

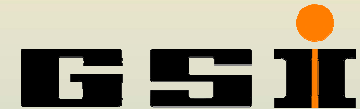
- Three-body correlations in ^{10}He and ^{13}Li decays



Kurchatov Institute

Unbound Nuclei Workshop, Pisa, 3-5 November 2008

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Gesellschaft
für Schwerionenforschung





Angular correlations

Two-body case

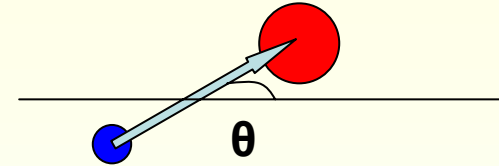
$$\hat{l}^2 = -\frac{1}{\sin \theta} \left\{ \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial^2}{\partial \varphi^2} \right\}$$

$$\hat{l}^2 Y_{lm}(\theta, \varphi) = l(l+1) Y_{lm}(\theta, \varphi)$$

$$\hat{H} = -\frac{\hbar^2}{2} \left[\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} l(l+1) \right]$$

centrifugal barrier

$$d\tau = \sqrt{E} dE \, d\Omega$$



Three-body case

$$\hat{K}^2(\Omega_\theta) = -\frac{\partial^2}{\partial \alpha^2} - 4 \cot 2\alpha \frac{\partial}{\partial \alpha} + \frac{1}{\cos^2 \alpha} \hat{l}^2(\Omega_x) + \frac{1}{\sin^2 \alpha} \hat{l}^2(\Omega_y)$$

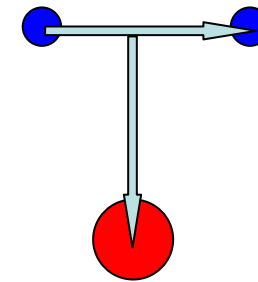
$$\rho^2 = r_x^2 + r_y^2 = \rho^2 (\sin^2 \alpha + \cos^2 \alpha); \quad \alpha = \arccos \frac{r_x}{\rho}$$

$$\mathbf{E} = \mathbf{E}_x + \mathbf{E}_y; \quad \alpha = \arccos \sqrt{\frac{E_x}{E}}; \quad \epsilon = \cos^2 \alpha = \frac{E_x}{E}$$

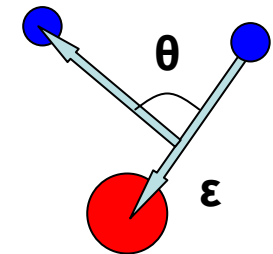
α - hyperangle

$$\hat{K}^2 \mathcal{Y}_K^{l_x, l_y, m_x, m_y}(\Omega_\theta) = K(K+4) \mathcal{Y}_K^{l_x, l_y, m_x, m_y}(\Omega_\theta)$$

three-body barrier



T system



Y system

$$\hat{H} = -\frac{\hbar^2}{2} \left[\frac{\partial^2}{\partial \rho^2} + \frac{5}{\rho} \frac{\partial}{\partial \rho} - \frac{1}{\rho^2} K(K+4) \right]$$

$$d\tau = E^2 dE \, \sqrt{\epsilon(1-\epsilon)} d\epsilon \, d\Omega_x \, d\Omega_y$$

Hyperspherical harmonics

$$\mathcal{Y}_K^{l_x, l_y, m_x, m_y}(\Omega_6) =$$

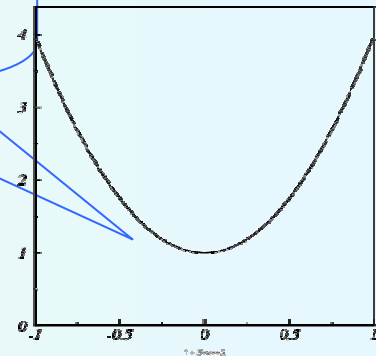
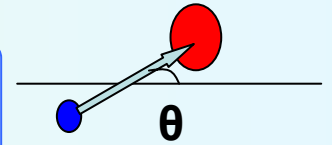
$$N_K^{l_x, l_y} \cos^{l_x}(\alpha) \sin^{l_y}(\alpha) P_n^{l_x+1/2, l_y+1/2}(\cos 2\alpha) Y_{l_x, m_x}(\Omega_x) Y_{l_y, m_y}(\Omega_y)$$

Quantum numbers: J^π , S , L , l_x , l_y , K - hypermomentum.

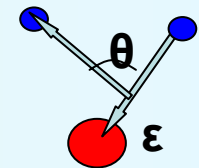
$$\mathcal{Y}_K^{l_x, l_y, L, M}(\Omega_6) = \sum_{m_x, m_y} \langle L, l_x, m_x, m_y | LM \rangle \mathcal{Y}_K^{l_x, l_y, m_x, m_y}(\Omega_6)$$

Transformation from T-system to Y-system
Reinal-Revai. coefficients

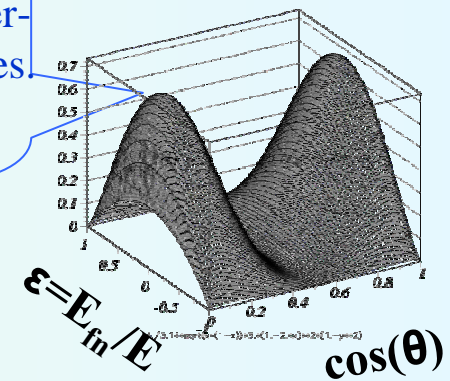
Expansion of a decay amplitude into the spherical-harmonic series
 Requires polarization.



Polarized $p_{3/2}$ shell



Expansion of a decay amplitude into the hyperspherical-harmonic series
 Internal correlations.



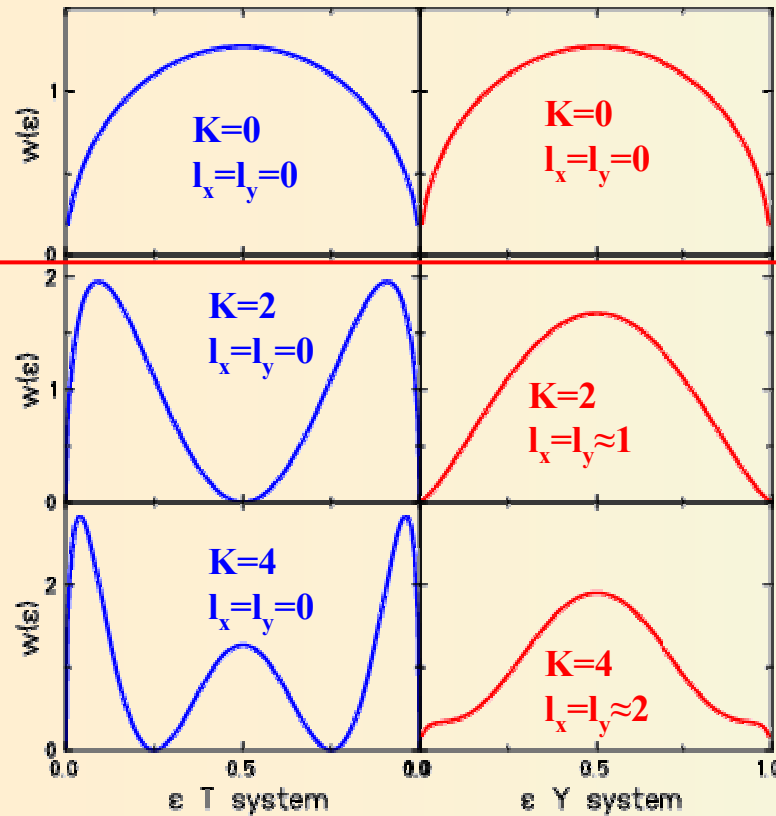
$K=4, l_x=1, l_y=1$

Two-body correlations:

^5He - L.V.Chulkov et al., Phys.Rev.Lett. 79, 201 (1997)

^{10}Li - H.Simon et al., Phys.Rev.Lett. 83, 496 (1999)

Non interacting particles



Recoil effect in the 3-body system.

$$\mathcal{Y}_K^{l_x, l_y, L, M}(\Omega_6) = \sum_{m_x, m_y} \langle l_x l_y m_x m_y | LM \rangle \mathcal{Y}_K^{l_x, l_y, m_x, m_y}(\Omega_6)$$

Harmonic $K=4, l_x=0, l_y=0$, T-system

$S=0, L=0,$ l_x and l_y	WF components in Y system.
2 and 2	87.0%
1 and 1	2.6%
0 and 0	10.4%

Harmonic $K=0, l_x=l_y=0$.
100% in Y and T system
when core is very heavy.

Favorite condition for a
three-body state: p- or d-
resonance in n+core
subsystem

3 neutrons, T-system,
 $K=4, l_x=l_y=0$. No AS.

$S=0, l_x, l_y$	WF in Y
2 and 2	50%
1 and 1	50%
0 and 0	0%

Assuming strong s-wave interaction between ^8He and neutron, intensive resonance at very low energy was predicted for ^{10}He :

S. Aoyama, Phys.Rev.Lett. 89(2002)_052501.

L.V.Grigorenko et al., Phys.Rev. C 77 (2008) 034611

TERTIARY AND GENERAL-ORDER COLLISIONS

L. M. DELVES
 Clarendon Laboratory, Oxford

Received 2 October 1958

Abstract: The general formalism of scattering theory is extended to cover reactions in which more than two particles exist in the entrance or exit channels, and the behaviour of the scattering matrix in this general case is discussed at arbitrary energies, in particular near thresholds. The energy dependence of observable quantities such as cross-sections and polarizations is given, and the occurrence and origin of the Wigner cusps and their counterpart in the general case are clearly seen; such singularities do not arise from the behaviour of the eigenphase-shifts.

Expansion of a decay amplitude into the hyper-spherical-harmonic series:

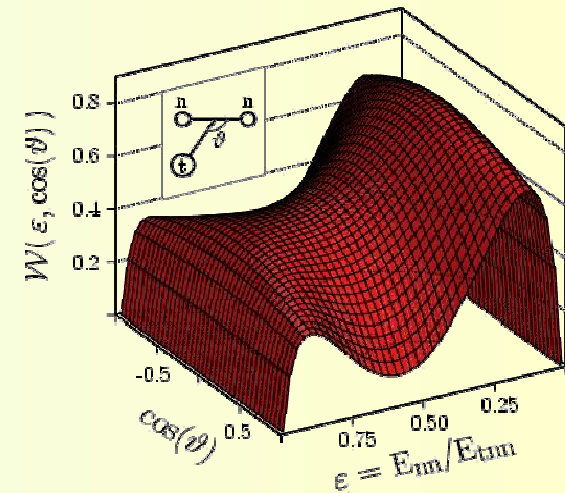
$$F_{IM} = \sum_{KLS} \sum_{l_x l_y} A_{l_x l_y}^{KLS}(E) \sum_{mv} C_{LmSv}^{IM} C_{\frac{1}{2}v_1 \frac{1}{2}v_2}^{Sv} y_{l_x l_y}^{KLM}(\mathbf{q}_{12}, \mathbf{q}_{3-12}).$$

Quantum numbers: \mathbf{K} , $\mathbf{I}^\pi = \mathbf{L} + \mathbf{S}$, $\mathbf{L} = l_x + l_y$, $\mathbf{S} = s_1 + s_2$

Phys.Rev.Lett. 91, 162504 (2003)

M.Meister *et al*

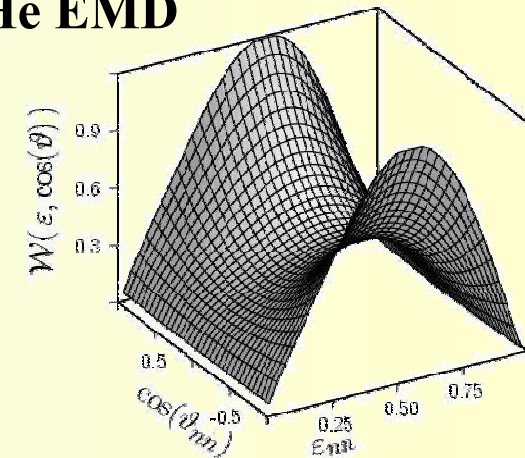
^5H



Nucl.Phys. A759, 23 (2005)

L.V.Chulkov *et al.*

^6He EMD



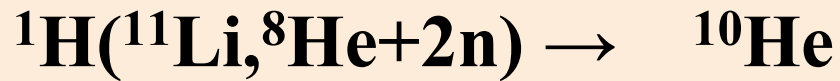
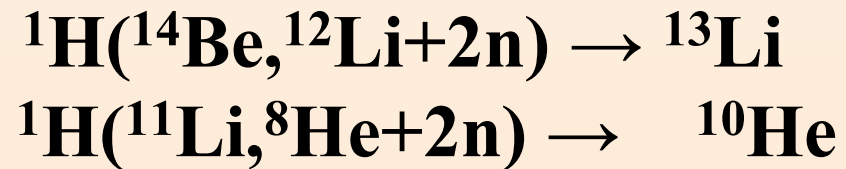
Assumptions. Ockham's Razor.

- Decay amplitude is antisymmetric under permutation of neutrons.
- Contribution from the states with the total spin of the two neutrons equal to 0 dominates.
- Partial angular momenta are coupled to zero.
- The hyperspherical-harmonic series is restricted to $K=0, 2, 4$
- No one more harmonic than necessary (Ockham-razor principle).



William of Ockham
(1287-1347)

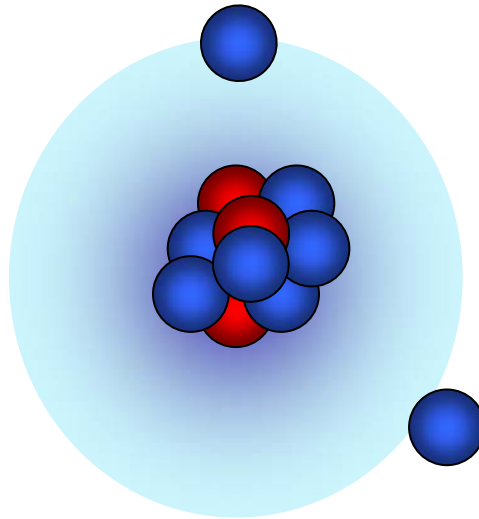
Now we come to the experiment where two unstable nuclides were obtained in proton knock out.



Few words about reaction mechanism:

Quasi free scattering on a nucleon inside halo nucleus (valence nucleon knockout)

Example with ^{11}Li



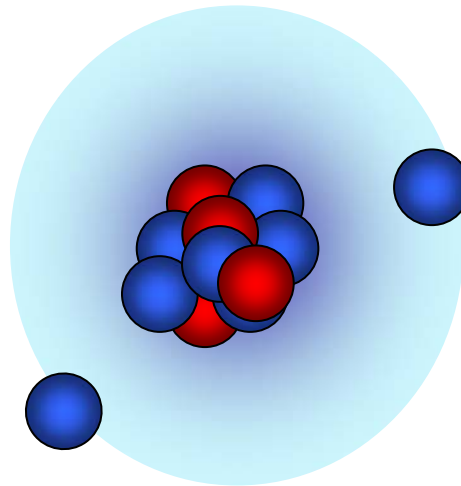
- Valence neutron knockout
- Small binding energy [<1 MeV]
- Remaining system is a spectator
- Momentum is transferred to the whole remaining system (^{10}Li).



Quasi free scattering on a nucleon inside halo nucleus (nucleon knockout from core)

Example with ^{14}Be

Shake-off mechanism of fragmentation.

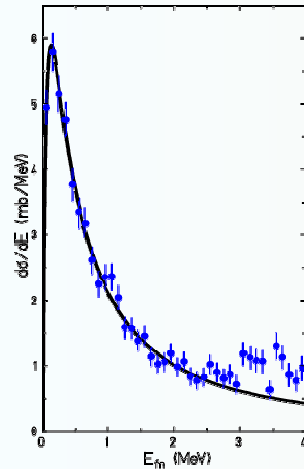


- Proton knockout from the core
- Proton strongly bound [~ 20 MeV]
- Remaining system can not be considered as spectator.
- Momentum is transferred to the remaining of the core (^{11}Li).

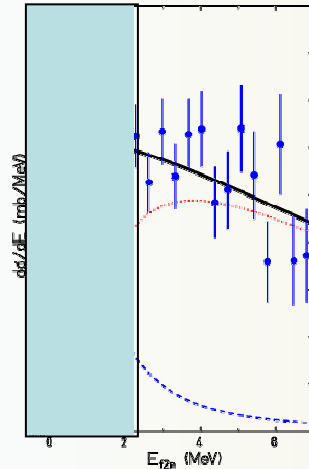


He¹⁰ and Li¹³. Twins?

¹²Li



¹³Li



¹²Li

$a = -13.7(1.6) \text{ fm}$, $\varepsilon = 1.47(19) \text{ MeV}$

derived in the fit is consistent with the experimental value for the two-neutron separation energy in ¹⁴Be of $S_{2n} = 1.26(13) \text{ MeV}$

No other states.

Virtual state

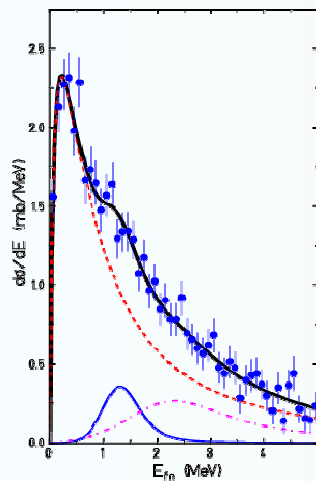
$$\frac{d\sigma}{dE_{fn}} \propto p_{fn} \left[\frac{1}{k^2 + p_{fn}^2} \right]^2 \left[\cos(\delta) + \frac{k}{p_{fn}} \sin(\delta) \right]^2,$$

$$p_{fn} \cot(\delta) = -\frac{1}{a} + \frac{1}{2} r_0 p_{fn}^2 + \mathcal{O}(p_{fn}^4).$$

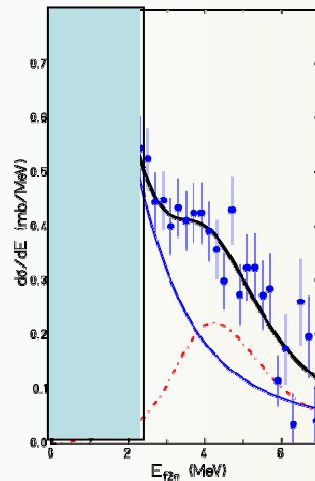
Phys. Lett. B 666 (2008) 434, Yu.Aksyutina, H.Johansson et al.

G.F.Bertsch et al., Phys.Rev. C57(1998)1356

⁹He



¹⁰He



⁹He

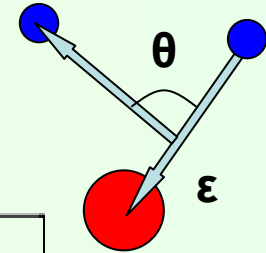
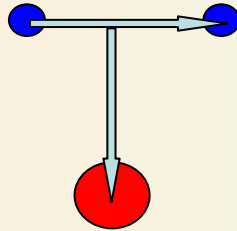
$a = -3.3(5) \text{ fm}$, $\varepsilon = 0.71(7) \text{ MeV}$

$E_1 = 1.28(10) \text{ MeV}$ $\Gamma = 0.15 \text{ MeV}$

$E_2 = 2.4 \text{ MeV}$ $\Gamma = 2 \text{ MeV}$

Yu.Aksyutina, H.Johansson et al. to be published

Ground state region 0-3 MeV.

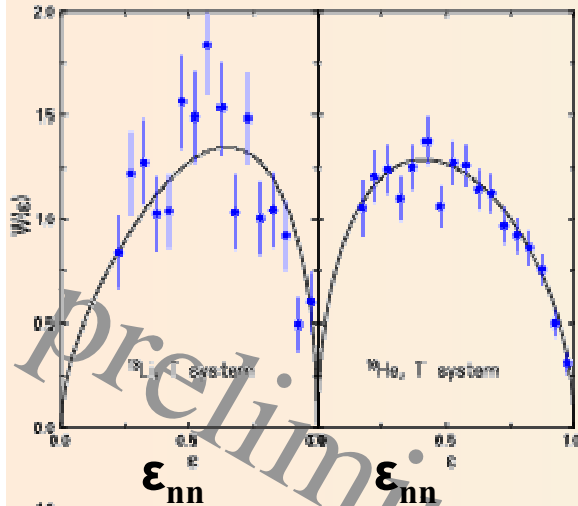


¹³Li

¹⁰He

¹³Li

¹⁰He



¹³Li

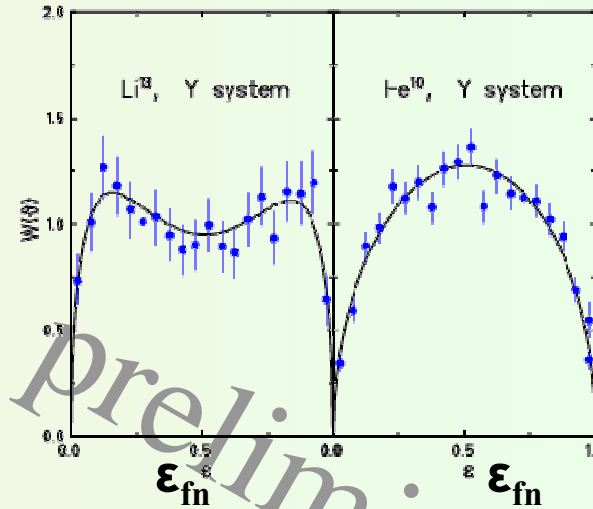
K=0 → 97.3%

K=2

lx=ly=0 → 0.8%

K=4

lx=ly=2 → 1.9%

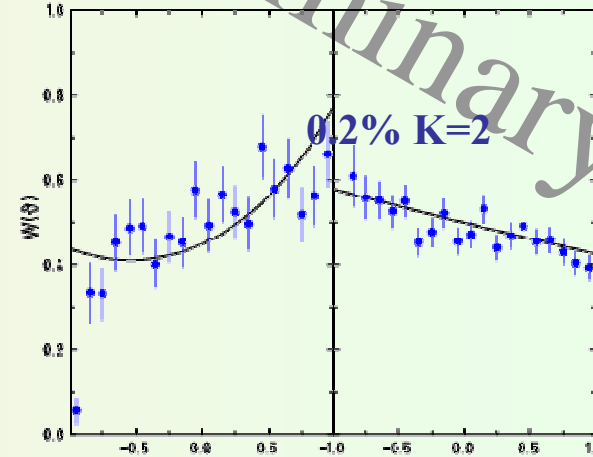


¹⁰He

K=0 → 99.8%

K=2

lx=ly=0 → 0.2%



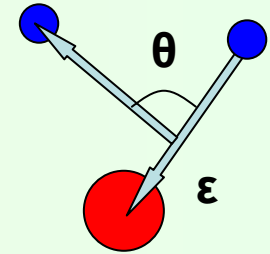
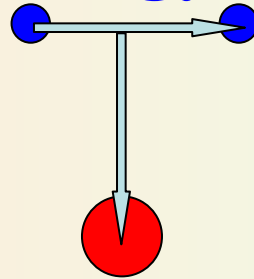
Cos(θ)

Cos(θ)

Cos(θ)

Cos(θ)

High energy region 3-7 MeV

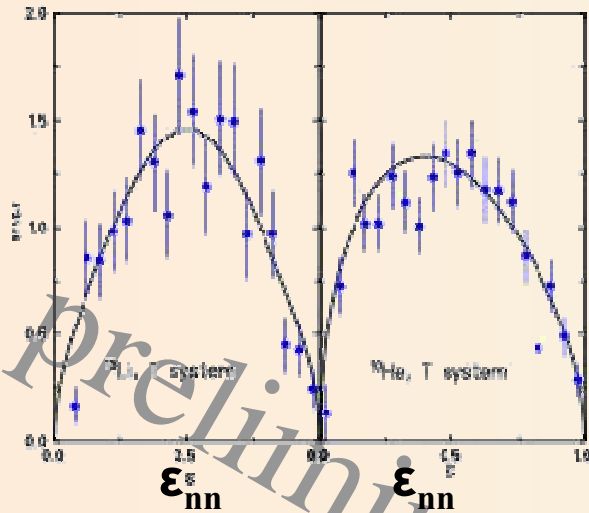


^{13}Li

^{10}He

^{13}Li

^{10}He

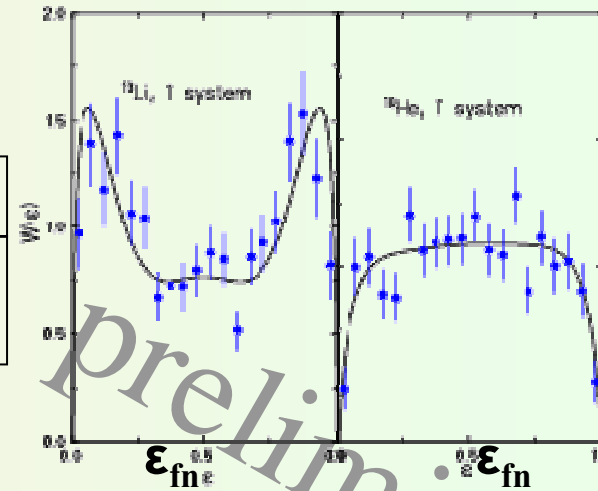


^{13}Li

$K=0 \rightarrow 75.0\%$

$K=4$

$l_x=l_y=2 \rightarrow 25.0\%$



^{10}He

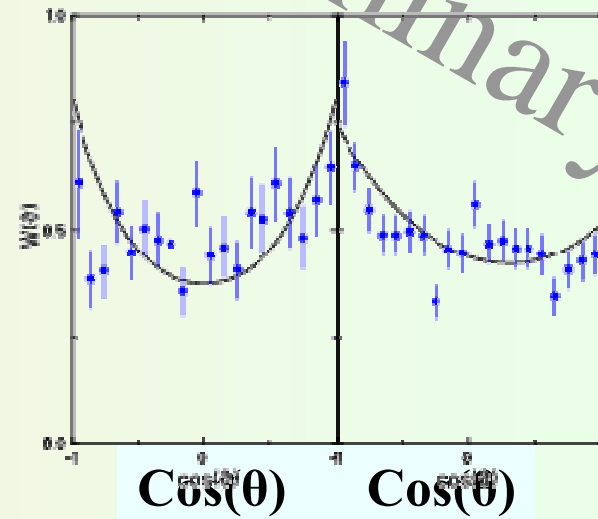
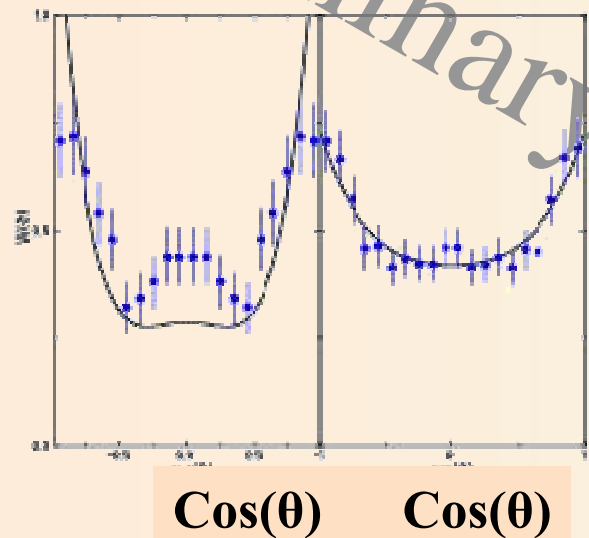
$K=0 \rightarrow 96.3\%$

$K=2$

$l_x=l_y=0 \rightarrow 0.5\%$

$K=4$

$l_x=l_y=2 \rightarrow 3.2\%$



Preliminary

Preliminary

Résumé

- ✿ Expansion of the decay amplitude into a hyperspherical-harmonic series allows to feel the presence of alien on a level of 0.1%.
- ✿ Essential difference exists between ^{13}Li and ^{10}He nuclides.
- ✿ Analysis has shown that a three-body resonance is improbable when only s-wave interactions exists in the binary subsystems.

