Dissociation of Relativistic Projectiles with the Continuum-Discretized Coupled-Channels Method

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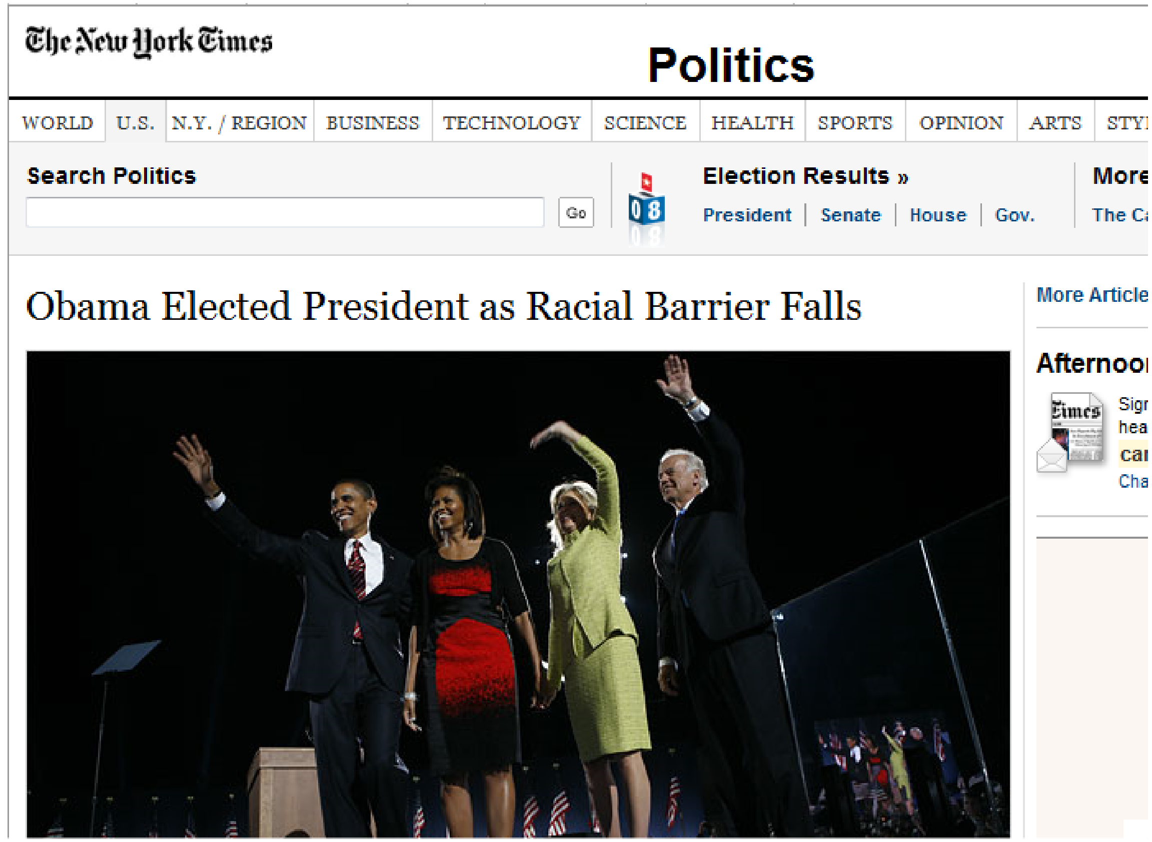
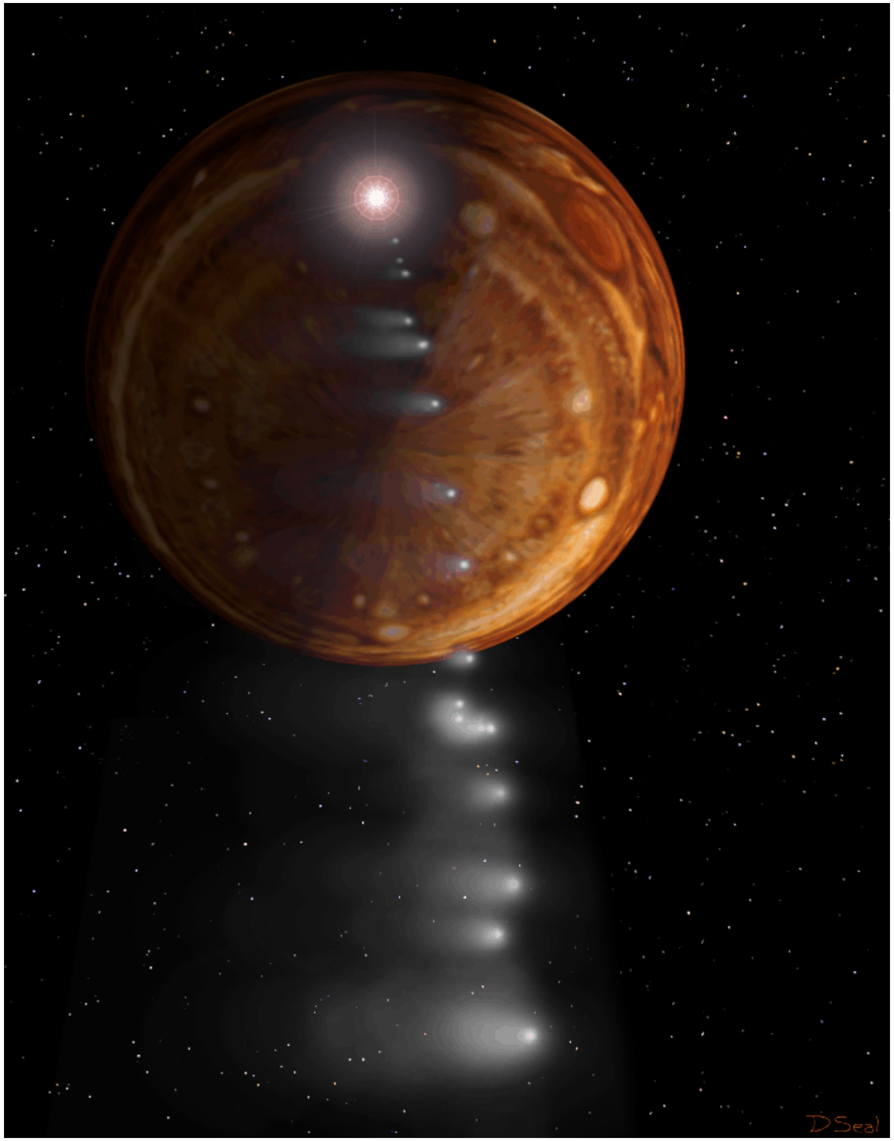


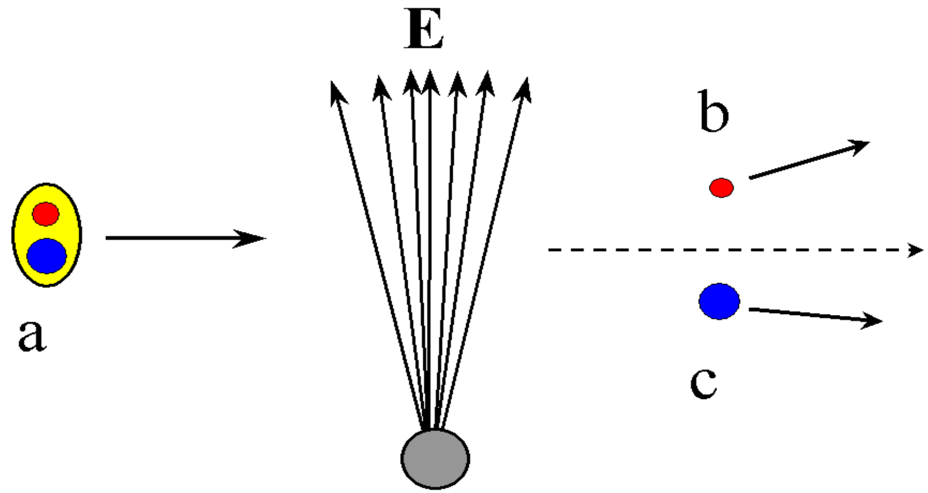
"I am very happy today… a huge victory for Democracy."
Angela Bonaccorso, Pisa, Italy, 11/05/2008

Why dissociation?

Ex: Coulomb dissociation



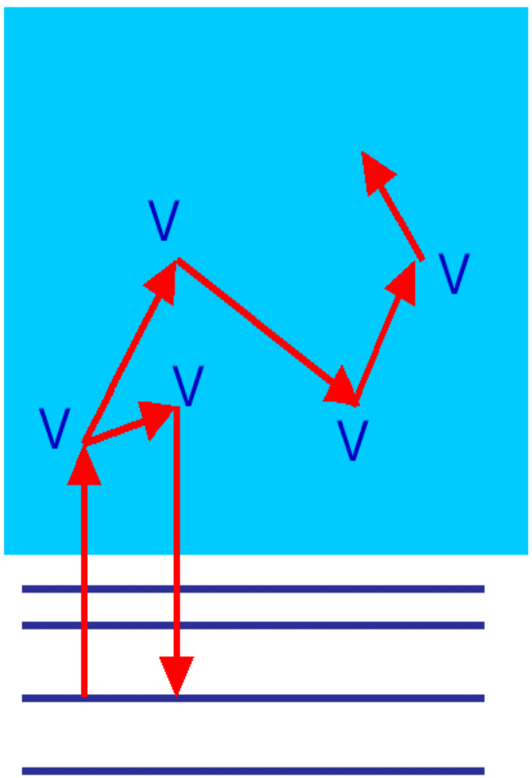
Shoemaker-Levy comet

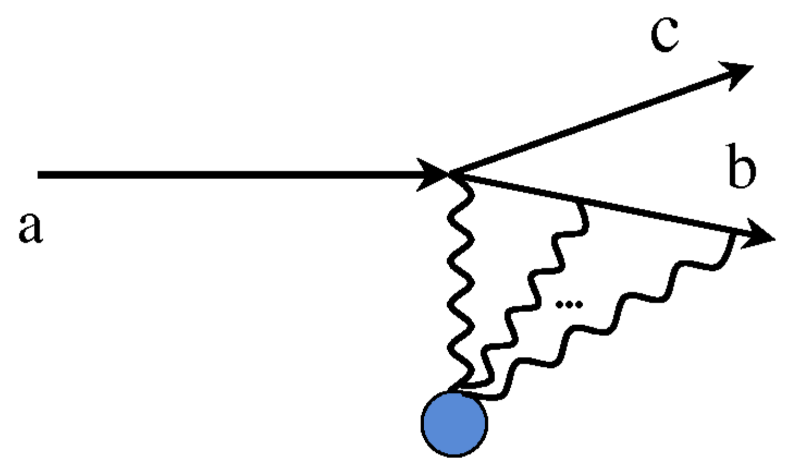


$$\frac{d\sigma}{dE_\gamma d\Omega} \propto \sigma_{\gamma + \mathrm{a} \rightarrow \mathrm{b} + \mathrm{c}}(E_\gamma)$$

- Baur, Bertulani, Rebel, NPA 458 (1986) 188
- Motobayashi, PRL 73 (1994) 2680 $^7Be(p,\gamma)^8B$

CDCC





Some CDCC projects

Incident Energy

200 – 300 A MeV

1. Eikonal CDCC
2. Eikonal CDCC with Relativistic Corrections

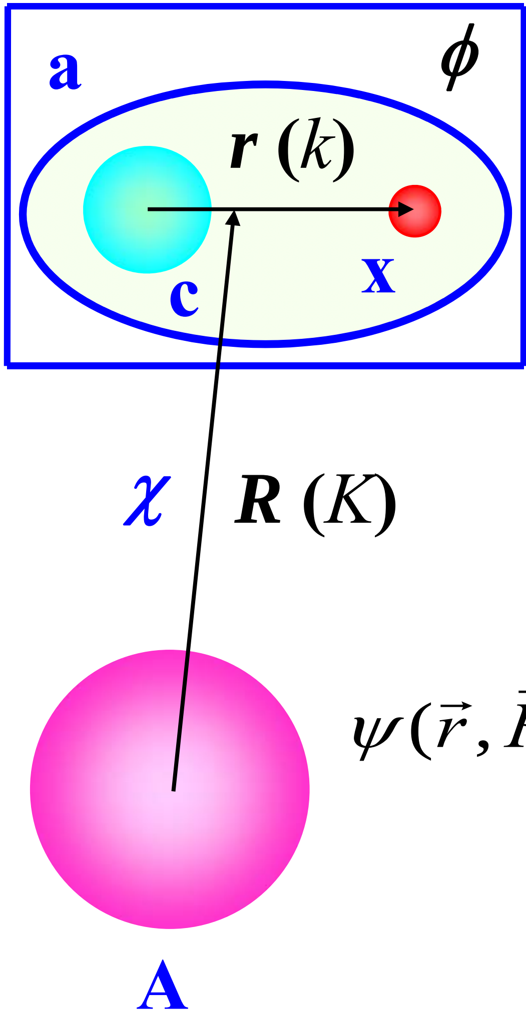
50 – 100 A MeV

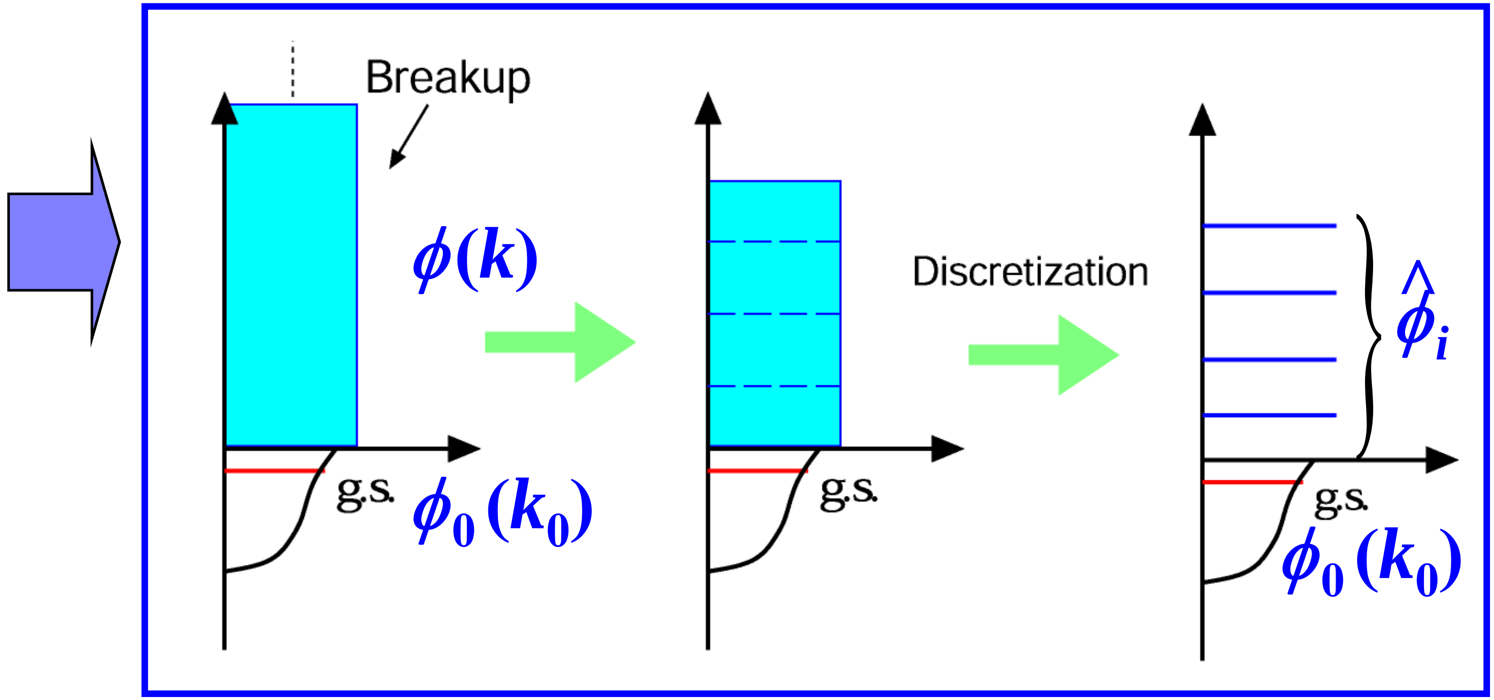
1. CDCC with microscopic optical potential
2. 4-body CDCC

< 10 A MeV

1. Transfer reaction with rearrangement
2. Incomplete (breakup) fusion with CDCC
3. Description of ternary processes in nucleosynthesis

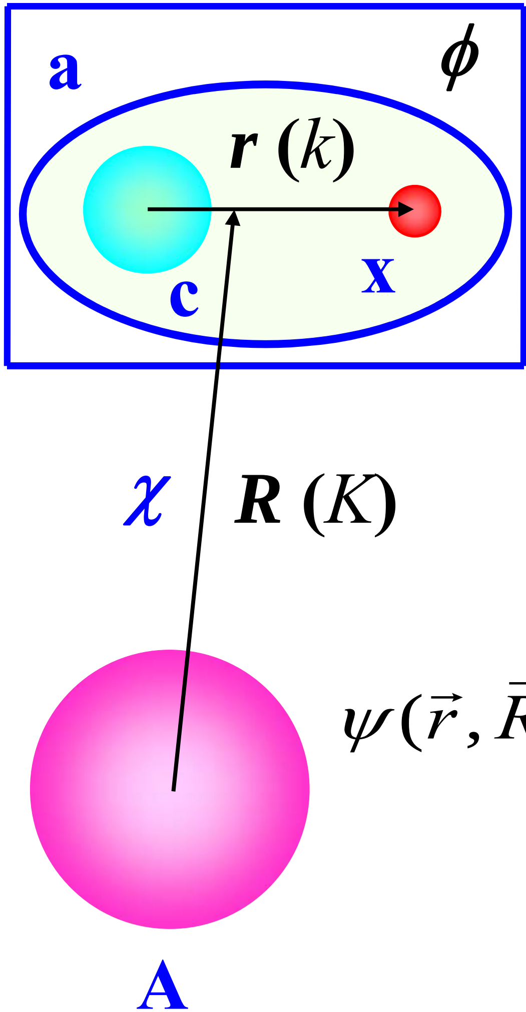
The Continuum-Discretized Coupled Channels method (CDCC)

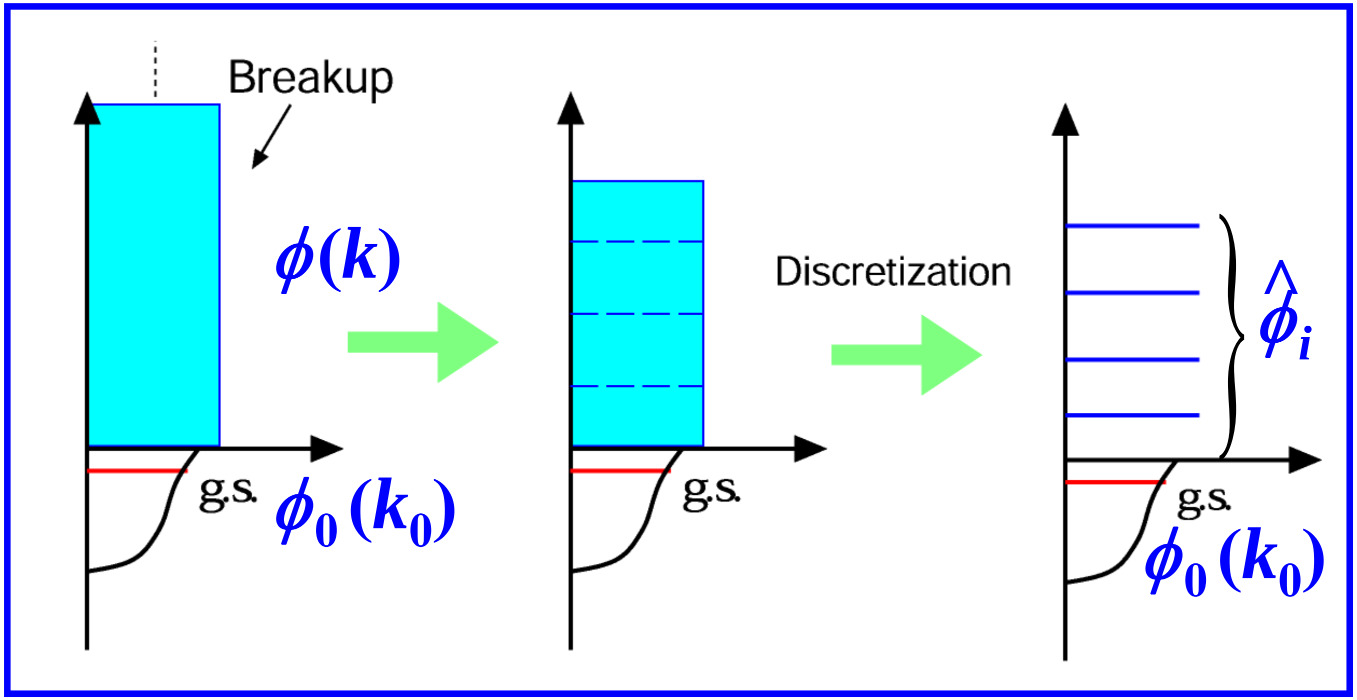




$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

The Continuum-Discretized Coupled Channels method (CDCC)





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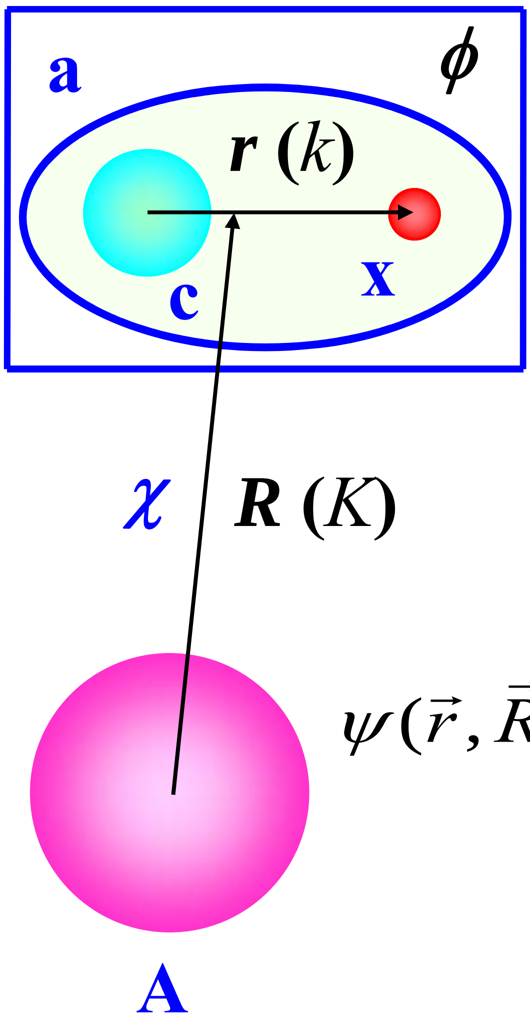

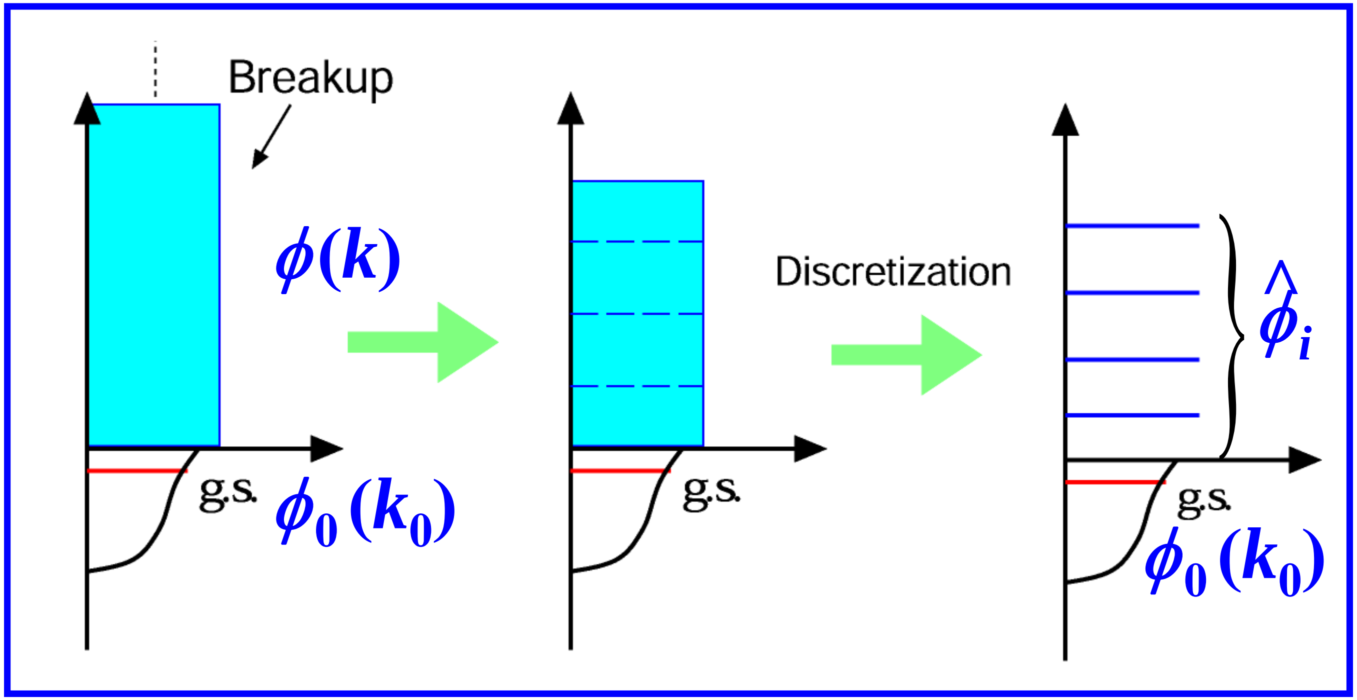

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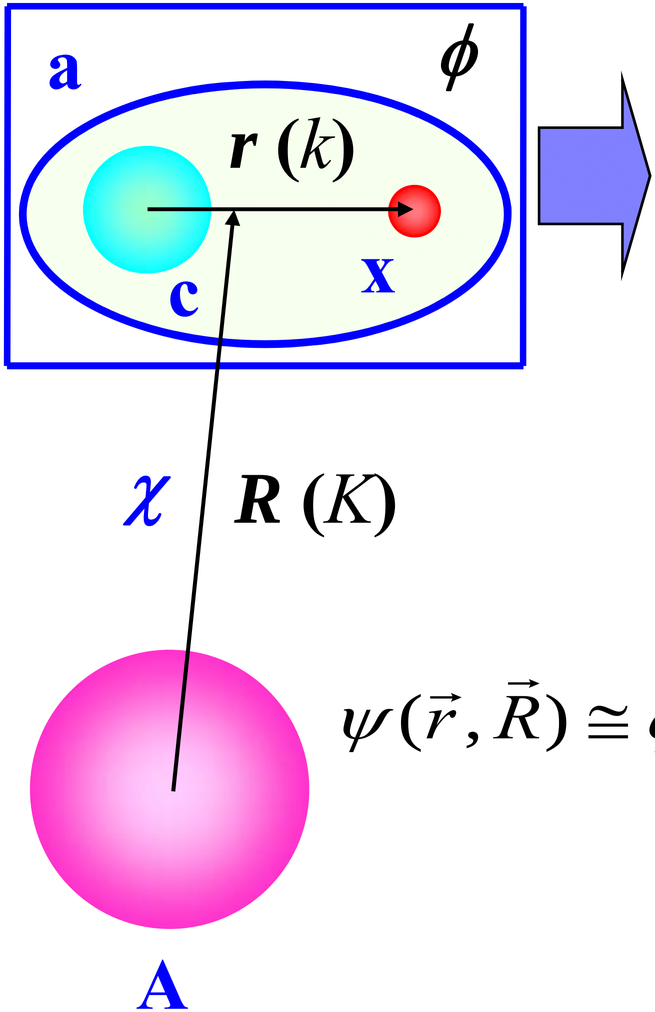
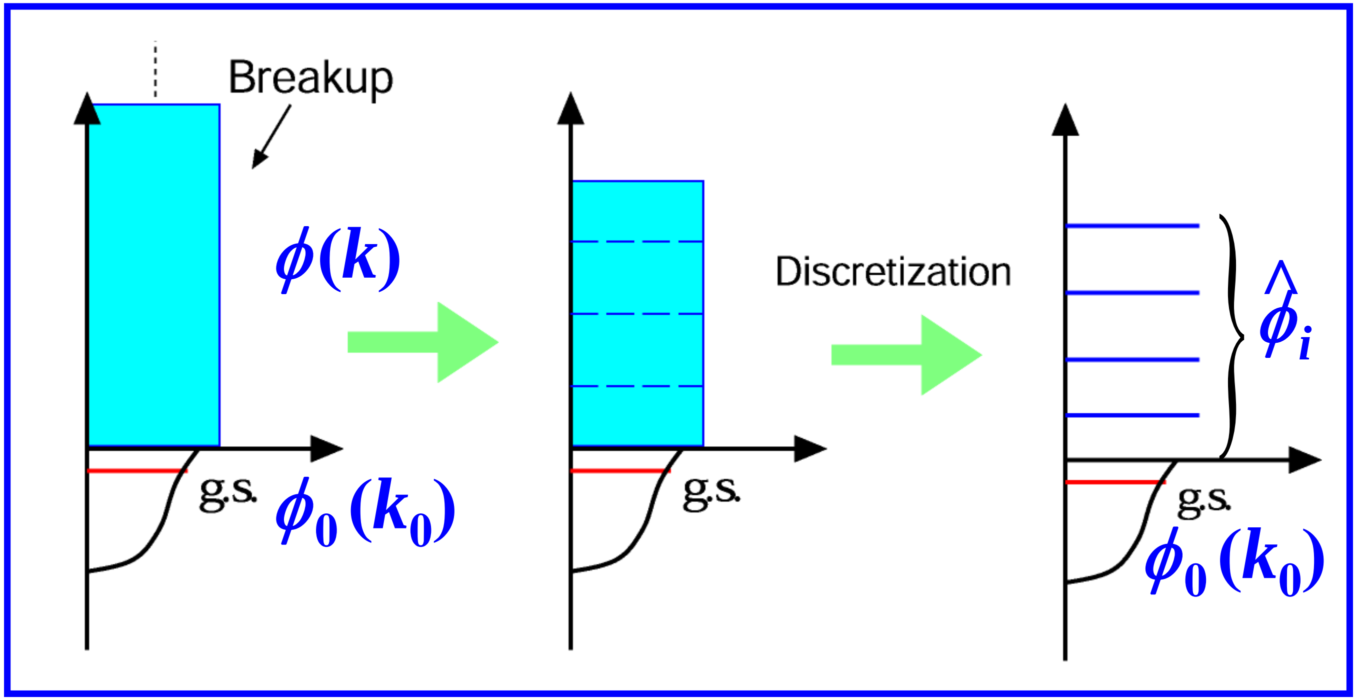
Truncation and Discretization

The Continuum-Discretized Coupled Channels method (CDCC)





$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \underline{\sum_{i=1}^{i_{\max}} \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})\chi(K,\vec{R})dk}$$

Truncation and Discretization

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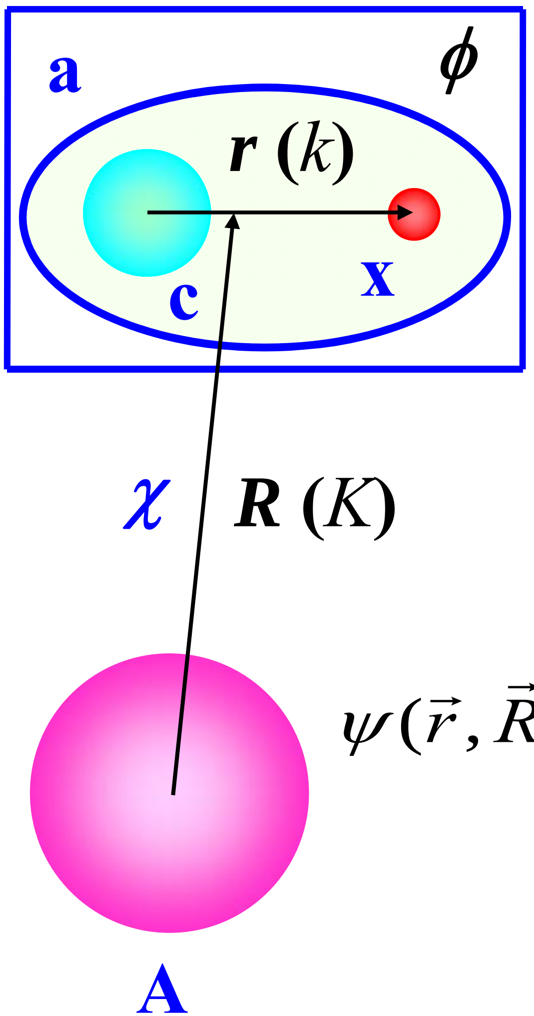

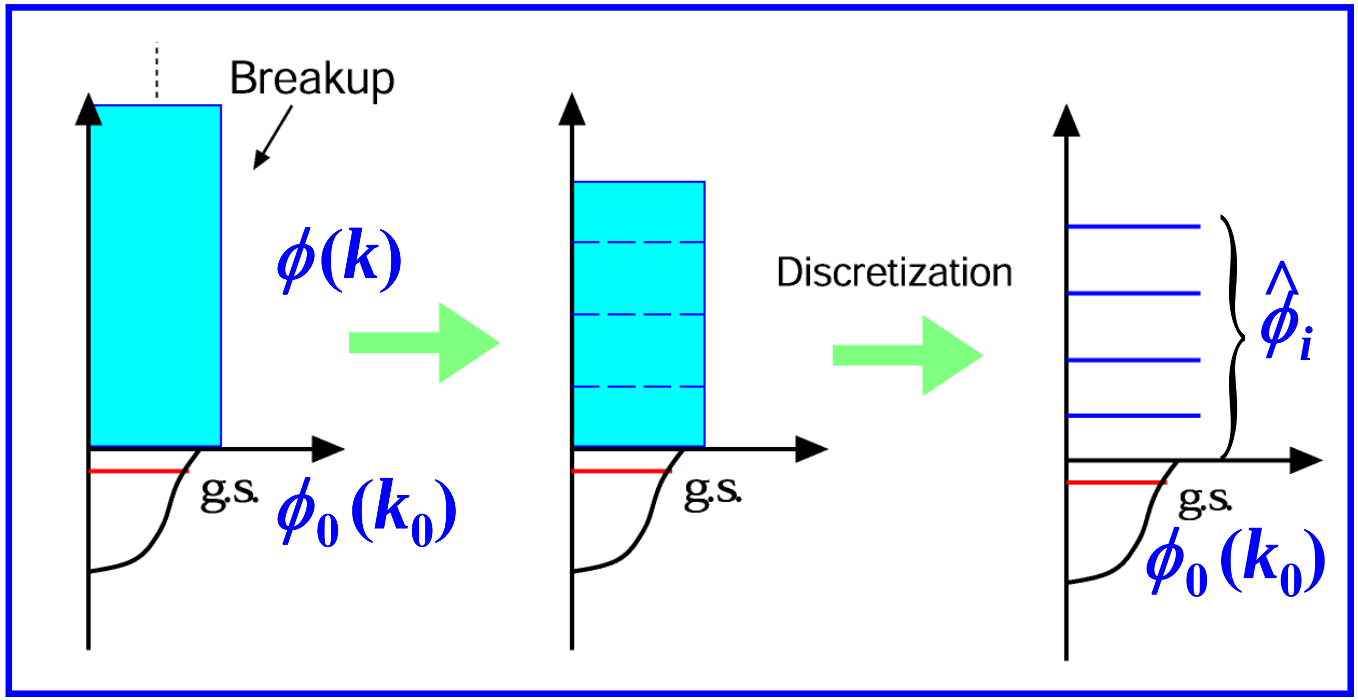

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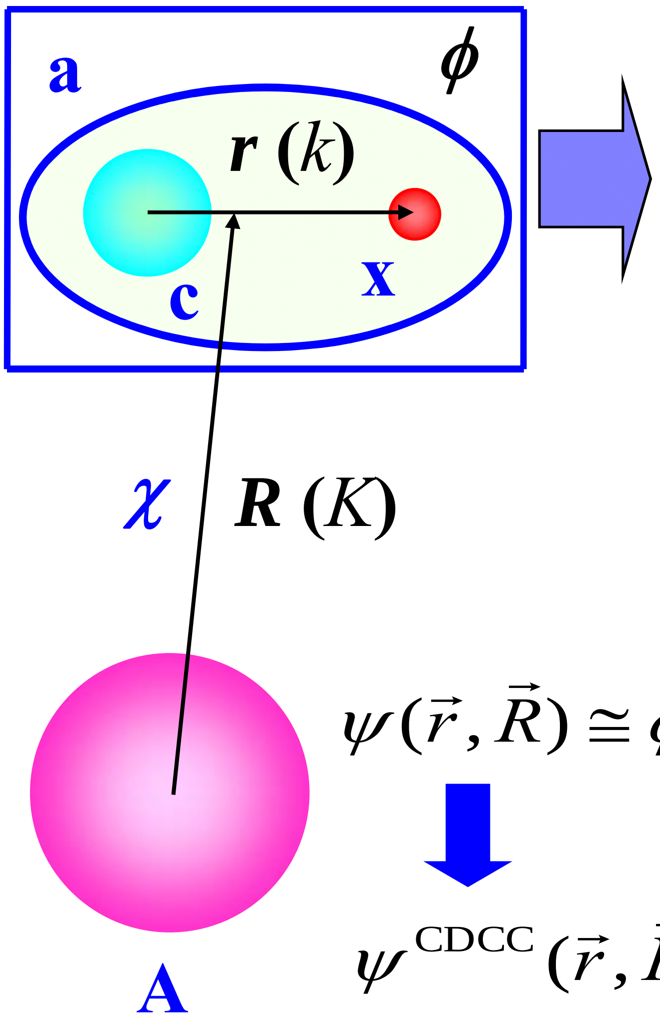

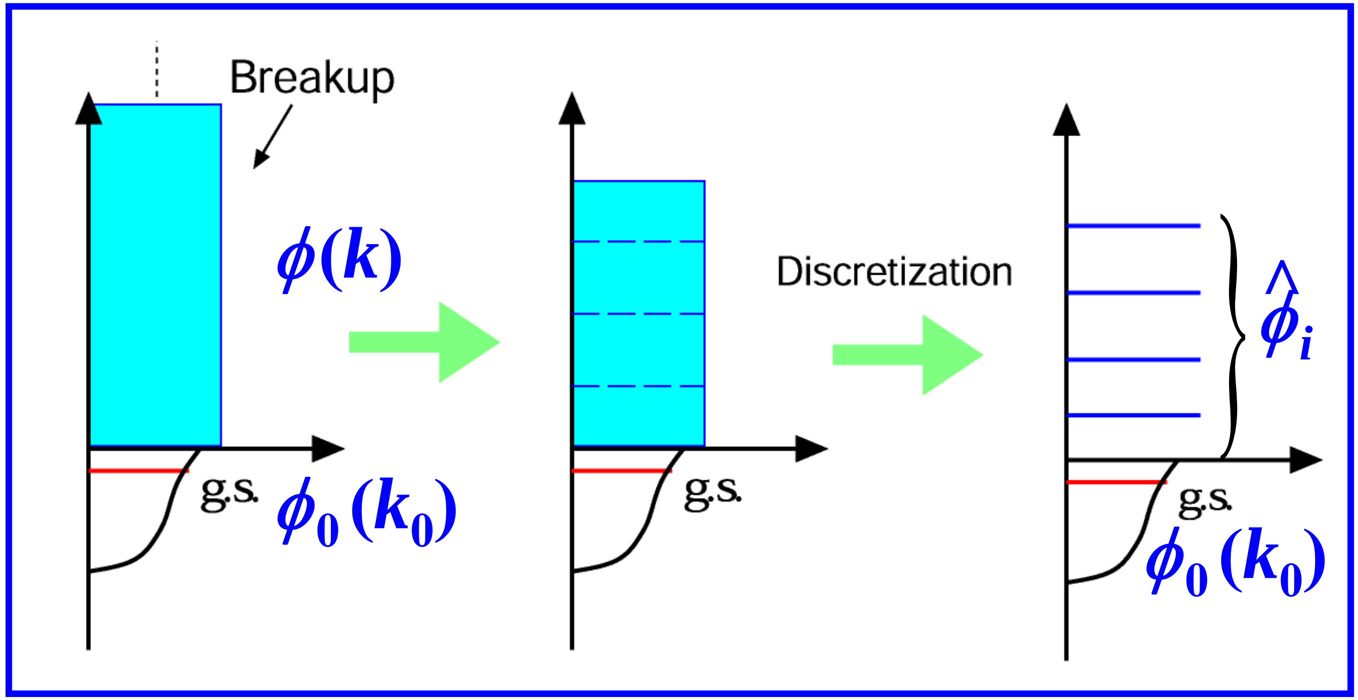

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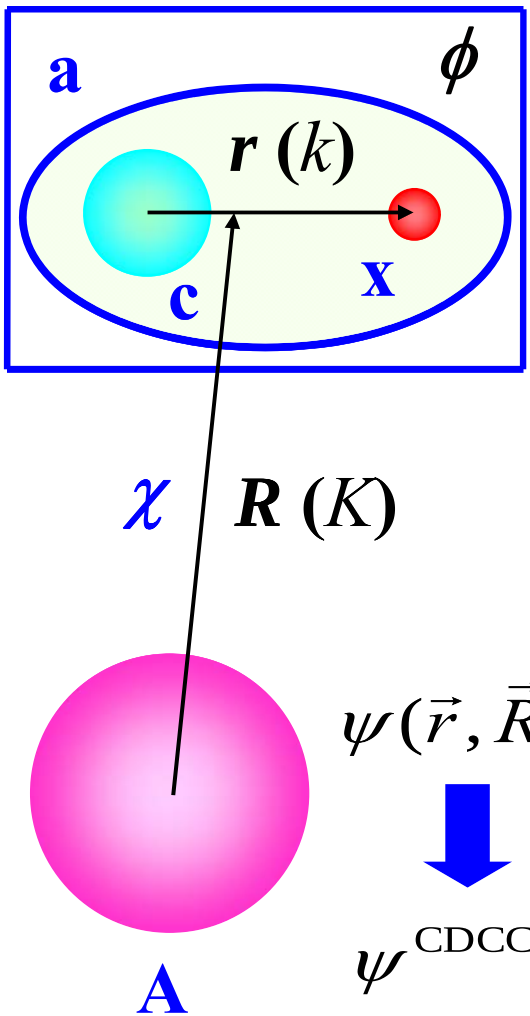
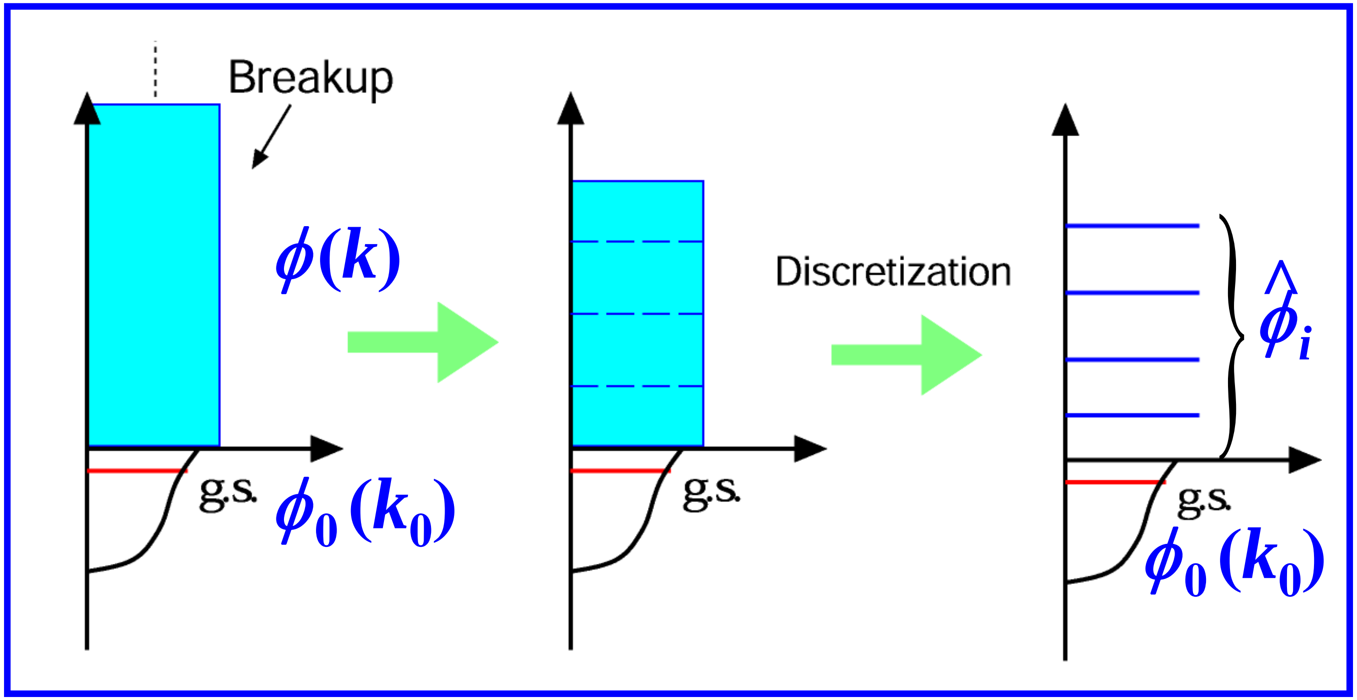
Truncation and Discretization

$$\psi^{\mathrm{CDCC}}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{\max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(K_i,\vec{R})$$

The Continuum-Discretized Coupled Channels method (CDCC)





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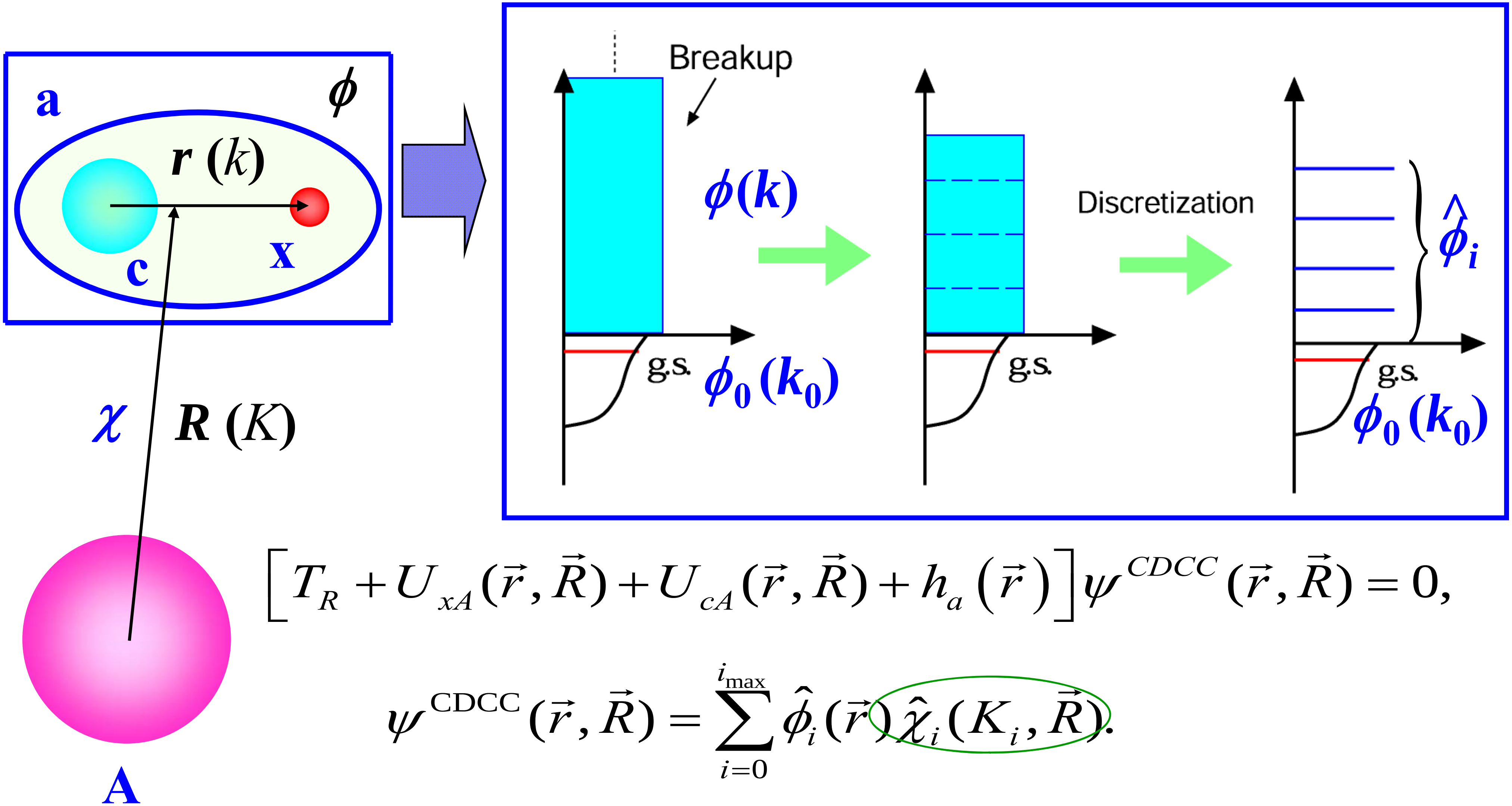
The Continuum-Discretized Coupled Channels method (CDCC)



$$\left[T_R + U_{xA}(\vec{r},\vec{R}) + U_{cA}(\vec{r},\vec{R}) + h_a\left(\vec{r}\right)\right]\psi^{CDCC}(\vec{r},\vec{R}) = 0,$$

$$\psi^{\mathrm{CDCC}}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{\max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(K_i,\vec{R}).$$

More about CDCC

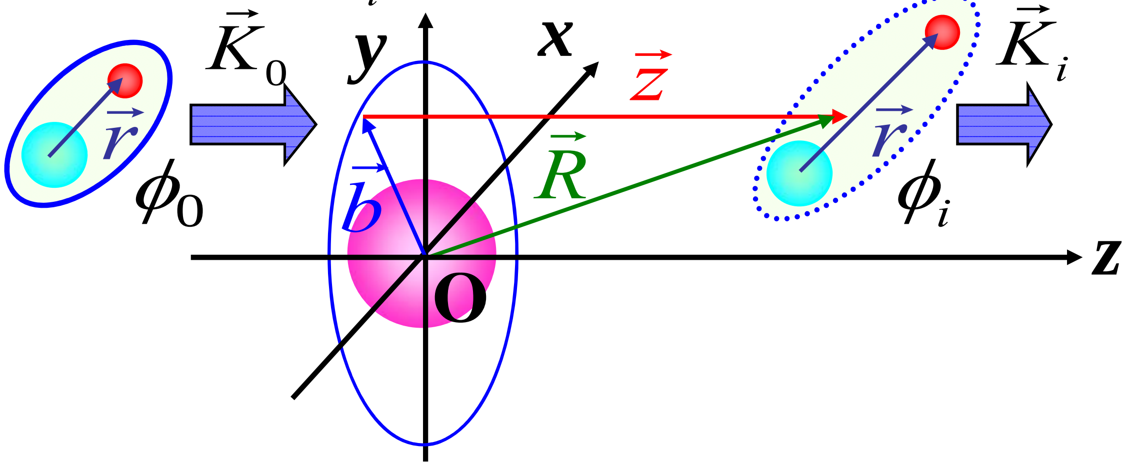
- **Review papers (*keystones*)**
  - ✓ *Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl.* ***89****, 1 (1986)*
  - ✓ *Austern, Iseri, Kamimura, Kawai, Rawitscher and Yahiro, Phys. Rep.* ***154*** *(1987) 126*
- Theoretical foundation based on the distorted-wave Faddeev formalism
  - ✓ *Austern, Yahiro and Kawai, PRL* ***63****, 2649 (1989)*
  - ✓ *Austern, Kawai and Yahiro, PRC* ***53****, 394 (1996)*
- **Nuclear BU** (real or virtual)**:**
  - ✓ ^{58}Ni($\vec{d}$,d) at 56, 80, 200, 400, 700 MeV; ^{208}Pb($\vec{d}$,d) at 56 MeV
  - ✓ (d,pn) on ^{12}C, ^{51}V, ^{118}Sn at 56 MeV; (^{3}He,dp) on ^{12}C, ^{51}V, ^{90}Zr at 90 MeV (^{3}He,dp) on ^{12}C, ^{28}Si, ^{58}Ni at 52 MeV
  - ✓ Elastic, inelastic, and BU processes of ^{6}Li and ^{7}Li on various stable nuclei.
- **Coulomb (and nuclear) BU:**
  - ✓ ^{208}Pb(^{6}Li,αd) at 156 MeV: *Hirabayashi and Sakuragi, PRL****69****, 1892 (1992)*
  - ✓ ^{208}Pb(^{8}B,p^{7}Be) at 44 and 83 A MeV, and ^{58}Ni(^{8}B,p^{7}Be) at 26 MeV: *Davids et al., PRC****63****, 065806 (2001),* *Tostevin et al., PRC****63,*** *024617 (2001)*

High Energies: Eikonal-CDCC

- **Eikonal approximation for scattering wave functions $\hat{\chi}_i(K_i, \vec{R})$:**

$$\psi^{\text{E-CDCC}} = \sum_i \hat{\phi}_i(\vec{r})\ \hat{\chi}_i(b,z)\exp(i\vec{K}_i\cdot\vec{R}),\ \ K_i = \sqrt{2\mu_R(E-\varepsilon_i)}/\hbar,$$

Energy conservation



$\Delta E << E$, $\theta << 1$ radian,
$|\nabla^2 \chi / \chi|_{\Delta r=\lambda} << 1$

Eikonal waves

$$\frac{i\hbar^2 K_i}{\mu_R}\frac{d}{dz}\hat{\chi}_i^{(b)}(z) = \sum_{i'} F_{ii'}^{(b)}(z)\ \hat{\chi}_{i'}^{(b)}(z)e^{i(K_{i'}-K_i)z}$$

- Boundary condition for $\hat{\chi}_i$

$$\hat{\chi}_i(b,z) \xrightarrow{z\to-\infty} \delta_{i,0}$$

$$F_{c'c}^{(b)}(Z) = \left\langle \Phi_{c'} \middle| U_{1A} + U_{2A} \middle| \Phi_c \right\rangle_{\mathbf{r}} e^{-i(m-m')\phi} = \sum_\lambda F_{c'c}^{(b);\lambda}(Z)$$

Inclusion of Coulomb distortion

- The total wave function is expanded as:

$$\psi^{\mathrm{E}} = \sum_c \phi_c(\vec{r})\ e^{-i(m-m_0)\varphi_R}\chi_c(b,z)\varphi_c^{\mathrm{C}},$$

where φ_c^{C} is the Coulomb wave function:

$$\varphi_c^{\mathrm{C}} = \underbrace{\frac{e^{-\pi\eta/2}}{(2\pi)^{3/2}}\Gamma(1+i\eta)}_{\equiv C} e^{i\vec{K}_c\cdot\vec{R}} F(-i\eta,1,i(K_c R-\vec{K}_c\cdot\vec{R})).$$

- Approximations:
  - $\nabla\varphi_c^{\mathrm{C}} \mathbin{//} \vec{z}$ for sufficiently large impact parameters
  - Local Semi-Classical Approximation to φ_c^{C}
  - Neglect of channel dependence of F

$$\frac{i\hbar^2}{\mu_R}\underline{K_c^{(b)}(z)}\frac{d}{dz}\chi_c^{(b)}(z) = \sum_{c'} F_{cc'}^{(b)}(z)\ \chi_{c'}^{(b)}(z)e^{i(K_{c'}-K_c)z}.$$

$$\boxed{\frac{\hbar^2}{2\mu_R}K_c^2(R) = E-\varepsilon_i-\frac{Z_{\mathrm{P}}Z_{\mathrm{T}}e^2}{R}.}$$

Hybrid CDCC

- **Scattering amplitude:** $f = f^{\mathrm{C}} + f^{\mathrm{N}}$, **where** f^{C} **is the Rutherford amplitude and**

$$f = \sum_L f_L \equiv \sum_L \frac{2\pi}{iK_i} \sqrt{\frac{2L+1}{4\pi}} i^m Y_{Lm}(\Omega)[S_{i,0}^{b(L)} - \delta_{i,0}]$$

- **Hybrid scattering amplitude is given by**

$$f_{i,0}^{\mathrm{H}} \equiv \sum_{L=0}^{L_{\mathrm{C}}} f_L + \sum_{L=L_{\mathrm{C}}+1}^{L_{\max}} f_L^{\mathrm{E}}$$

- S-matrix

$$\hat{\chi}_i(b(L), z) \xrightarrow{z \to \infty} S_{i,0}^{b(L)}$$

- **the use of f_L^{E} is made only for sufficiently large *L***
- **hybrid method suitable for intermediate energies**

Ogata, Yahiro, Iseri, Matsumoto, Kamimura, PRC68, 064609 (2003)

Relativity

Relativity and strong interaction of composite particles

Ex: Dirac particle (proton) + nucleus

- meson exchange, two-nucleon interaction
- mean field approximation
- U_0 (ω exchange), U_S (2π exchange)

$$\left[E - V_C - U_0 - \beta\left(mc^2 + U_S\right)\right]\Psi = -i\hbar c\boldsymbol{\alpha}\cdot\nabla\Psi$$

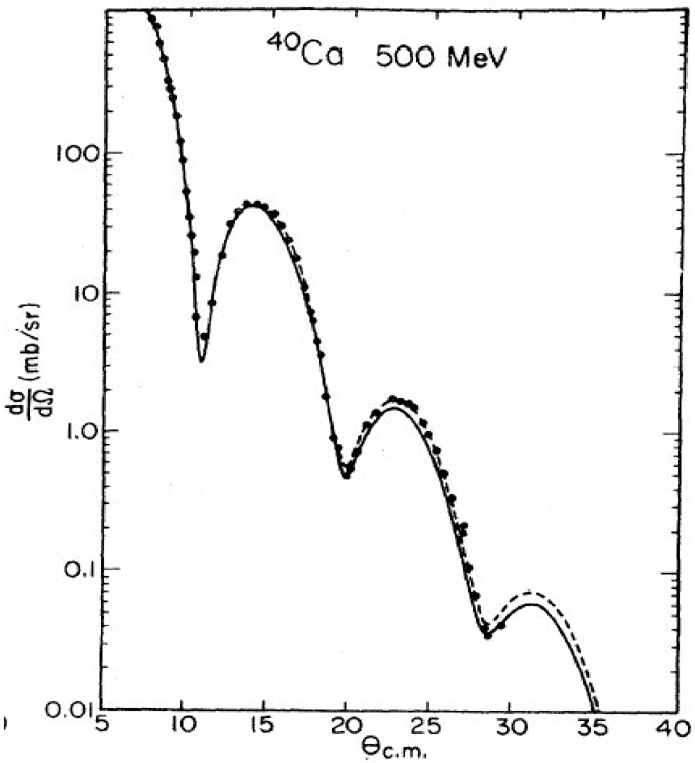
non-relativistic reduction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{cent} + \left(\frac{\hbar}{2mc}\right)^2 \frac{1}{r}\frac{d}{dr}U_{SO}\,\boldsymbol{\sigma}\cdot\mathbf{L}\right]\phi = E\phi$$

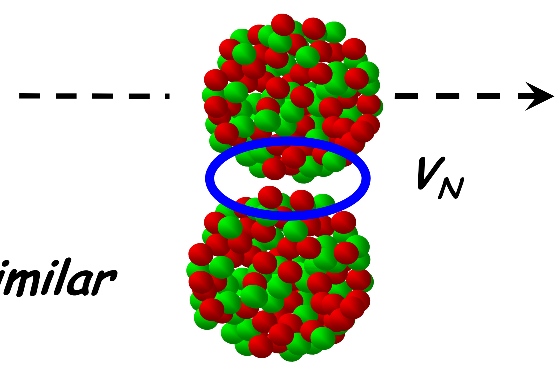
$$U_{cent} = m^*\left(U_0 + U_S\right) + \cdots$$

$$m^* = 1 - \frac{U_0 - U_S}{2mc^2} + \cdots$$

$$U_{SO} = U_0 - U_S + \cdots$$



Ray, Hoffmann, PRC 31, 538 (1985)

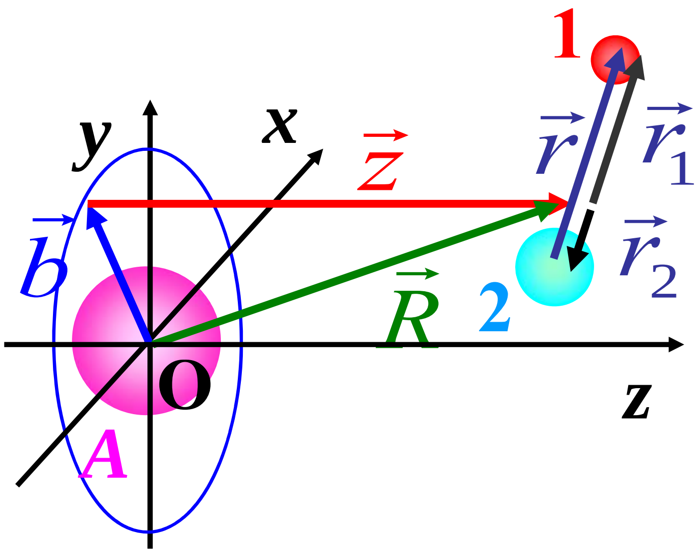


- ***For 2 colliding nuclei, similar theory does not exist***

Relativity in peripheral nuclear collisions

- **<u>Kinematical correction</u>:**
  - ✓ Usually only this correction is included in existing codes.

- **<u>Dynamical correction</u>:**
  - ✓ Based on (relativistic) Lienard-Wiechert expression of the EM field:



$$V_{\rm em}^{i}\left(\mathbf{R},\mathbf{r}_i\right)=Z_iZ_Ae^2\left(\phi^i-\frac{\mathbf{v}}{2mc^2}\left[\hat{\mathbf{p}}\phi^i+\phi^i\hat{\mathbf{p}}\right]\right),$$

$$\phi^i=\frac{\gamma}{\left[\left(\mathbf{r}_{i\perp}-\mathbf{R}_\perp\right)^2+\gamma^2\left(z_i-Z\right)^2\right]^{1/2}},$$

$$\gamma=\left[1-\left(\frac{v}{c}\right)^2\right]^{-1/2}.$$

*Winther , Alder, NP**A319**, 518 (1979)*
*Esbensen and Bertulani, PRC**65**, 024605 (2002)*
*Bertulani, PRL**94**, 072701 (2005).*

Relativistic E1 and E2 interactions

$$V_{E1,\mu} = \sqrt{\frac{2\pi}{3}} r Y_{1\mu}(\hat{\mathbf{r}}) \frac{\gamma e Z_A e_{eff}^{(1)}}{\left(b^2 + \gamma^2 Z^2\right)^{3/2}} \begin{cases} \mp b & (\mu = \pm 1) \\ \sqrt{2} Z & (\mu = 0) \end{cases}$$

$$V_{E2,\mu} = \sqrt{\frac{3\pi}{10}} r^2 Y_{2\mu}(\hat{\mathbf{r}}) \frac{\gamma e Z_A e_{eff}^{(2)}}{\left(b^2 + \gamma^2 Z^2\right)^{5/2}} \begin{cases} b^2 & (\mu = \pm 2) \\ \mp 2\gamma^2 b Z & (\mu = \pm 1) \\ \sqrt{2/3}\left(2\gamma^2 Z^2 - b^2\right) & (\mu = 0) \end{cases}$$

*Bertulani, PRL**94**, 072701 (2005)*

- **<u>Inclusion of such potential with Lorentz contraction</u>:**
  - ✓ Easy in cylindrical coordinate system (classical calculation)
  - ✓ Difficult in spherical coordinate system (QM calculation)

*c.f. Esbensen and Bertulani, PRC**65**, 024605 (2002)*

Relativistic E-CDCC

- **Form factor of non-rel. E-CDCC:**

$$F_{c'c}^{(b)}(Z) = \left\langle \Phi_{c'} \left| U_{1A} + U_{2A} \right| \Phi_c \right\rangle_{\mathbf{r}} e^{-i(m-m')\phi} = \sum_{\lambda} F_{c'c}^{(b);\lambda}(Z)$$

- **Rel. corr. to the form factor:**

$$F_{c'c}^{(b);\lambda}(Z) \to f_{\lambda,m'-m} \gamma F_{c'c}^{(b)\lambda}(\gamma Z)$$

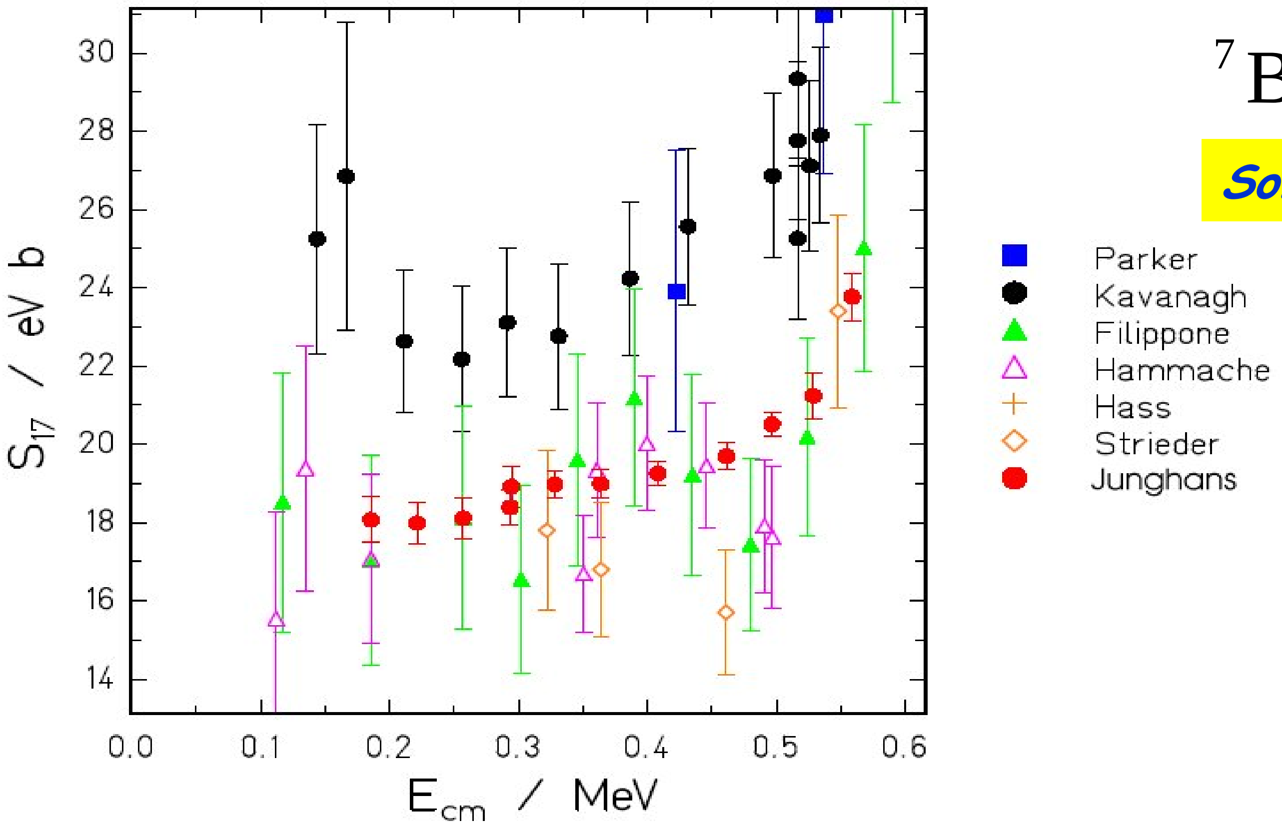
$$f_{\lambda,m'-m}^{\text{Coul}} = \begin{cases} 1/\gamma & (\lambda=1,\ m'-m=0) \\ \gamma & (\lambda=2,\ m'-m=\pm 1) \\ 1 & (\text{otherwise}) \end{cases} \qquad f_{\lambda,m'-m}^{\text{nucl}} = 1$$

- **Assumptions:**
  - ✓ Point charges for 1, 2 and A
  - ✓ Neglecting far-field ($r_i > R$) contribution
  - ✓ Correction to nuclear form factor —

Feshbach and Zabek,
Ann. of Phys. **107** (1977) 110

Numerical Results

Applications: (a) proton halo ^{8}B



$$^{7}\mathrm{Be}(\mathrm{p},\gamma)^{8}\mathrm{B}$$

Solar neutrinos and ν-oscillations

(b) neutron halo: ^{11}Be

Numerical Calculations

- **Reaction:**

  ^{208}Pb(^{8}B, ^{7}Be+p) at 250 A MeV and 100 A MeV

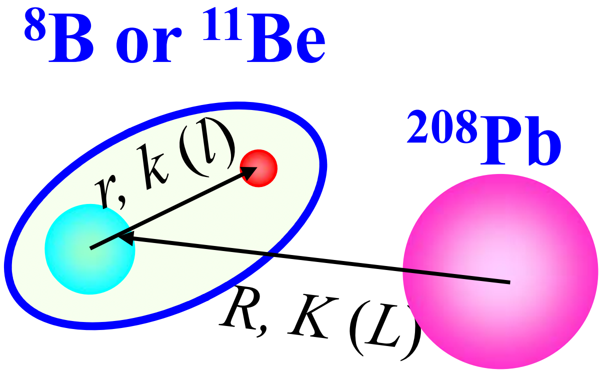
  ^{208}Pb(^{11}Be, ^{10}Be+n) at 250 A MeV and 100 A MeV

- **Projectile wave function and distorting potential**

  M. S. Hussein *et al*., PL**B640**, 91 (2006). (standard Wodds-Saxon)

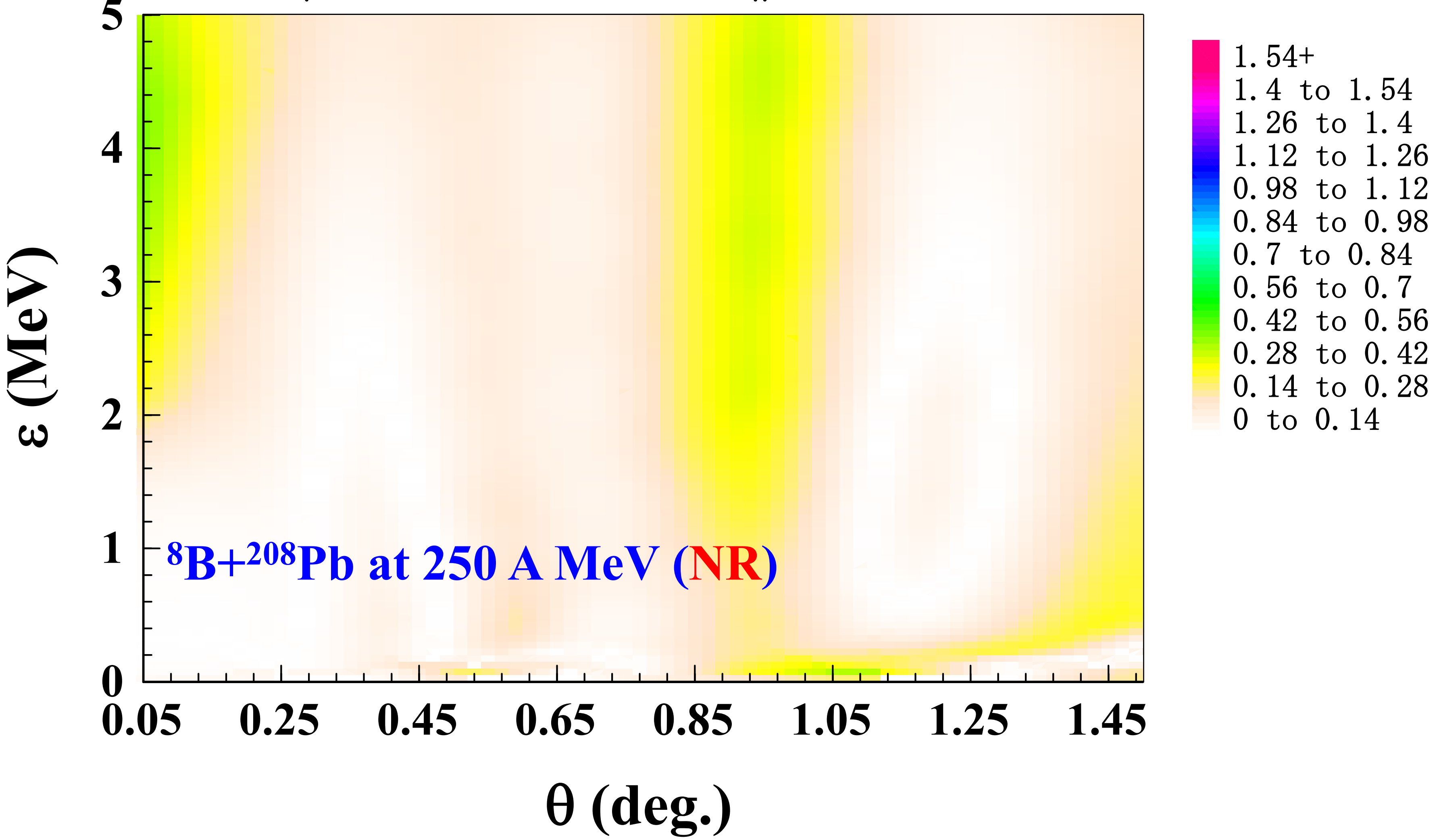
- **Modelspace:**

^{8}B	^{11}Be
l_{max}= 3	l_{max}= 3
N_s=20, $N_{p\text{-}d}$=10 , N_f=5	$N_{s,p}$=20, N_d=10 , N_f=5
ε_{max}=10 MeV	ε_{max}=10 MeV
r_{max}= 200 fm	r_{max}= 200 fm
R_{max}= 500 fm	R_{max}= 450 fm
N_{ch} = 138	N_{ch} = 166

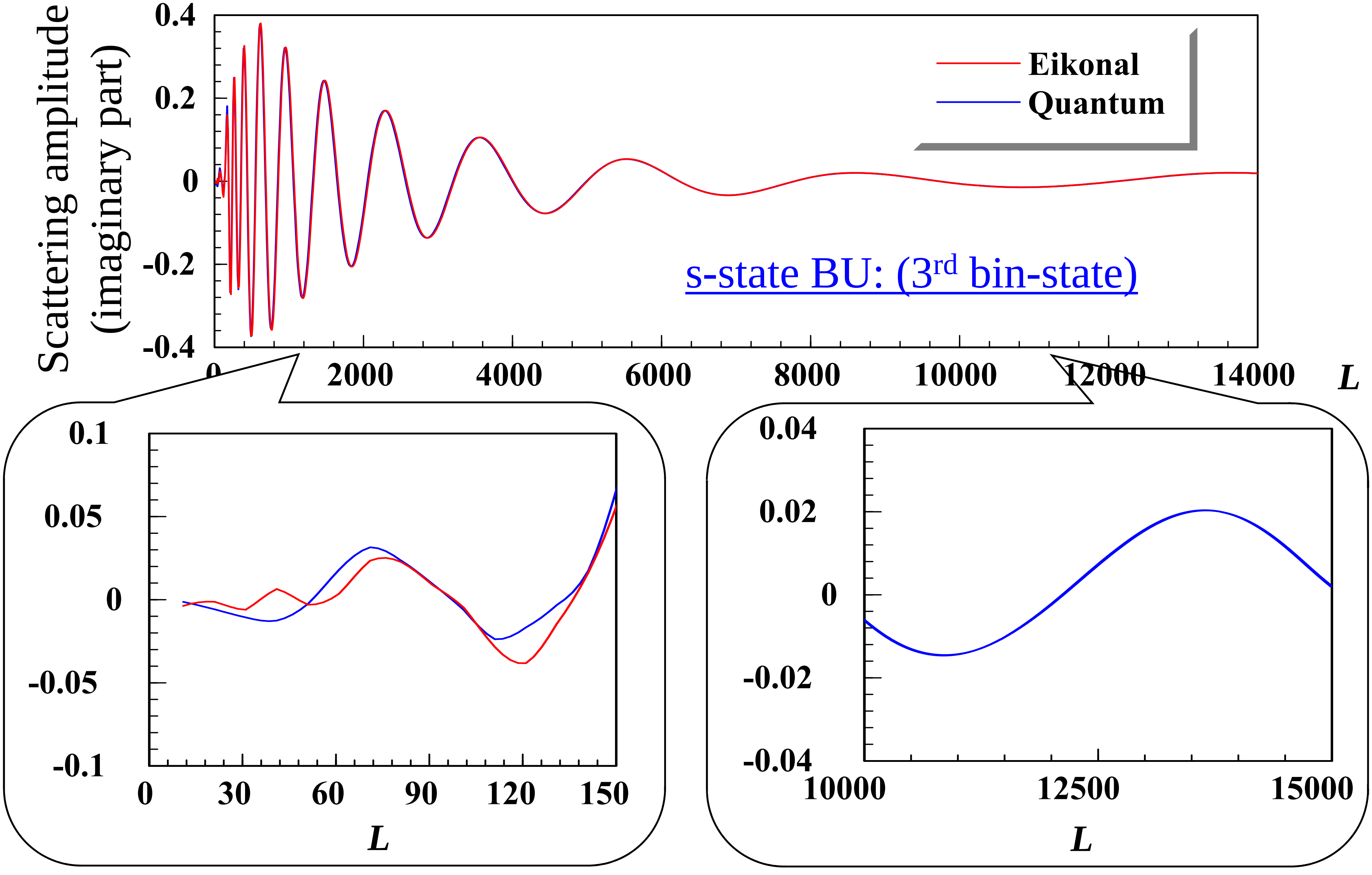


Relative difference of the QM and EK

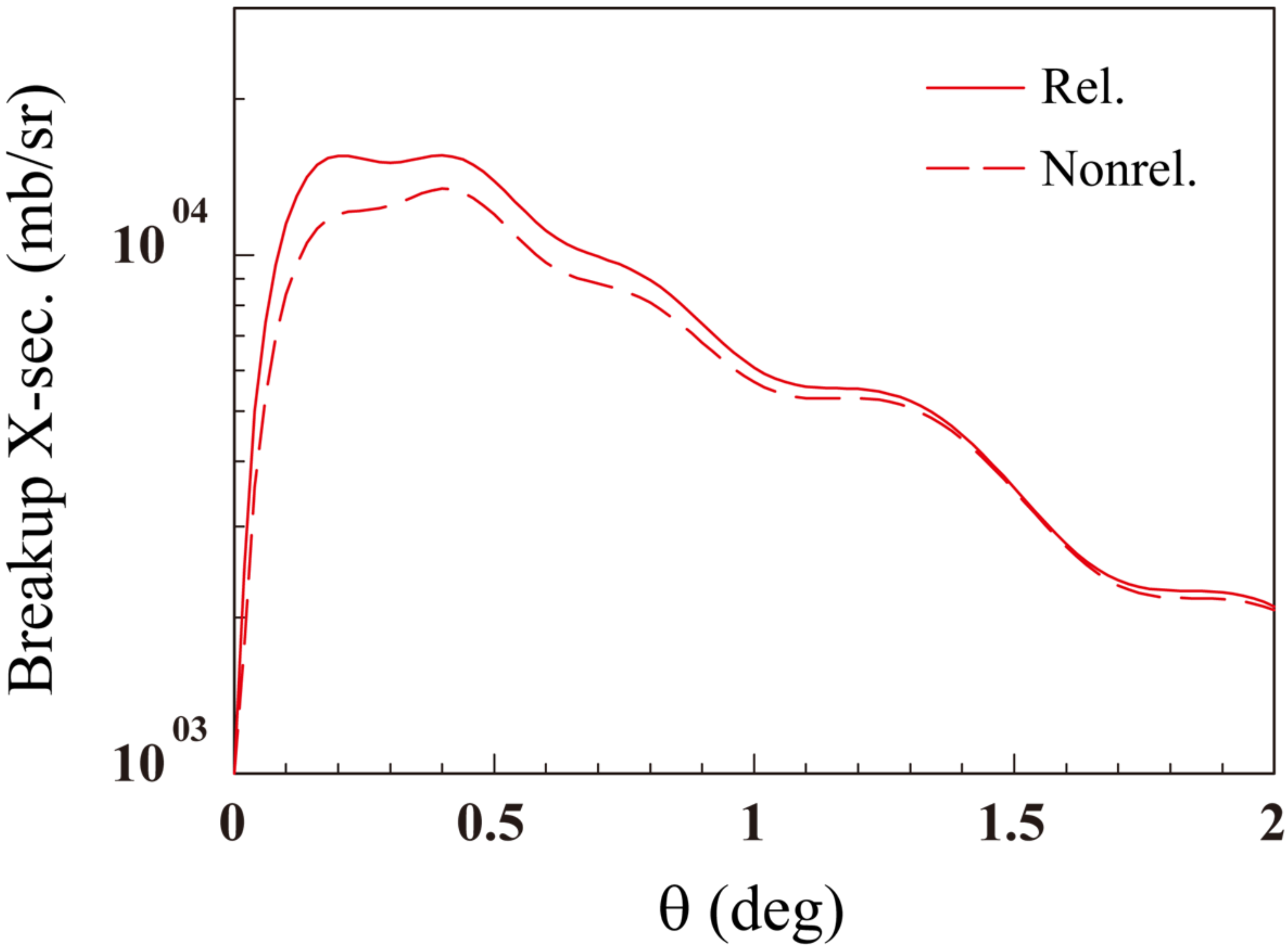
$$\left| \frac{d^2\sigma_{BU}^{QM}}{d\Omega d\varepsilon} - \frac{d^2\sigma_{BU}^{EK}}{d\Omega d\varepsilon} \right| \Big/ \frac{d^2\sigma_{BU}^{EK}}{d\Omega d\varepsilon}$$



Comparison between f^{E} and f^{Q}



^{8}B breakup by ^{208}Pb at 250 A MeV



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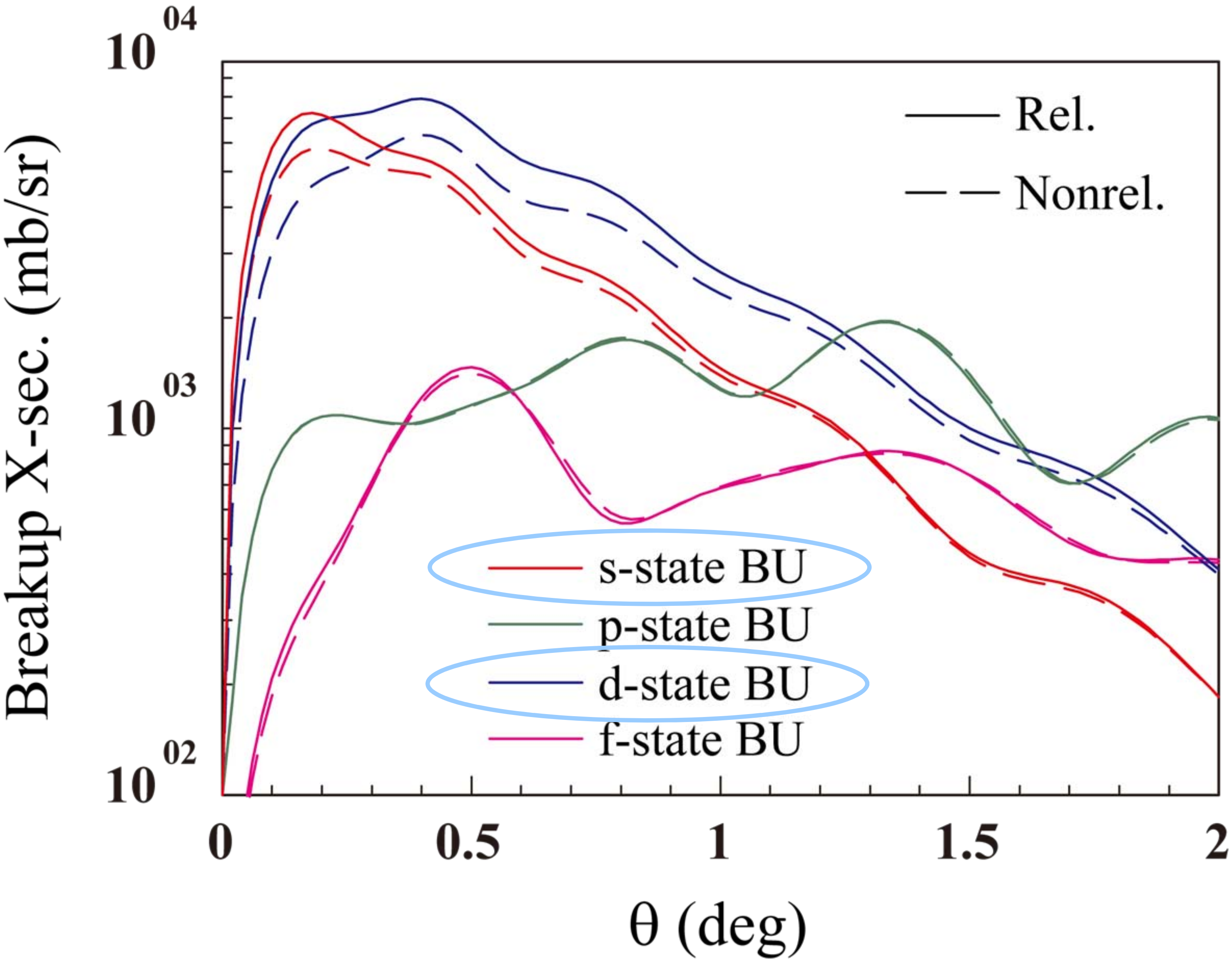
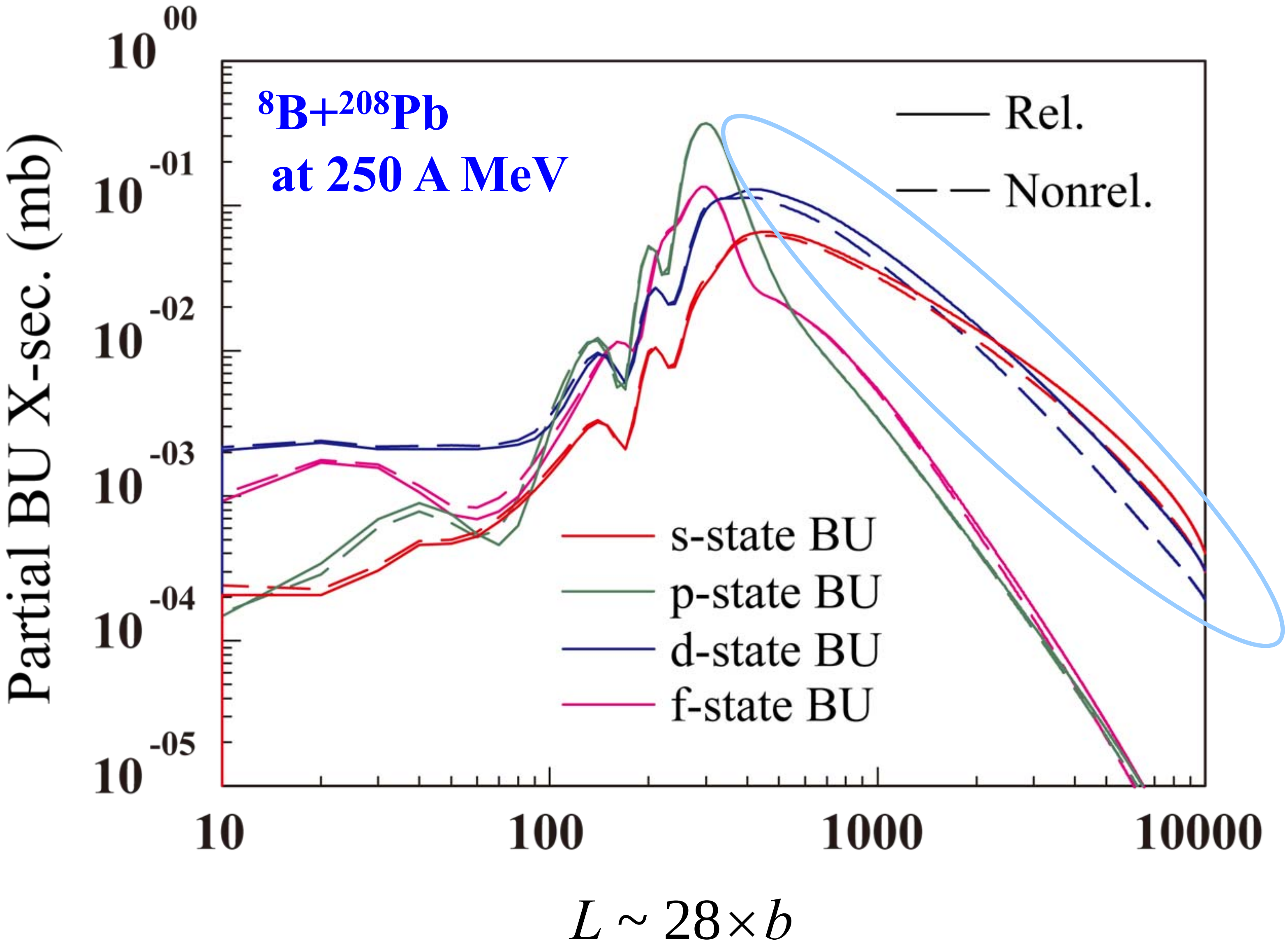
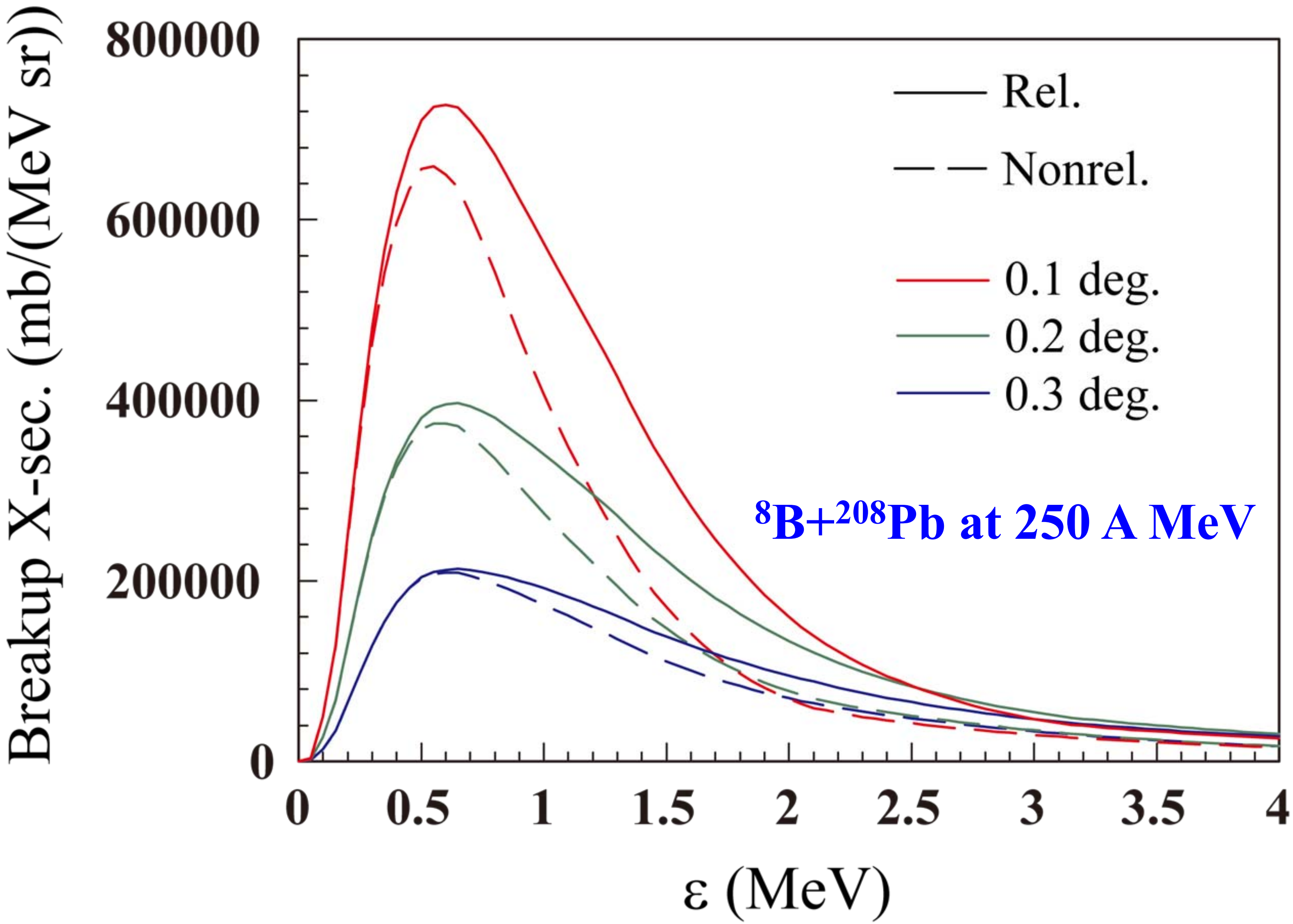


Partial breakup X-sec.

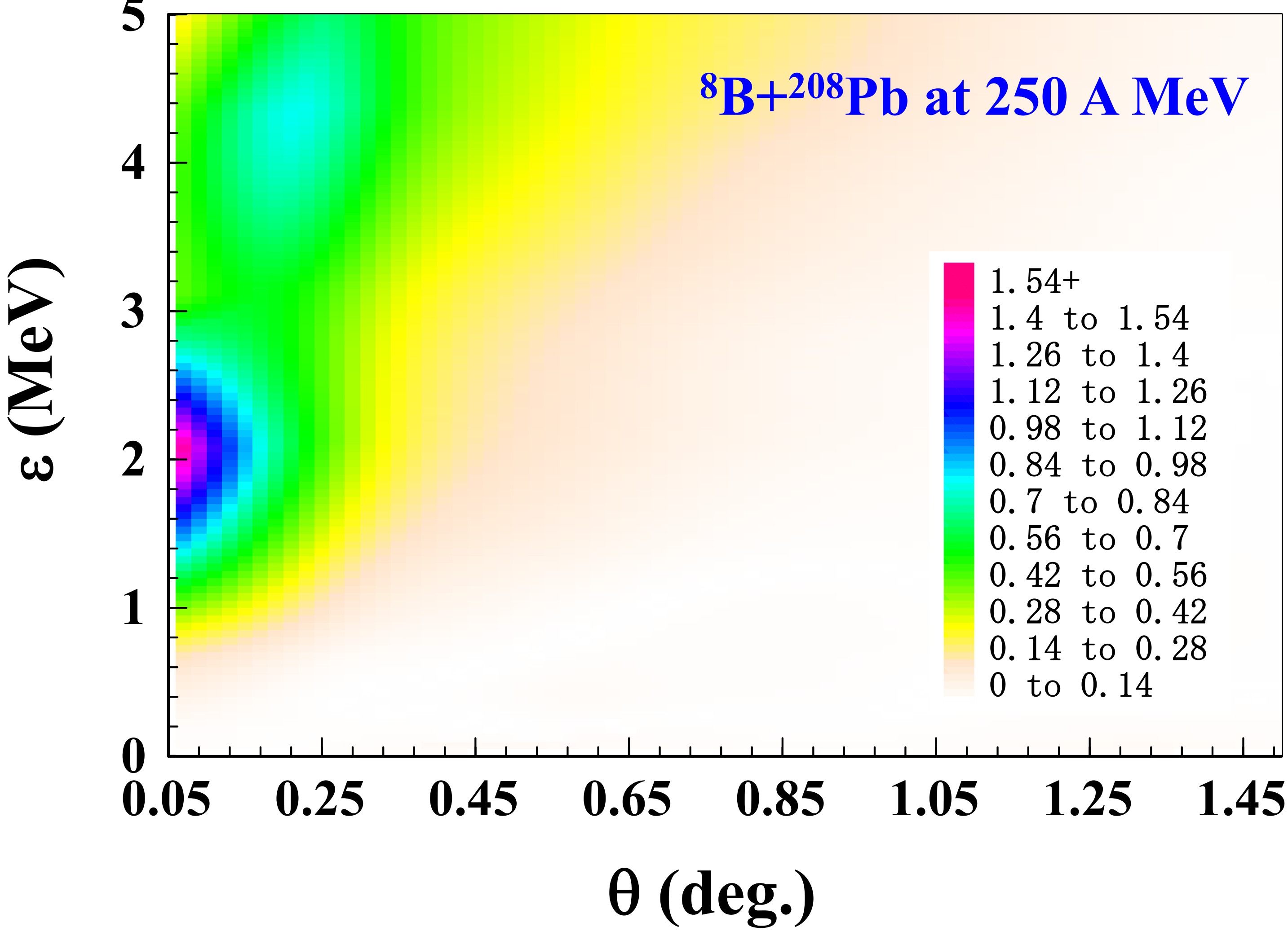


BU energy spectra

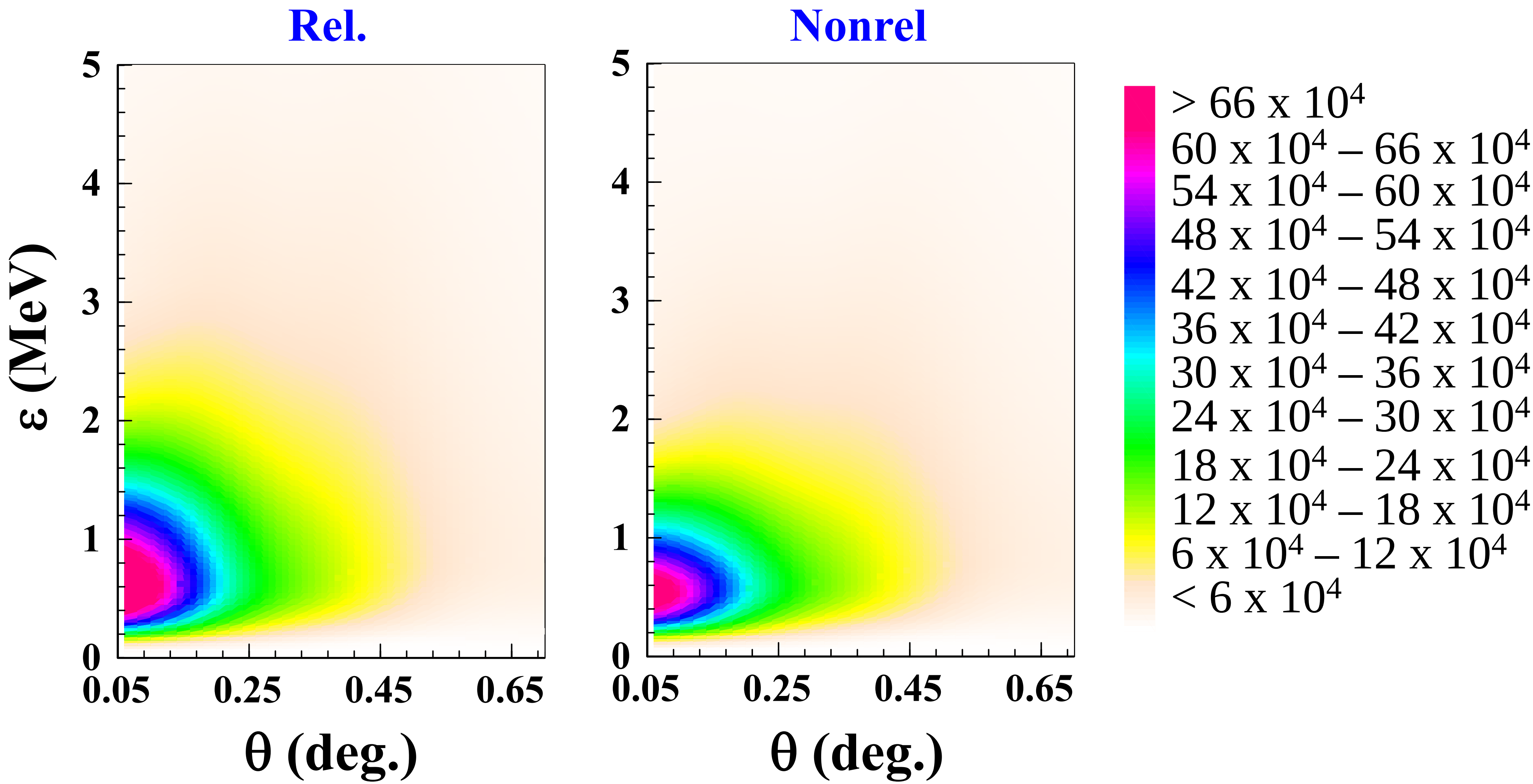


Relative difference

$$\left| \frac{d^2\sigma_{\rm BU}^{\rm Rel}}{d\Omega d\varepsilon} - \frac{d^2\sigma_{\rm BU}^{\rm Non\text{-}rel}}{d\Omega d\varepsilon} \right| \Big/ \frac{d^2\sigma_{\rm BU}^{\rm Non\text{-}rel}}{d\Omega d\varepsilon}$$



Comparison of the DDX



First-order eikonal aproximation

$$\psi_c^{(b)}(z) = e^{\frac{1}{i\hbar v_c}\int_{-\infty}^{z} F'^{(b)}_{cc}(Z')dZ'} \left[\frac{1}{i\hbar v_c} \int_{-\infty}^{z} \sum_{c' \neq c} \left\{ F'^{(b)}_{cc'}(Z') \psi_{c'}^{(b)}(z') e^{i\left(K_{c'}^{(b)} - K_c^{(b)}\right)Z'} \right\} \times e^{\frac{1}{i\hbar v_c}\int_{-\infty}^{z'} F'^{(b)}_{cc}(Z'')dZ''} dZ' + \delta_{c0} \right]$$

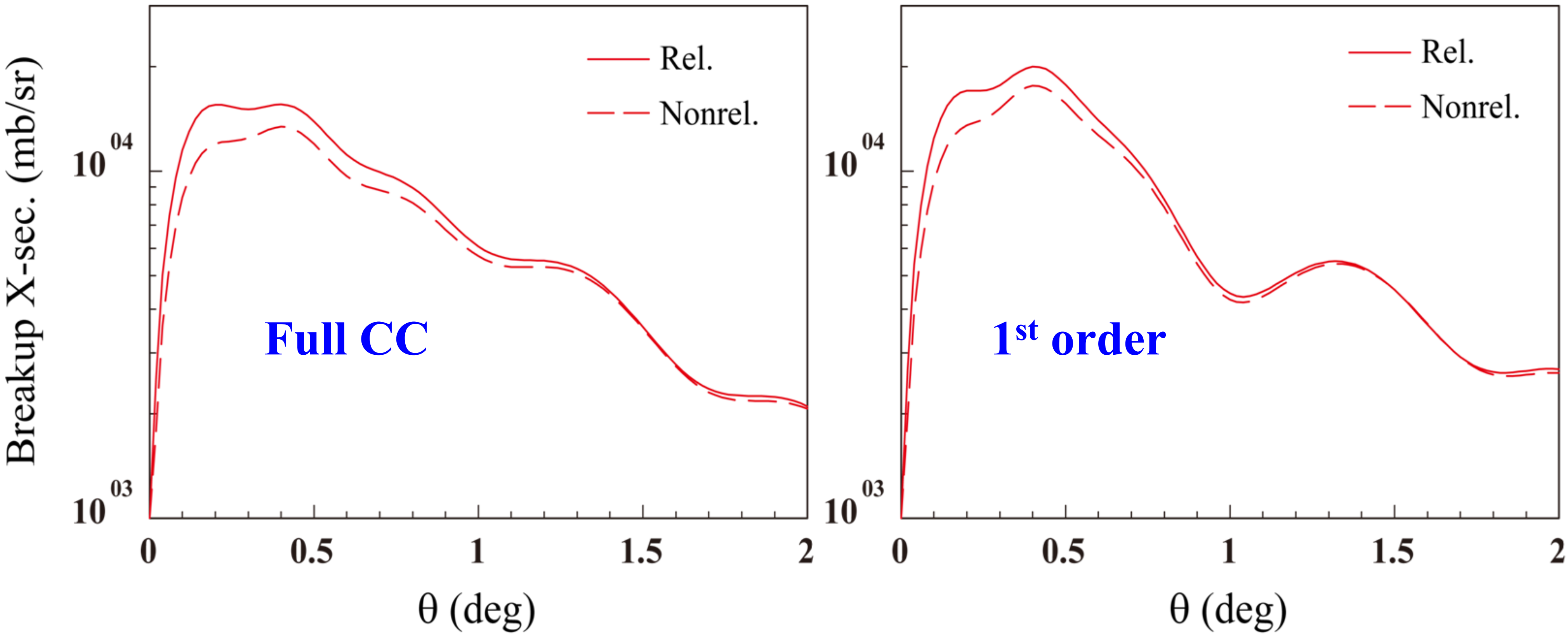
- **<u>S-matrix with 1st order calculation</u>:**
  - ✓ Elastic component

$$S_0^{(b)} \equiv \psi_0^{(b)}(\infty) = \exp\left[\frac{1}{i\hbar v_0} \int_{-\infty}^{z} F'^{(b)}_{00}(Z')dZ' \right]$$

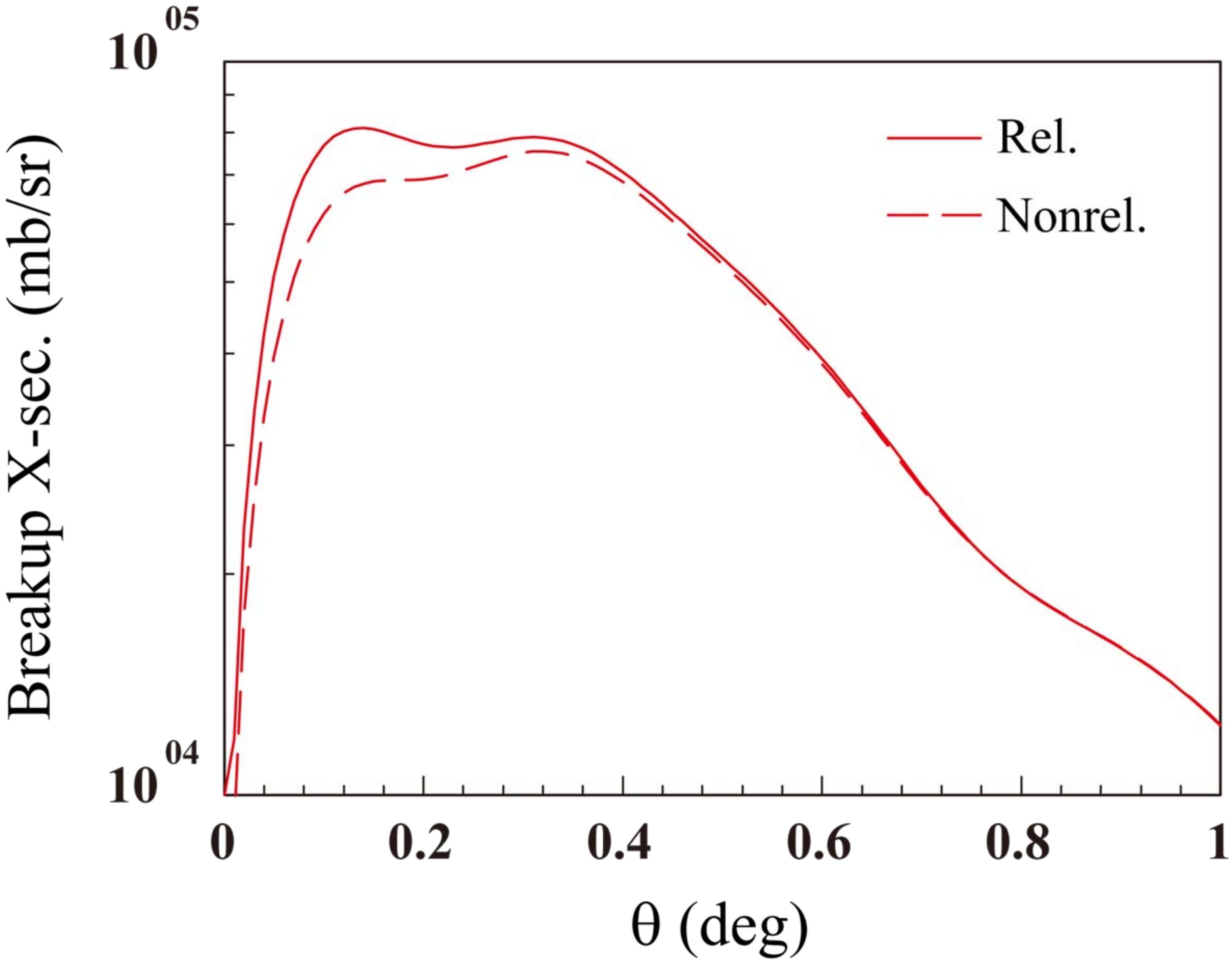
  - ✓ Non-elastic component

$$S_c^{(b)} = \frac{1}{i\hbar v_c} \int_{-\infty}^{\infty} F'^{(b)}_{c0}(Z') \psi_0^{(b)}(z') e^{i\left(K_0^{(b)} - K_c^{(b)}\right)Z'} dZ'$$

Full CC vs. 1st order perturbation

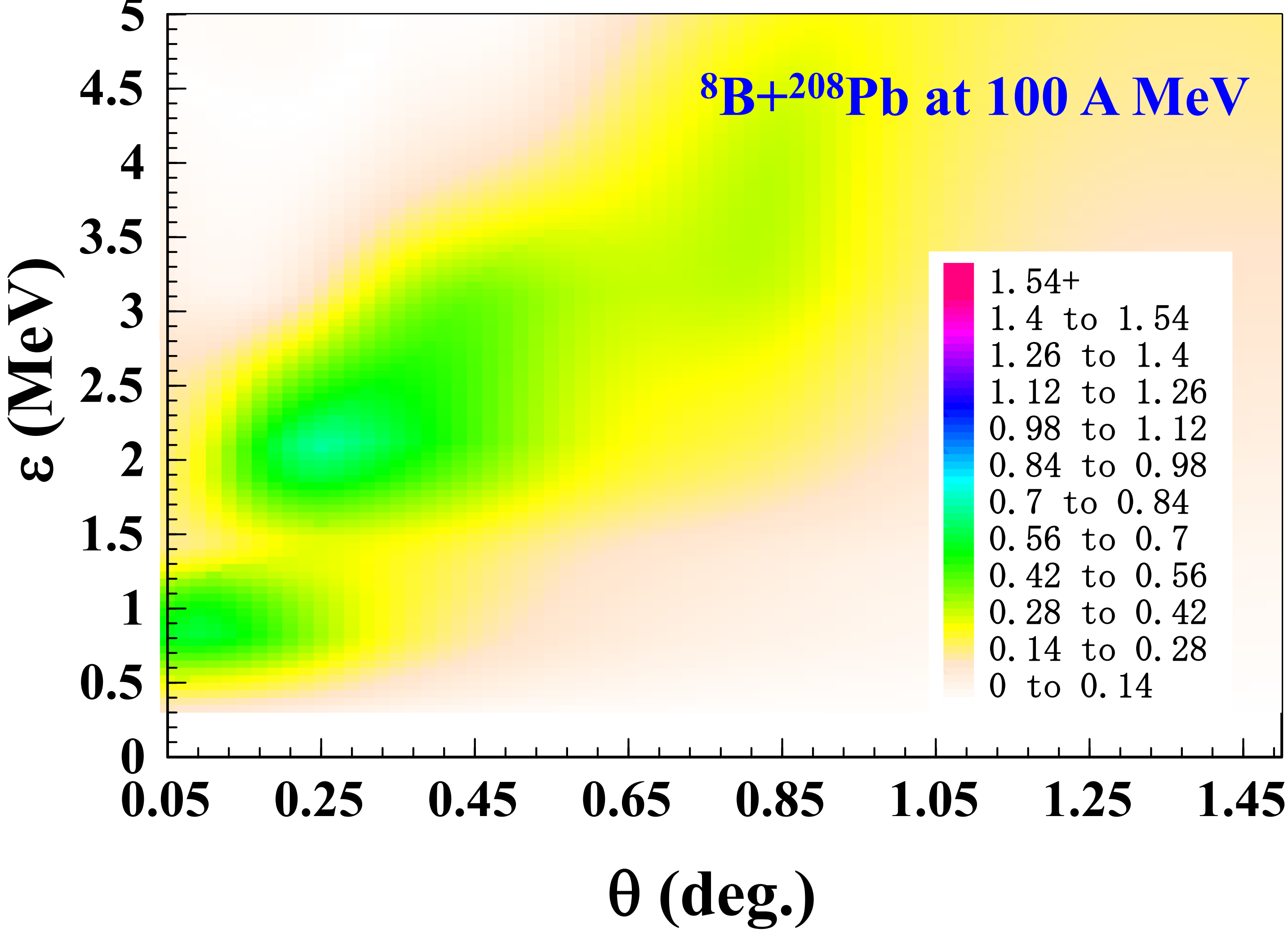


^{11}Be breakup by ^{208}Pb at 250 A MeV

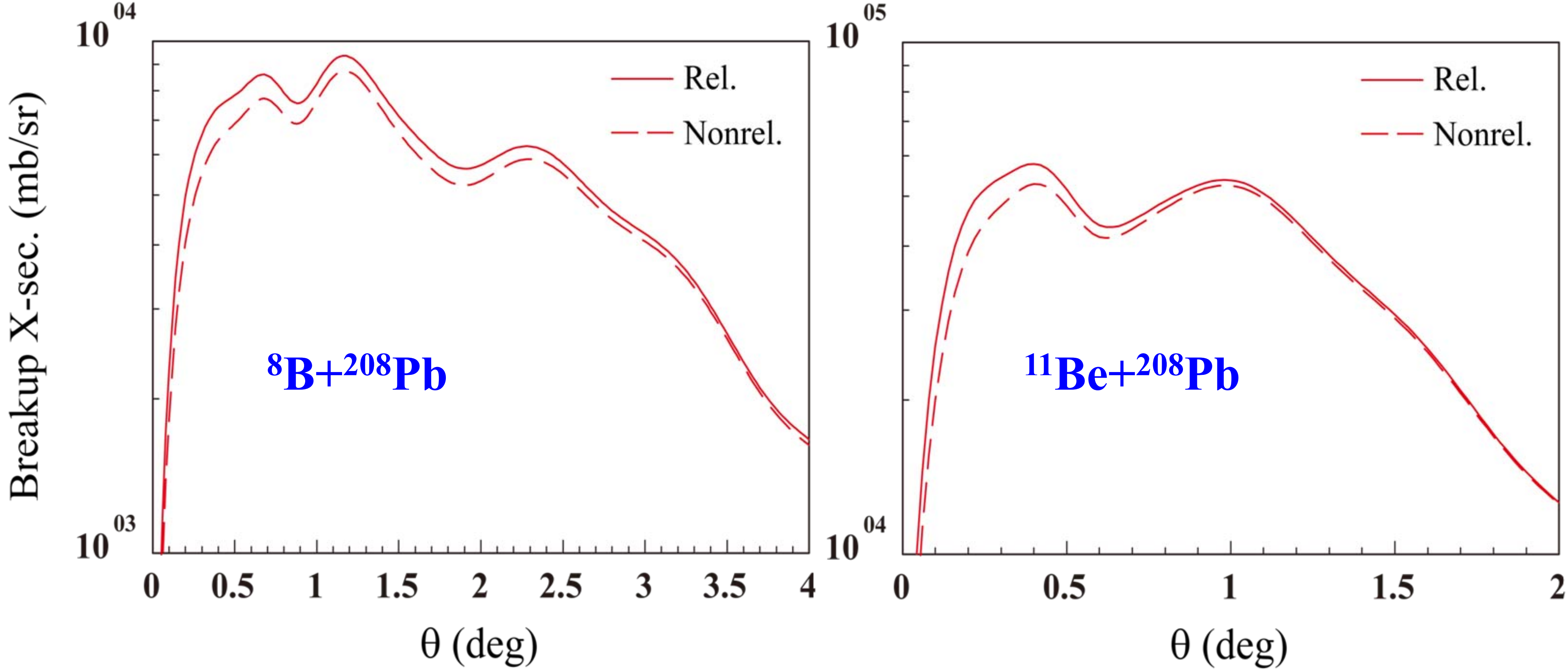


Relative difference

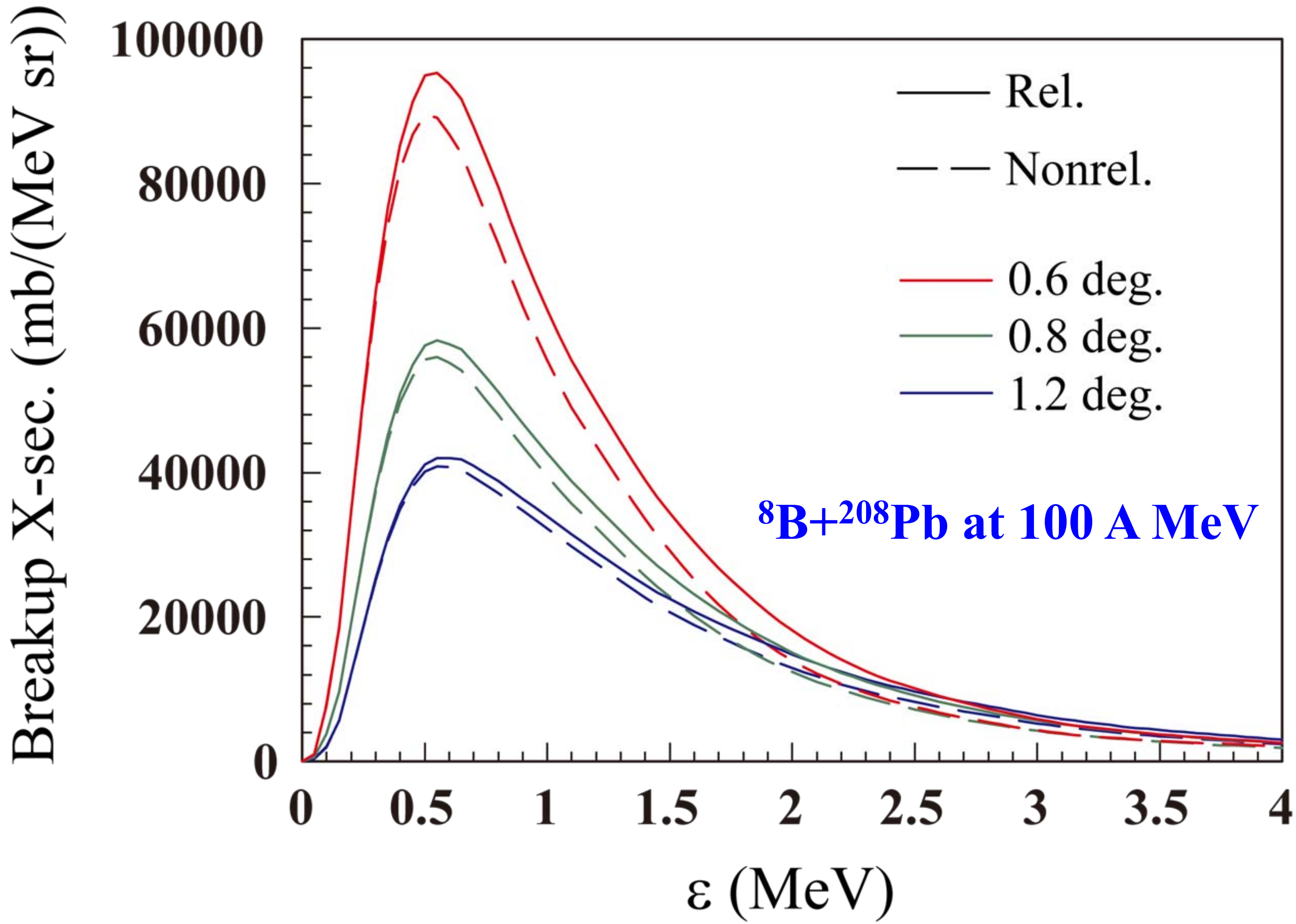
$$\left|\frac{d^2\sigma^{\text{Rel}}_{\text{BU}}}{d\Omega d\varepsilon} - \frac{d^2\sigma^{\text{Non-rel}}_{\text{BU}}}{d\Omega d\varepsilon}\right| \Big/ \frac{d^2\sigma^{\text{Non-rel}}_{\text{BU}}}{d\Omega d\varepsilon}$$



Results at 100 A MeV



BU energy spectra



Summary

- **Dynamical relativistic effect is evaluated for ^{8}B- and ^{11}Be-breakup reactions by ^{208}Pb at 250 and 100 A MeV by means of E-CDCC.**
  - ✓ Significant enhancement of the BU X-sec. of about 20 – 30 % is found at forward angles at 250 A MeV.
  - ✓ At 100 A MeV relativistic effects are found to be about 10%.

- **To draw a definite conclusion, at 100 A MeV in particular, the assumption used must be examined.**
  - ✓ Point charge, non-farside calc., correction to nuclear FF, QM effects etc.