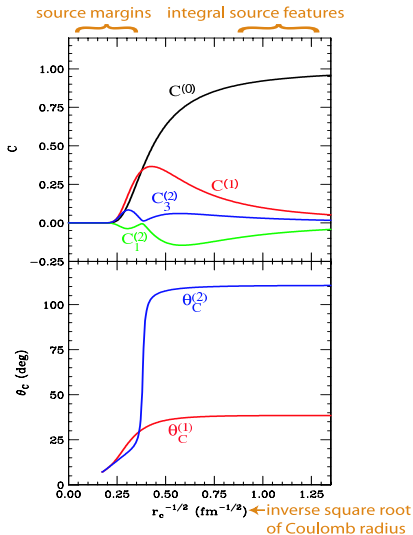
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Coulomb Correlation



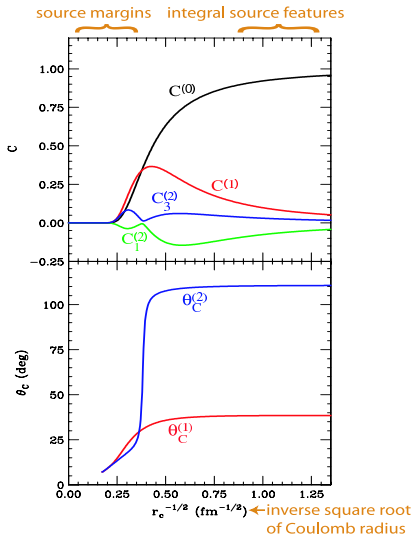
$$r_c^{-1/2} \propto q$$

For more schematic sources, one or more correlation values vanish and/or angles exhibit less variation.

90° jump associated with K_2 sign change and prolate-oblate transition



Coulomb Correlation



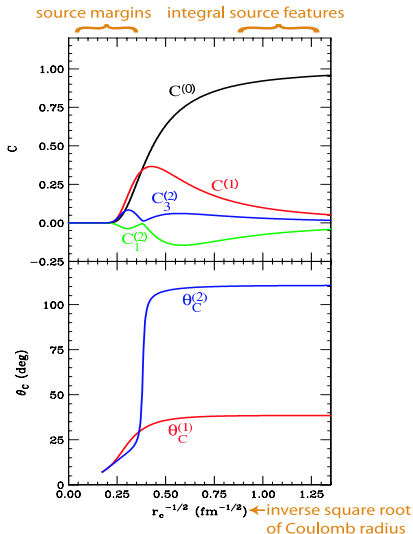
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Imaging

Assumption: $\ell \leq 2$ cartesian coefficients measured at 80 values of $r_c^{-1/2}$ subject to an r.m.s. error of 0.015.

No of restored values, with the region of $r < 17$ fm: 5 for $\ell = 0$, 4 for $\ell = 1$ and 3 for $\ell = 2$.

[Return to source](#)

$r < 17$ fm Source Characteristics:

	Unit	Restored	Original
$4\pi \int dr r^2 S^{(0)}$		0.99 ± 0.05	1.00
$\langle x \rangle$	fm	2.47 ± 0.11	2.45
$\langle z \rangle$	fm	4.25 ± 0.13	3.90
$\langle (x - \langle x \rangle)^2 \rangle^{1/2}$	fm	3.80 ± 0.24	3.90
$\langle y^2 \rangle^{1/2}$	fm	3.81 ± 0.22	3.91
$\langle (z - \langle z \rangle)^2 \rangle^{1/2}$	fm	5.54 ± 0.19	5.60
$\langle (x - \langle x \rangle)(z - \langle z \rangle) \rangle$	fm ²	2.23 ± 1.49	-0.41



Summary

- Relative correlations give access to space-time geometry of emission.
- Cartesian harmonic coefficients allow for a systematic quantification of anisotropic correlation functions.
- The correlation coefficients are directly related to the analogous respective coefficients for the relative source.
- Features of the source anisotropies may be, to an extent, read off straight from the correlation anisotropies. Otherwise, they can be imaged.

nucl-th/0501003

Collaborators: S. Pratt, D. Brown, G. Verde...



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