

Unbound Nuclei Workshop

Spectroscopy of unbound states using the transfer to the continuum method

Antonio M. Moro

Universidad de Sevilla

Motivation

- Many ongoing experiments and other planned for the new experimental facilities involve transfer of weakly bound systems, including transfer to unbound states
 - Knock-out experiments at GSI (see talks by H. Simon and L. Chulkov)
 - LOI for Spiral 2: *Unbound states of neutron-rich isotopes via direct reactions*
- In these reactions, coupling to the continuum plays a dominant role.
- Extraction of meaningful physical information requires (i) a proper treatment of the continuum and (ii) suitable reaction theories.

Motivation (continued...)

The transfer to the continuum (TC) method provides an appropriate framework that incorporates continuum effects into a general theory for transfer reactions:

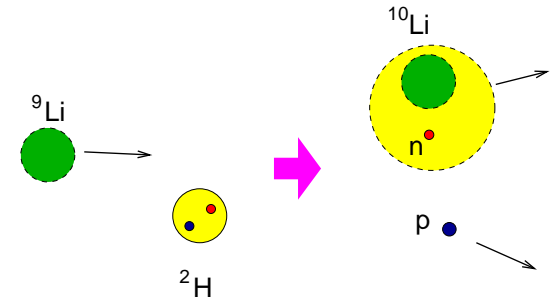
1. Spectroscopy to unbound states (e.g. ${}^9\text{Li}(d,p){}^{10}\text{Li}$)
2. Spectroscopy to bound states: (e.g. ${}^{10}\text{Be}(d,p){}^{11}\text{Be}$)
3. Interpretation of reaction mechanisms (e.g. ${}^6\text{He}+{}^{208}\text{Pb}$).

Part I: Spectroscopy to unbound states: the ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

The Experiment:

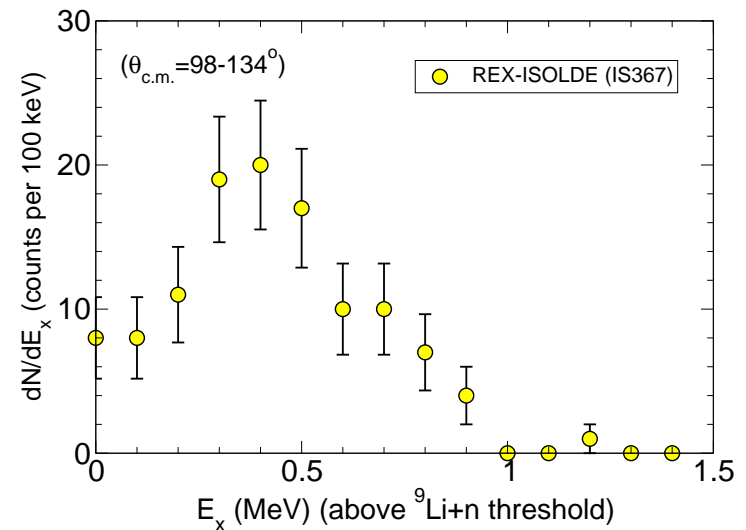
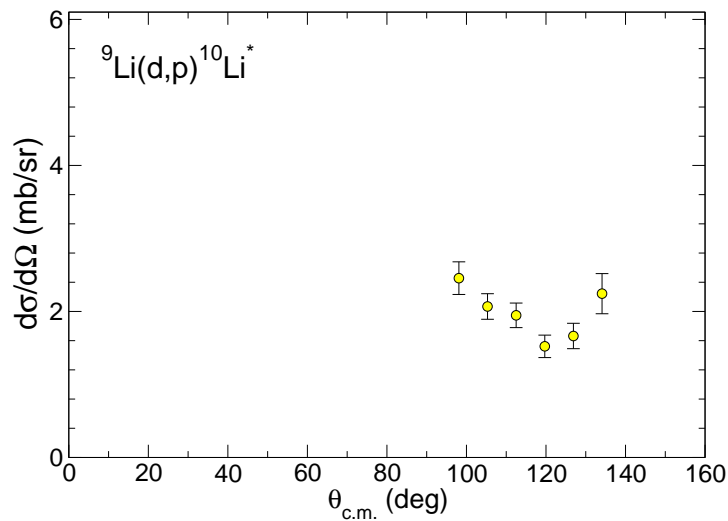
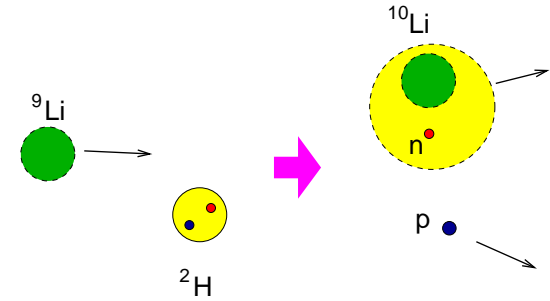
- REX-ISOLDE
- ${}^9\text{Li}$ beam on D target at $E = 2.36$ MeV/u
- The experiment provided angular and energy distributions for protons $\Rightarrow {}^{10}\text{Li}$.



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

The Experiment:

- REX-ISOLDE
- ${}^9\text{Li}$ beam on D target at $E = 2.36$ MeV/u
- The experiment provided angular and energy distributions for protons $\Rightarrow {}^{10}\text{Li}$.



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

MOTIVATION: *What spectroscopic information can be obtained from the ${}^{10}\text{Li}$ distributions?*

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

MOTIVATION: *What spectroscopic information can be obtained from the ${}^{10}\text{Li}$ distributions?*

IMPORTANT QUESTIONS TO ADDRESS:

- Q: What is the mechanism producing protons?
A: Direct (compound estimated to be very small)

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

MOTIVATION: *What spectroscopic information can be obtained from the ${}^{10}\text{Li}$ distributions?*

IMPORTANT QUESTIONS TO ADDRESS:

- **Q:** What is the mechanism producing protons?

- **A:** Direct (compound estimated to be very small)

- **Q:** How does the reaction mechanism affect the ${}^{10}\text{Li}$ energy spectrum? (i.e. relationship between observed bump and continuum structures)

- **A:** Requires a proper reaction calculation; not just a fit to data

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

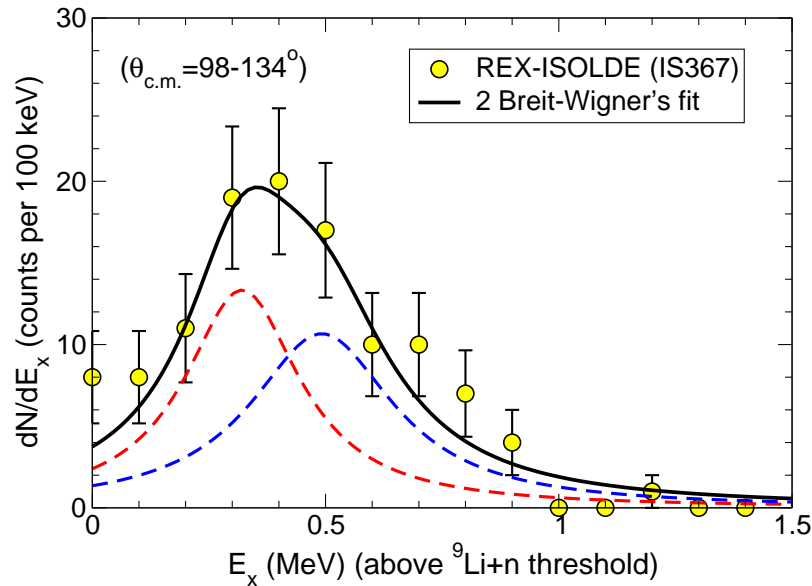
Breit-Wigner analysis:

$$\frac{d\sigma}{dE_x} \propto \frac{\Gamma}{(E - E_r)^2 + \frac{1}{4}\Gamma^2}$$

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Breit-Wigner analysis:

$$\frac{d\sigma}{dE_x} \propto \frac{\Gamma}{(E - E_r)^2 + \frac{1}{4}\Gamma^2}$$

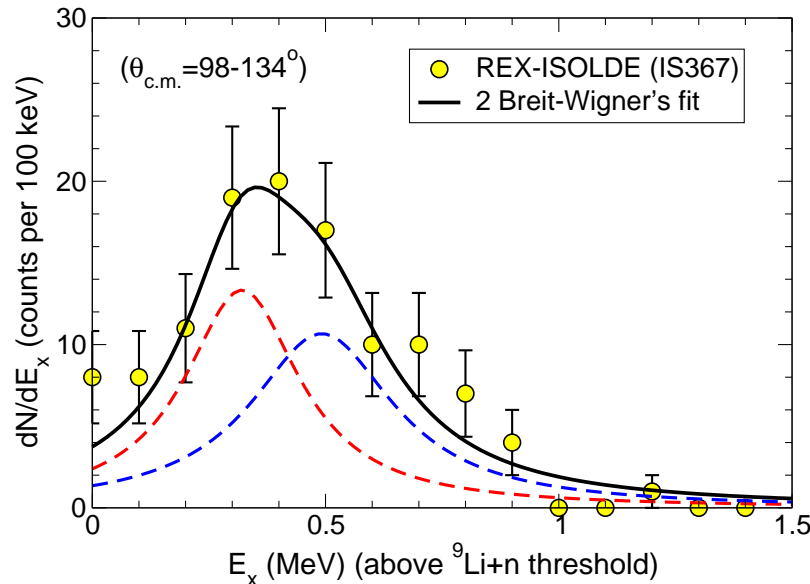


$$E_r^{(1)} = 0.32 \text{ MeV} \quad \Gamma^{(1)} = 0.30 \text{ MeV}$$
$$E_r^{(2)} = 0.49 \text{ MeV} \quad \Gamma^{(2)} = 0.37 \text{ MeV}$$

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Breit-Wigner analysis:

$$\frac{d\sigma}{dE_x} \propto \frac{\Gamma}{(E - E_r)^2 + \frac{1}{4}\Gamma^2}$$



$$E_r^{(1)} = 0.32 \text{ MeV} \quad \Gamma^{(1)} = 0.30 \text{ MeV}$$
$$E_r^{(2)} = 0.49 \text{ MeV} \quad \Gamma^{(2)} = 0.37 \text{ MeV}$$

This procedure is questionable, because (i) very ambiguous, (ii) ignores the effect of the reaction on the observed shapes, and (iii) the observed peak does not necessarily follow the simple Breit-Wigner shape

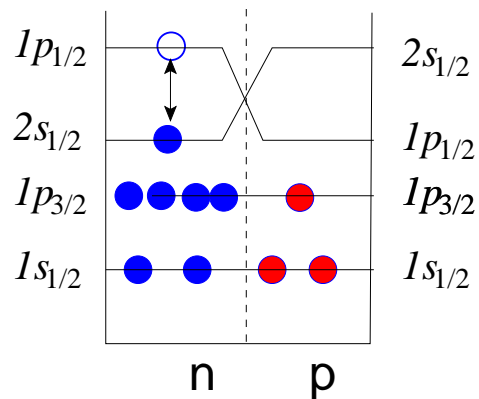
Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

${}^{10}\text{Li}$



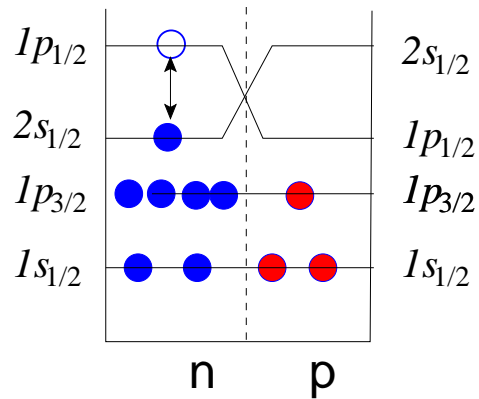
$${}^9\text{Li} (3/2^-) \otimes 2s_{1/2} = {}^{10}\text{Li} (1^- / 2^-)$$

$${}^9\text{Li} (3/2^-) \otimes 1p_{1/2} = {}^{10}\text{Li} (1^+ / 2^+)$$

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

STRUCTURE + REACTION

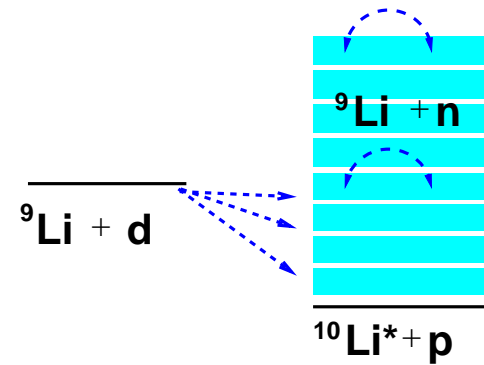
${}^{10}\text{Li}$



$${}^9\text{Li} (3/2^-) \otimes 2s_{1/2} = {}^{10}\text{Li} (1^- / 2^-)$$

$${}^9\text{Li} (3/2^-) \otimes 1p_{1/2} = {}^{10}\text{Li} (1^+ / 2^+)$$

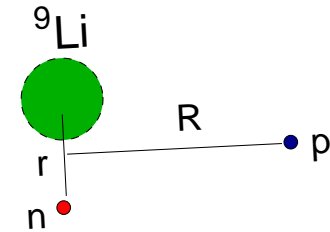
Transfer to the continuum
(direct mechanism)



The transfer to the continuum (TC) amplitude

- Exact scattering amplitude (prior form):

$$T_{if} = \langle \Psi_f^{(-)} | V_{n+{}^9\text{Li}} + U_{p+{}^9\text{Li}} - U_\alpha | \chi_d^{(+)} \phi_d \rangle$$



- Numerical evaluation of T:

- ❖ $\Psi_f^{(-)}$ calculated in 3-body model: p+n+ ${}^9\text{Li}$
- ❖ $\Psi_f^{(-)} \approx \Psi_f^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) = \sum_i \chi_{p-{}^{10}\text{Li}}^i(\mathbf{R}) \phi_{{}^{10}\text{Li}}^i(\mathbf{r})$
[$\phi_{{}^{10}\text{Li}}^i(\mathbf{r})$: continuum bins for n- ${}^9\text{Li}$ unbound states]
- ❖ $U_\alpha(\mathbf{R})$ taken to reproduce the elastic data *

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Some features of the TC/CDCC amplitude:

- Final states are represented by normalizable states (continuum bins) \Rightarrow the evaluation of T is analogous to the case of bound states.
- Resembles semiclassical TC method of Bonaccorso & Brink, but without straight line (eikonal) assumption (purely quantum-mechanical).
- Finite-range (no zero-range approximation).
- Both resonant and non-resonant continuum included and treated on equal footing.
- Continuum-continuum couplings (FSI) included in $\Psi_f^{(-)}$.
- $\Psi_f^{(-)}$ generated from p - n and p - ${}^9\text{Li}$ (does not require p - ${}^{10}\text{Li}$ potential).

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

${}^{10}\text{Li}$ structure: neutron- ${}^9\text{Li}$ core potential model: (Garrido *et al*, NPA700(2002)117)

$$V_{nc}(r) = \begin{cases} -V_{s12} \exp(-r^2/a^2) & (s_{1/2}) \\ -V_{p12} \exp(-r^2/a^2) & (p_{1/2}) \\ -V_{p32} \exp(-r^2/a^2) & (p_{3/2}) \end{cases}$$

- $a=2$ fm \Rightarrow correct rms for ${}^9\text{Li}$.
- $V_{p32} \Rightarrow$ separation energy.

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

${}^{10}\text{Li}$ structure: neutron- ${}^9\text{Li}$ core potential model: (Garrido *et al*, NPA700(2002)117)

$$V_{nc}(r) = \begin{cases} -V_{s12} \exp(-r^2/a^2) & (s_{1/2}) \\ -V_{p12} \exp(-r^2/a^2) & (p_{1/2}) \\ -V_{p32} \exp(-r^2/a^2) & (p_{3/2}) \end{cases}$$

- $a=2$ fm \Rightarrow correct rms for ${}^9\text{Li}$.
- $V_{p32} \Rightarrow$ separation energy.

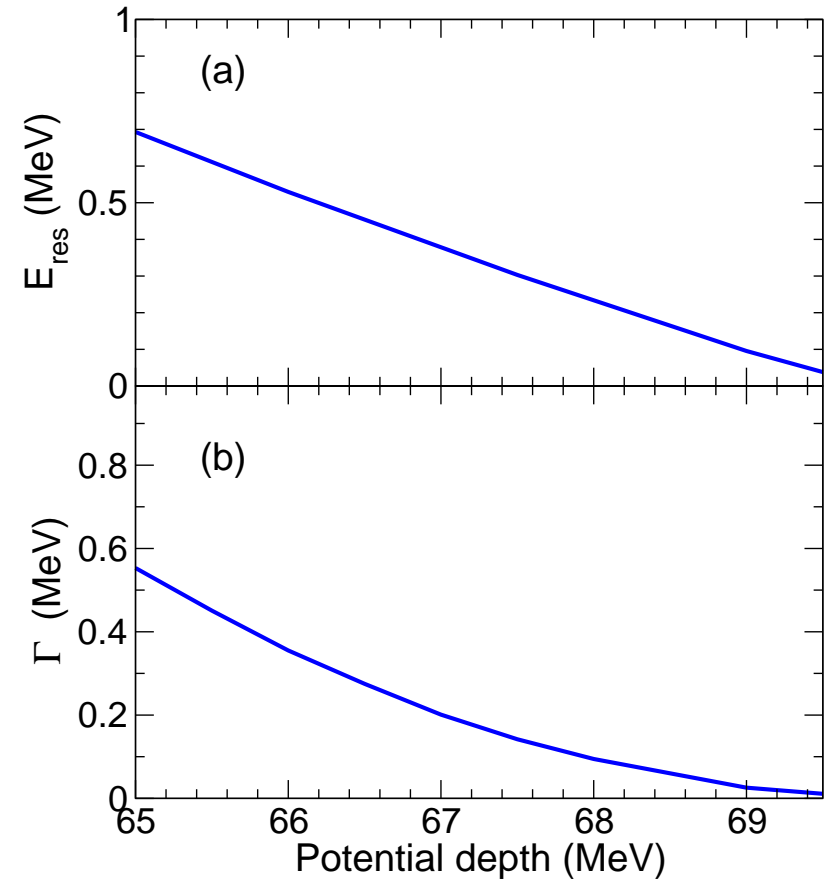
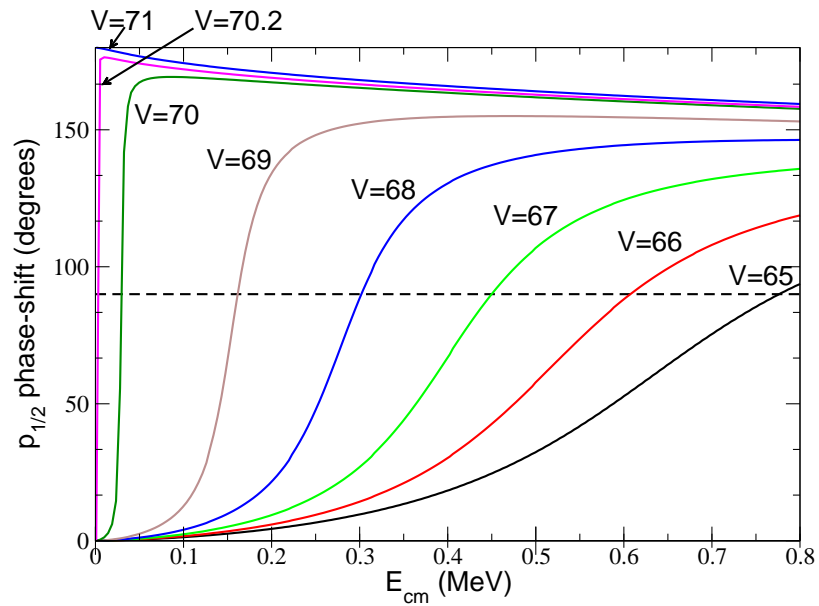
Two free parameters: V_{s12} , V_{p12}

- $V_{s12} \leftrightarrow E_s \leftrightarrow a_s$
- $V_{p12} \leftrightarrow E_{res} \leftrightarrow \Gamma$

No spin-spin interaction $\Rightarrow 1^+/2^+$ and $1^-/2^-$ degenerated

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

$p_{1/2}$ resonance



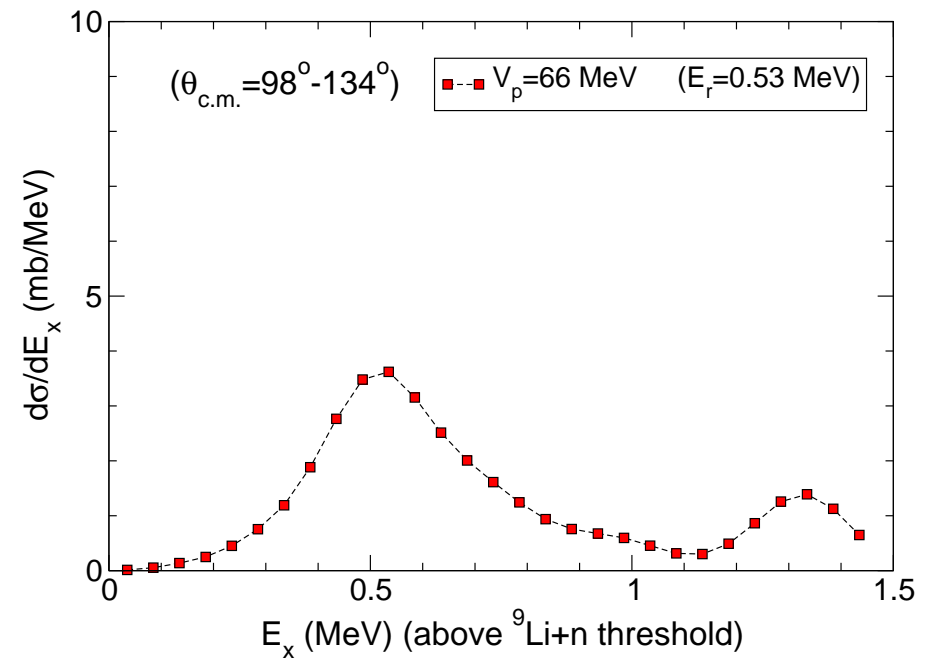
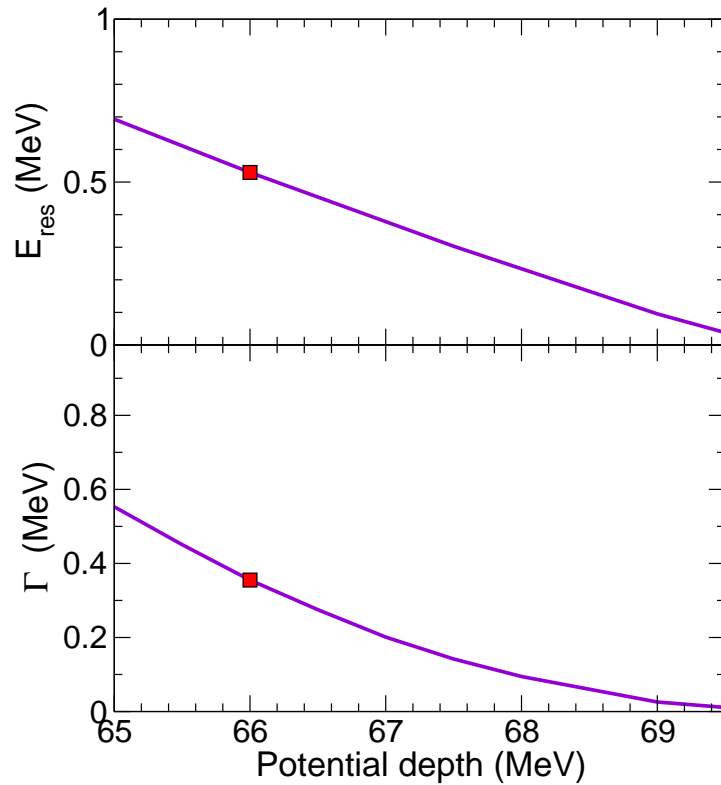
- $E_{res} \rightarrow \delta(E_{res}) = \frac{\pi}{2}$

- $\frac{2}{\Gamma} = \frac{d\delta}{dE}$

Interplay between structure and reaction

Structure: $p_{1/2}$ resonance

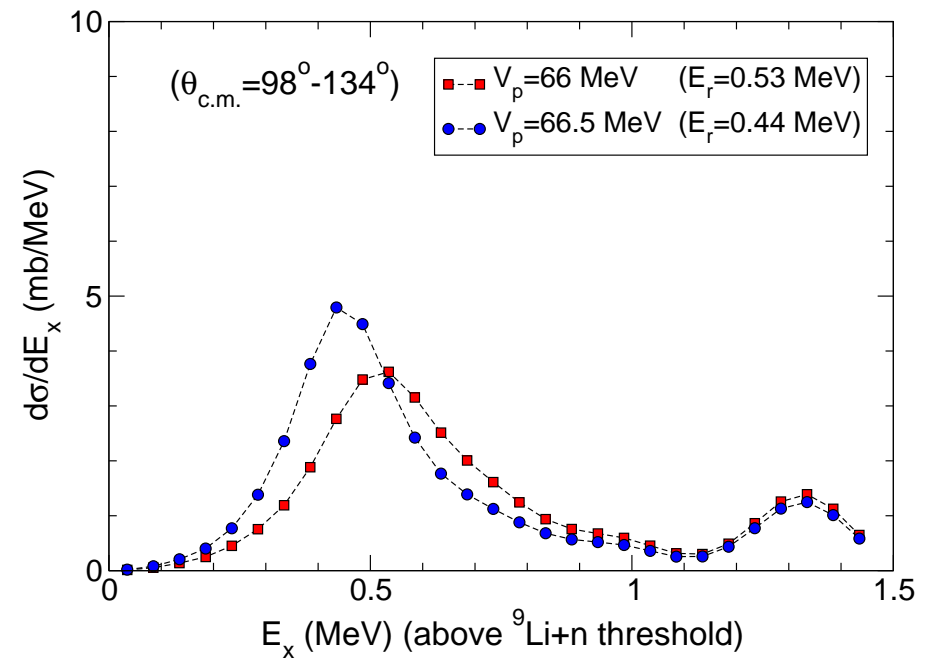
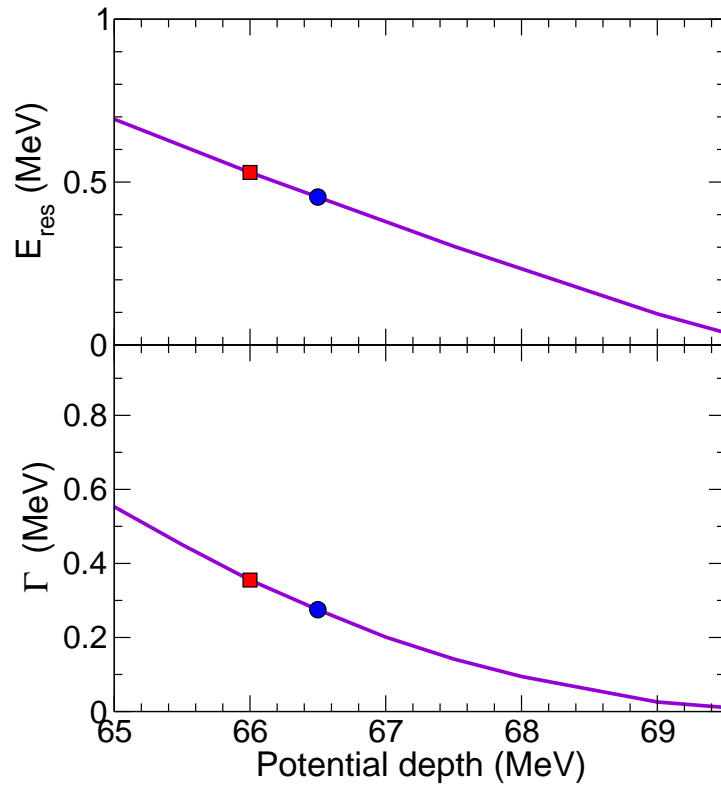
Reaction



Interplay between structure and reaction

Structure: $p_{1/2}$ resonance

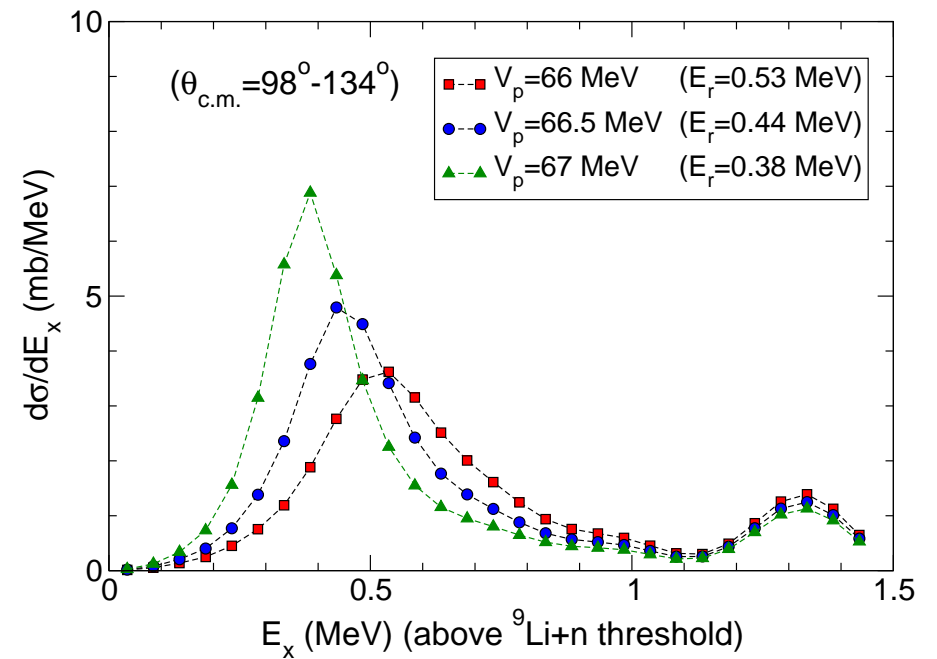
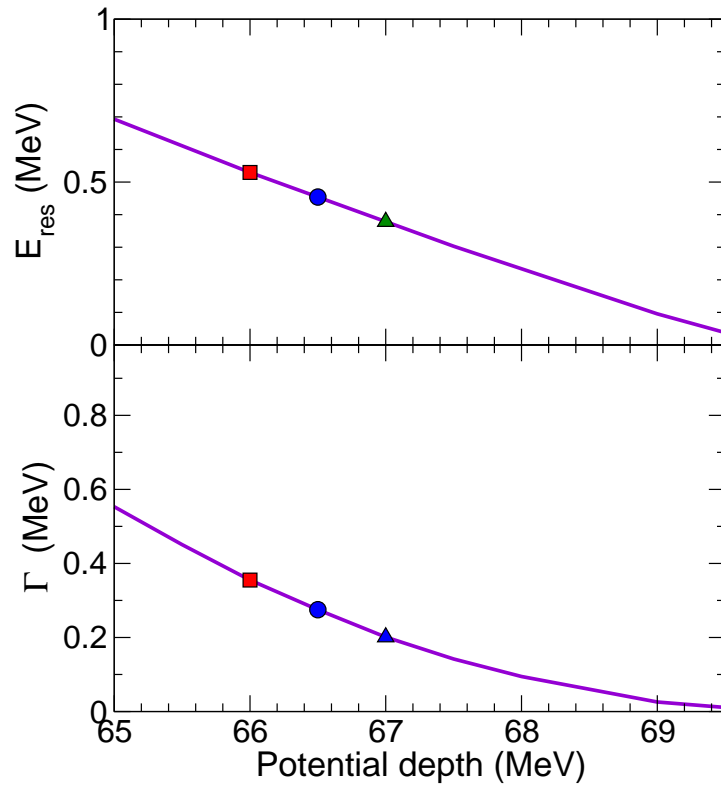
Reaction



Interplay between structure and reaction

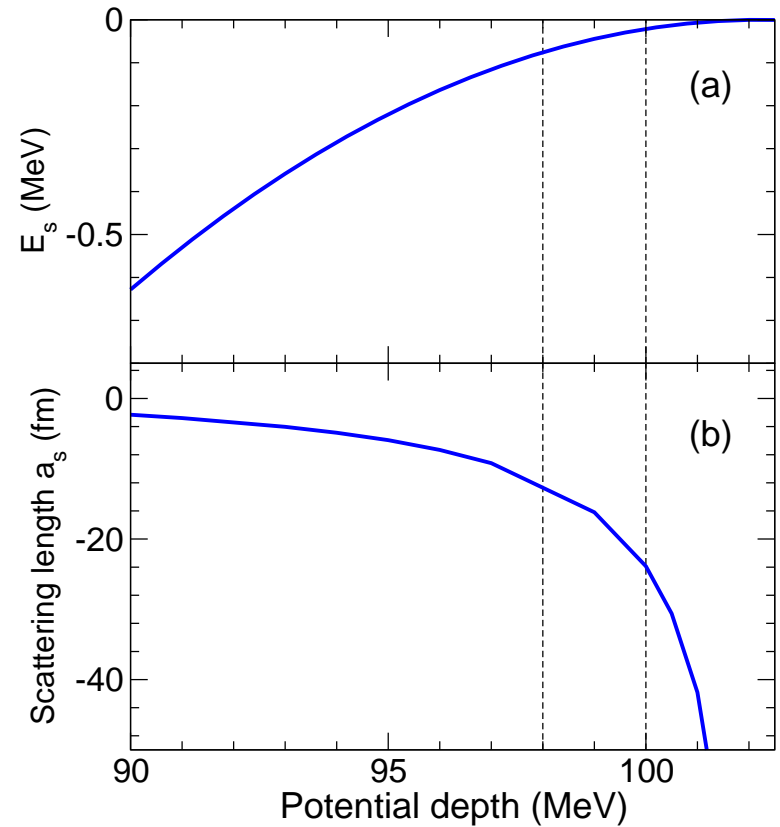
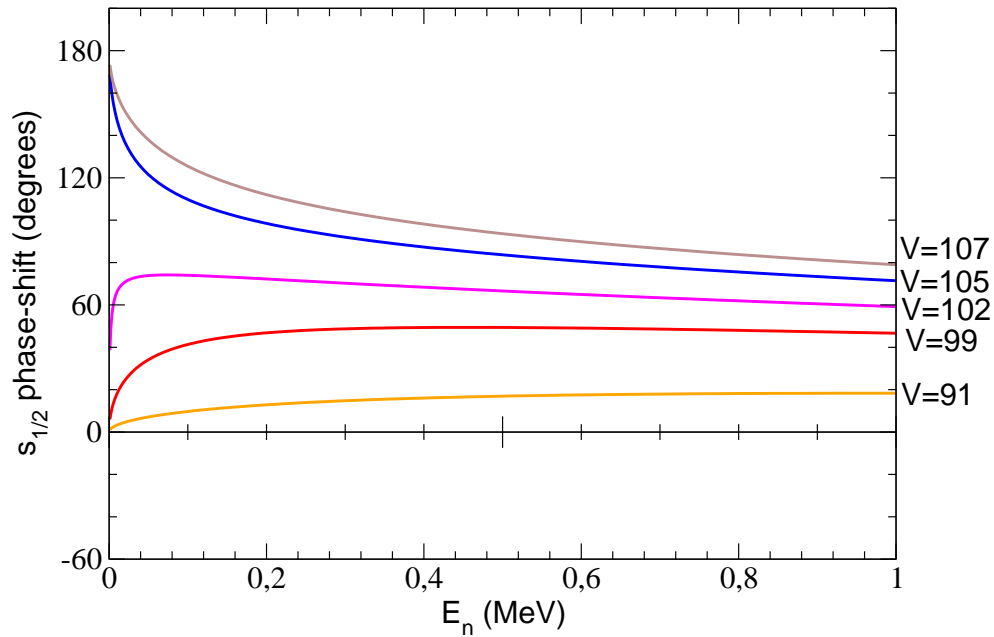
Structure: $p_{1/2}$ resonance

Reaction



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

V_s (virtual state)



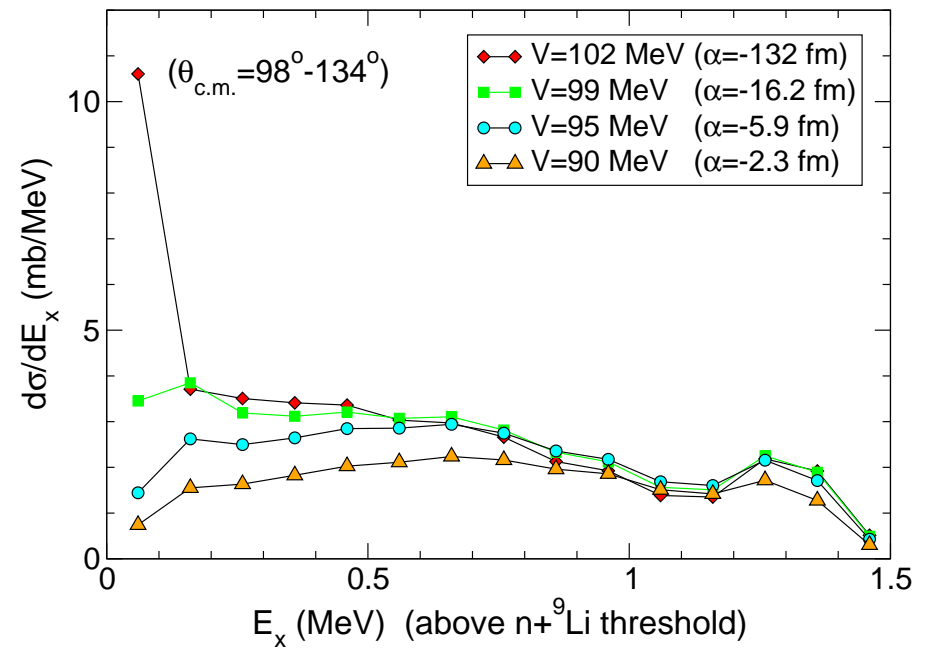
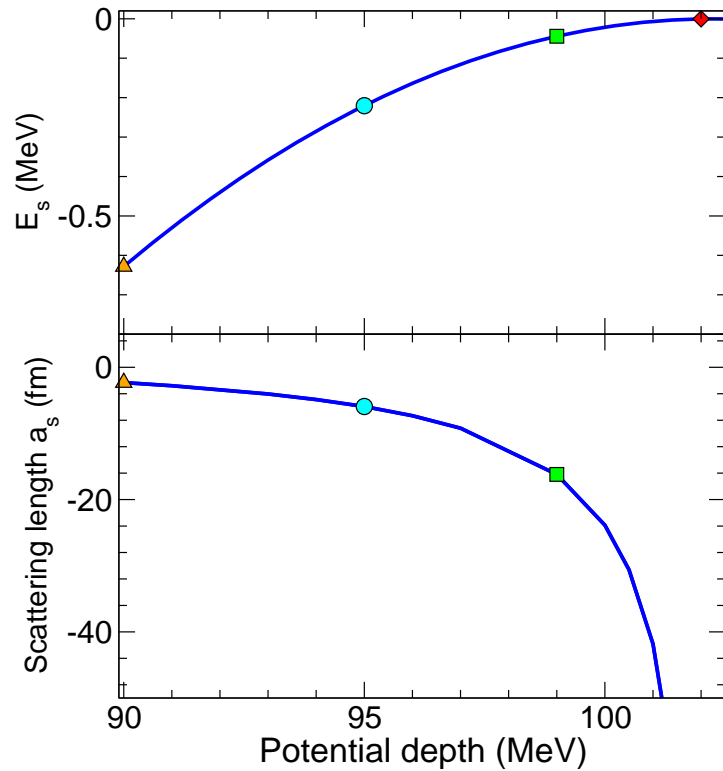
Scattering length:

$$a_s = - \lim_{k \rightarrow 0} \tan \frac{\delta(k)}{k}$$

Interplay between structure and reaction

Structure: $s_{1/2}$ v.s.

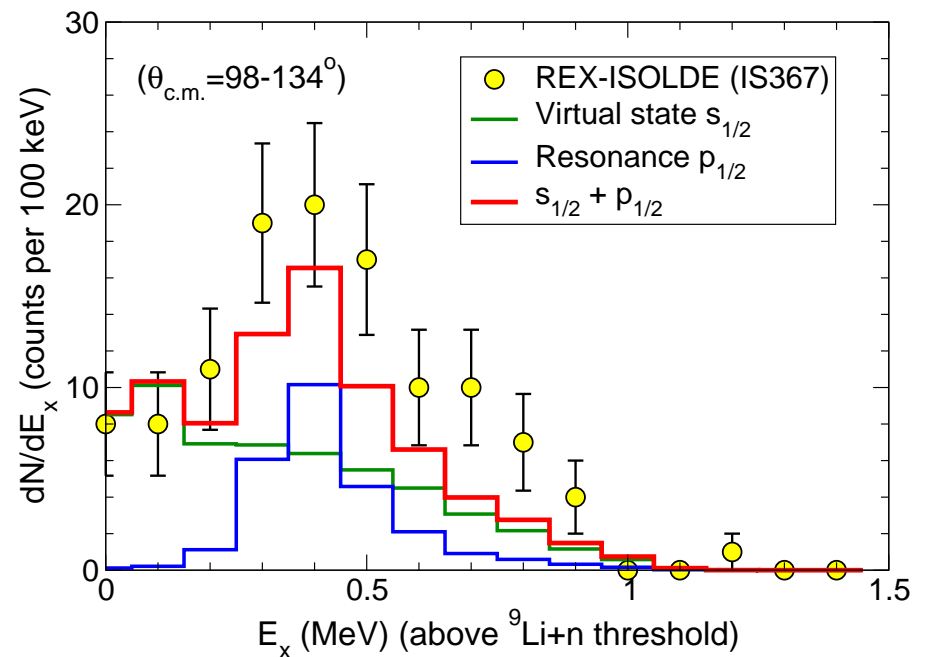
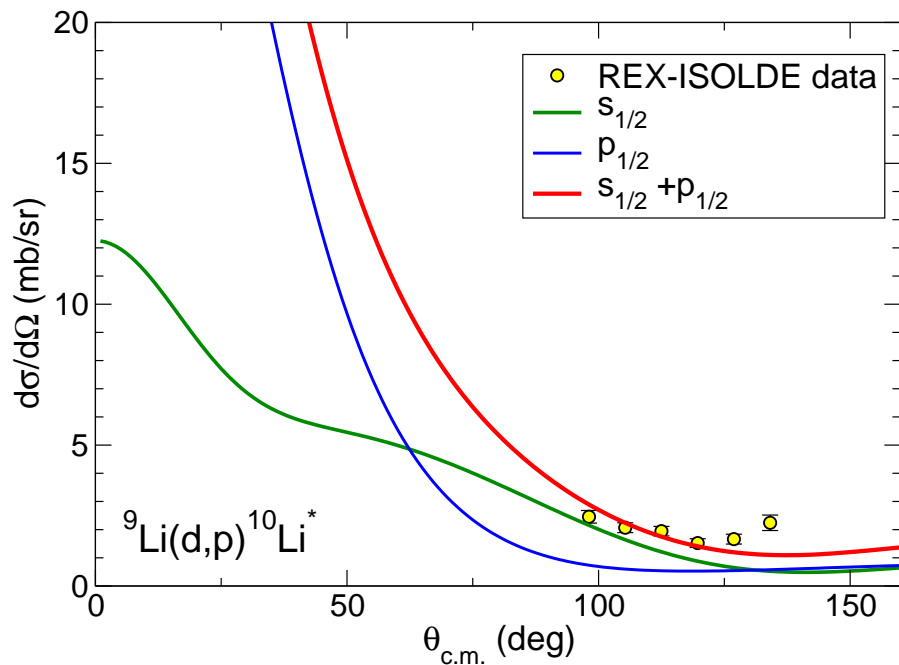
Reaction



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$

BEST FIT RESULTS: HP.Jeppesen *et al*, PLB642 (2006) 449

- $p_{1/2}$ resonance ($1^{+}/2^{+}$): $E_r \simeq 0.38$ MeV, $\Gamma = 0.2$ MeV
- $s_{1/2}$ virtual state ($1^{-}/2^{-}$): $a_s \simeq -24$ fm



Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

	(1)	(2)	${}^9\text{Li}(d,p){}^{10}\text{Li}$
Virtual state	$a_s = -30^{+12}_{-31}$	$a_s = -22.4(4.8)$	$a_s \simeq -24$ fm
p -resonance	$E_r = 0.510(44)$ $\Gamma = 0.54(16)$	$E_r = 0.566(14)$ $\Gamma = 0.548(30)$	$E_r = 0.38$ MeV $\Gamma = 0.20$ MeV
d -resonance	$E_r = 1.486(88)$ $\Gamma < 2.2$	- -	- -

(1) Simon *et al*, NPA791 (2007) 267,
based on the analysis of ${}^{11}\text{Li}$ fragmentation on ${}^{12}\text{C}$ at 264 MeV/u.

(2) Aksyutina *et al*, PLB666 (2008) 420,
based on the analysis of ${}^{11}\text{Li}$ fragmentation on H target

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Some conclusions from the analysis of ${}^9\text{Li}(d,p){}^{10}\text{Li}$ data:

- The transfer to the continuum approach provides a convenient theoretical framework to extract spectroscopic information from transfer reactions leading to unbound states.
- The analysis of the ${}^9\text{Li}(d,p){}^{10}\text{Li}$ reaction confirms that the low-energy continuum of ${}^{10}\text{Li}$ is dominated by a low-energy virtual state ($1^-/2^-$) and a p -wave resonance ($1^+/2^+$).

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Some conclusions from the analysis of ${}^9\text{Li}(d,p){}^{10}\text{Li}$ data:

- The transfer to the continuum approach provides a convenient theoretical framework to extract spectroscopic information from transfer reactions leading to unbound states.
- The analysis of the ${}^9\text{Li}(d,p){}^{10}\text{Li}$ reaction confirms that the low-energy continuum of ${}^{10}\text{Li}$ is dominated by a low-energy virtual state ($1^-/2^-$) and a p -wave resonance ($1^+/2^+$).

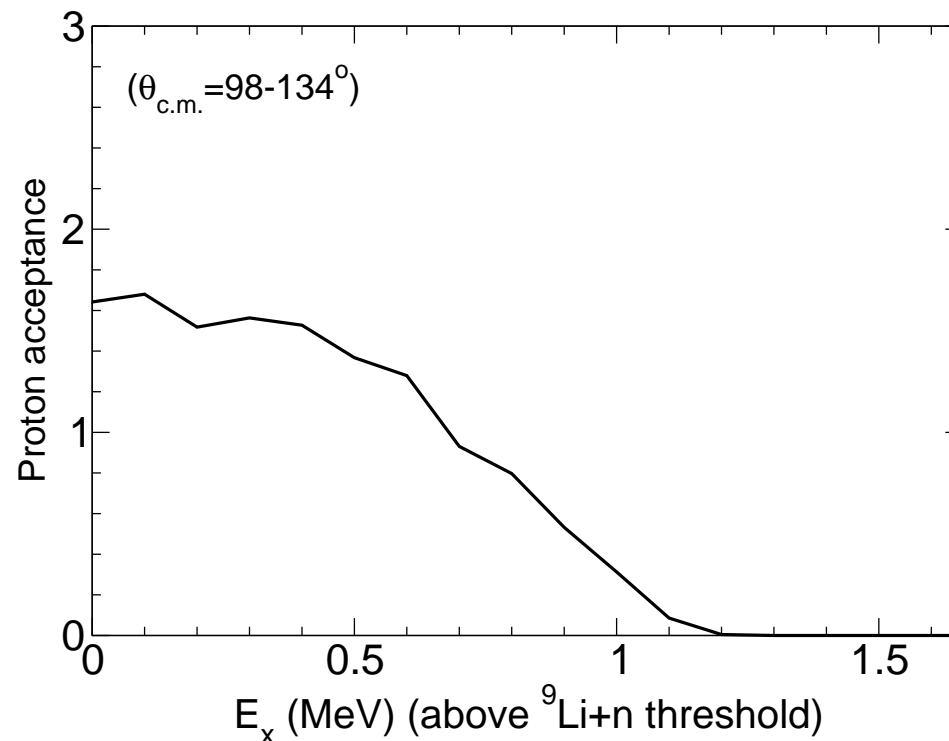
Open problems and future work

- Identification of the $1^+/2^+$ and $1^-/2^-$ doublets \Rightarrow better acceptance and statistics at high excitation energies.
- What is the role of core excitation (${}^9\text{Li}^*$) and how can it be incorporated?
- No evidence of d -wave resonance below $E_x < 1$ MeV, but could appear at higher energies

Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Identification of continuum structures above 1 MeV was not possible because:

- Available kinetic energy
- Proton acceptance



Part II: Spectroscopy to bound states: the $^{10}\text{Be}(d,p)^{11}\text{Be}$ case

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Motivation:

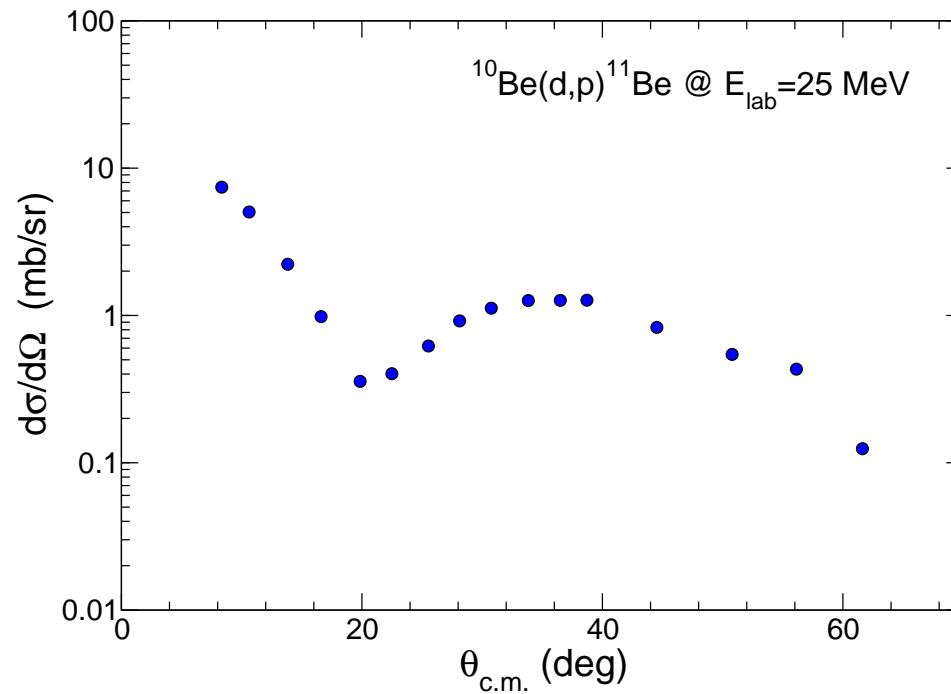
Transfer leading to bound states with weakly bound nuclei can be also affected by dynamical couplings with continuum states.

Example: $^{10}\text{Be}(d,p)^{11}\text{Be}_{\text{gs}}$ can proceed through deuteron and ^{11}Be continuum.



Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Experimental data: $^{10}\text{Be}(d,p)^{11}\text{Be}_{\text{gs}}$

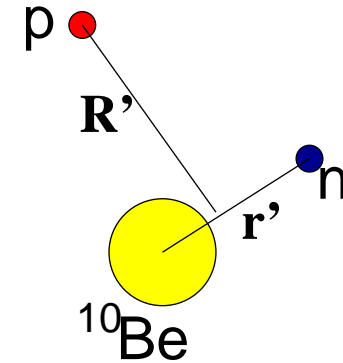


Zwieglinski et al, NPA315 (1979) 124

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Exact expression for the transition amplitude (post form):

$$T^{\text{exact}} = \langle \chi_p(\mathbf{R}') \phi_{^{11}\text{Be}}(\mathbf{r}') | V_{pn} + U_{p-^{10}\text{Be}} - U_\beta | \Psi_d^{(+)} \rangle$$



- $\Psi_d^{(+)}$: exact 3-body WF.
- U_β : arbitrary potential for final channel.

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

DWBA transition amplitude (post form)

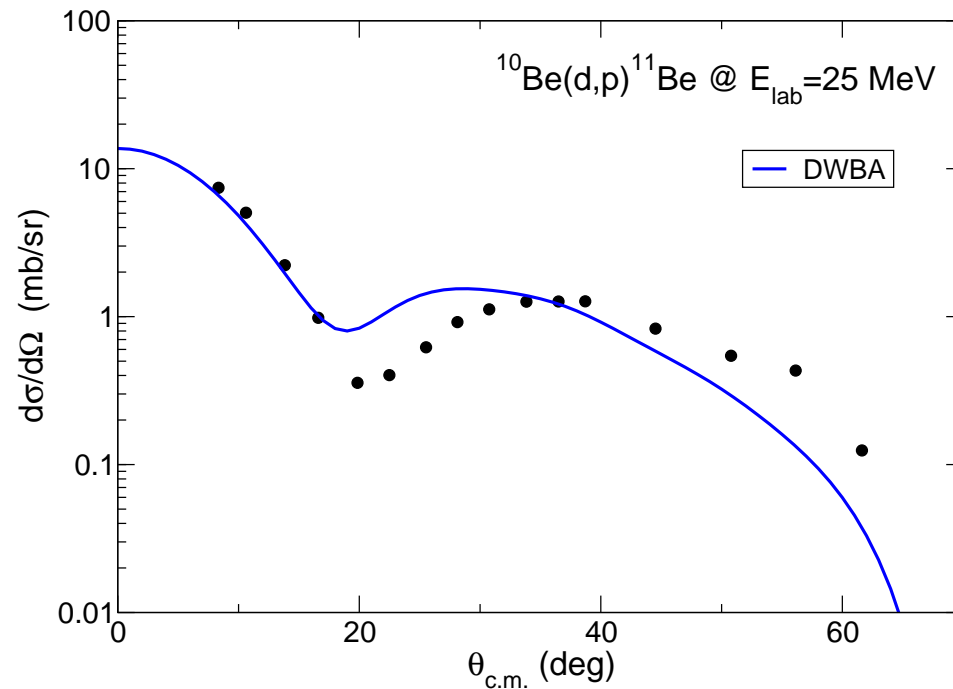
In the exact transition amplitude,

- $\Psi_d^{(+)} \approx \chi_d(\mathbf{R})\phi_d(\mathbf{r})$
- $U_\beta = U_{\text{opt}}(\mathbf{R}')$

So, continuum effects are ignored in both the entrance and exit channels

$$T^{\text{DWBA}} = \langle \chi_p \phi_{^{11}\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} - U_{\text{opt}} | \chi_d \phi_d \rangle$$

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$



⇒ *DWBA does not reproduce satisfactorily the data*

⇒ *Spectroscopic information extracted with DWBA can be misleading*

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

Exact transition amplitude:

$$T^{\text{exact}} = \langle \chi_p \phi_{11\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} - U_\beta | \Psi_d^{(+)} \rangle$$

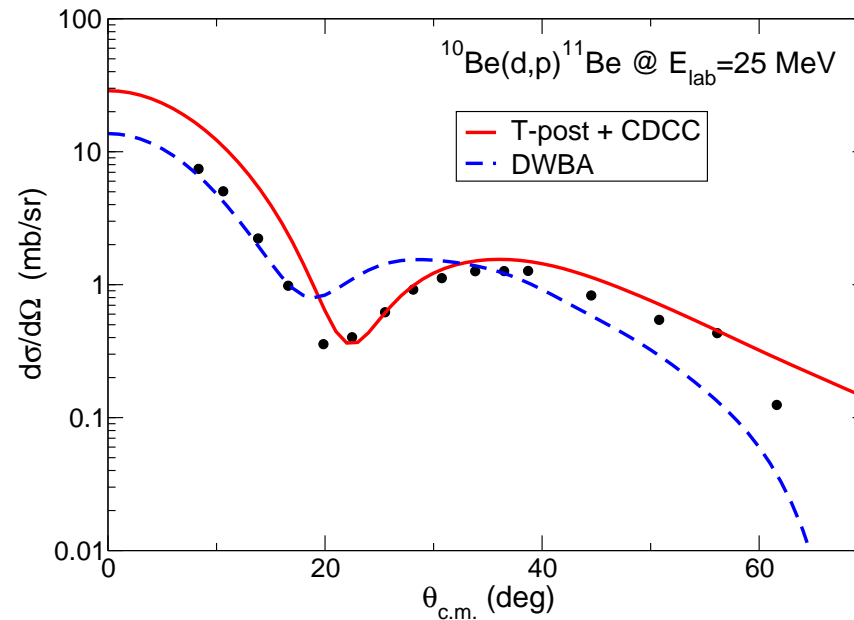
Evaluation within the CDCC approximation:

- $\Psi_d^{(+)} \approx \Psi_d^{\text{CDCC}} = \sum_i \chi_d^i(\mathbf{R}) \phi_d^i(\mathbf{r})$
- $U_\beta = \langle \phi_{11\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} | \phi_{11\text{Be}} \rangle \equiv U_{00}(R')$

$$T_{\text{post}}^{\text{CDCC}} = \langle \chi_p \phi_{11\text{Be}} | V_{pn} + U_{p-^{10}\text{Be}} - U_{00} | \Psi_d^{\text{CDCC}} \rangle$$

Spectroscopy to bound states: $^{10}\text{Be}(d,p)^{11}\text{Be}$

DWBA versus CDCC transition amplitude:



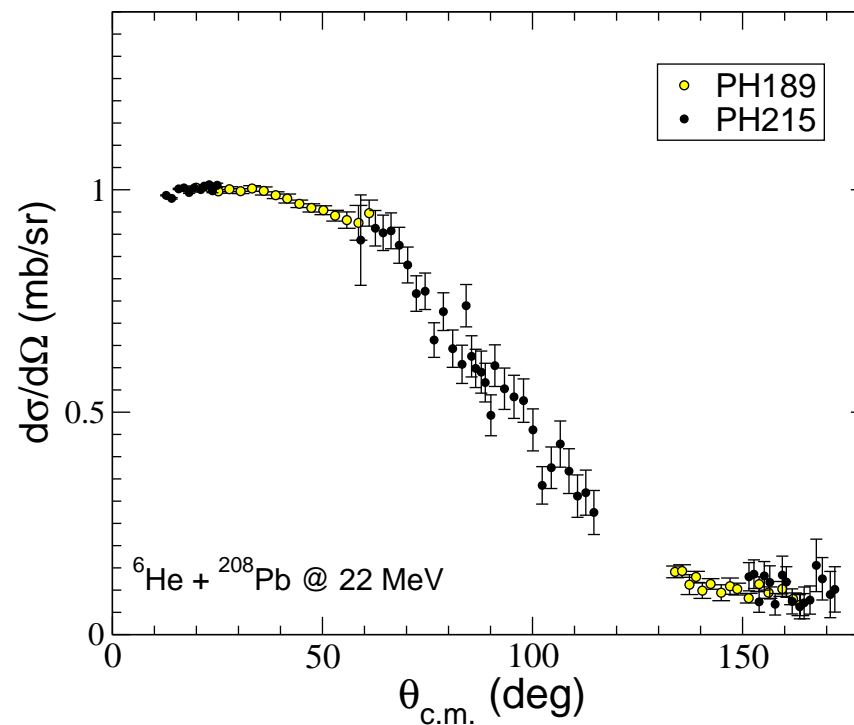
- ➡ Including deuteron continuum improves agreement with data.
- ➡ Spectroscopic factor is probably too small (but similar to the analysis of Timofeyuk and Johnson, *Phys. Rev. C* **59**, 1545 (1999))

Part III: Reaction mechanisms in ${}^6\text{He}$ induced reactions

Sevilla-Huelva-LLN ${}^6\text{He} + {}^{208}\text{Pb}$ experiment at LLN

MOTIVATION:

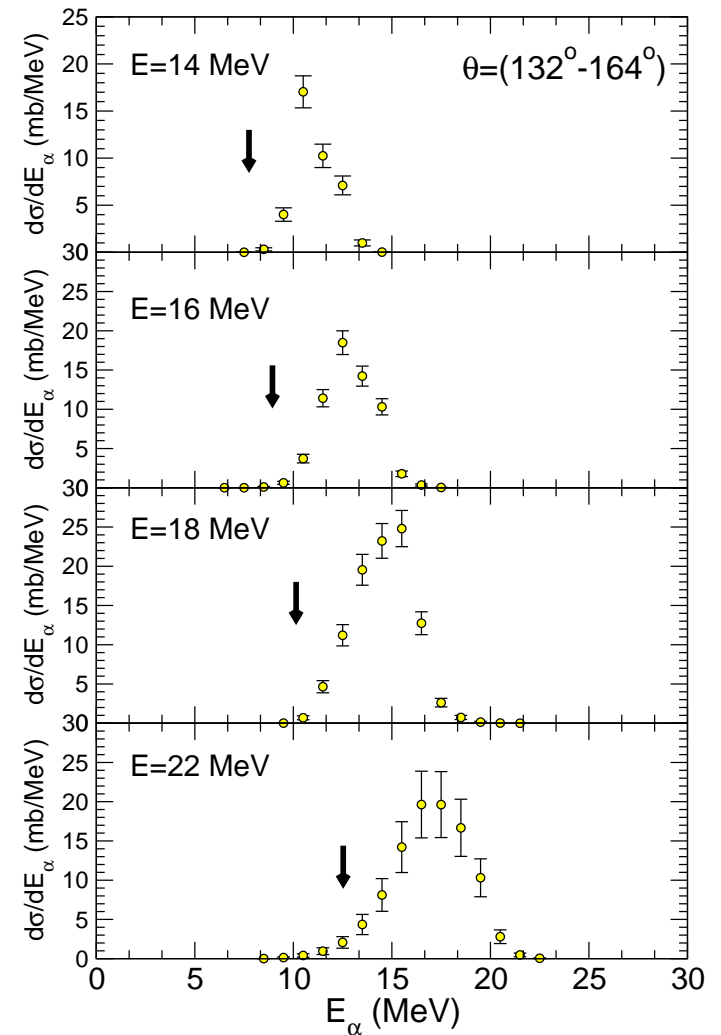
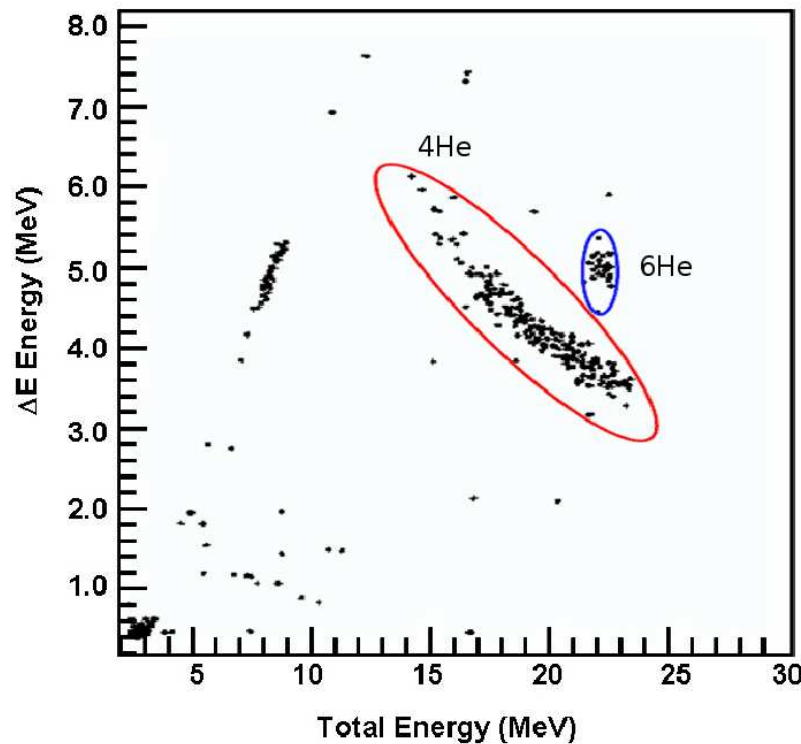
- Study the effect of Coulomb dipole polarizability on elastic scattering (previously studied for stable nuclei: d, ${}^6,{}^7\text{Li}$...)



Sevilla-Huelva-Madrid ${}^6\text{He}+{}^{208}\text{Pb}$ experiment at LLN

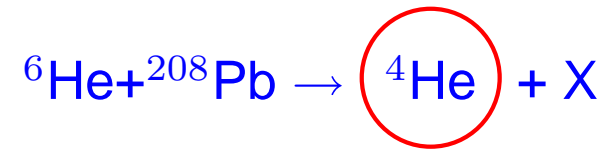
Breakup: angular and energy distributions of α particles (neutrons not recorded)

- Large yield (transfer/breakup?)
- α particles post-accelerated



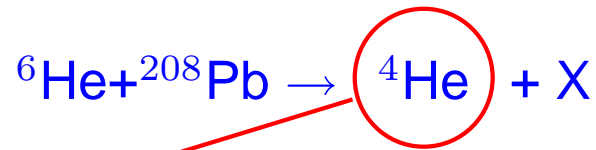
${}^6\text{He} + {}^{208}\text{Pb}$: breakup

What is the mechanism responsible for the production of α 's ?

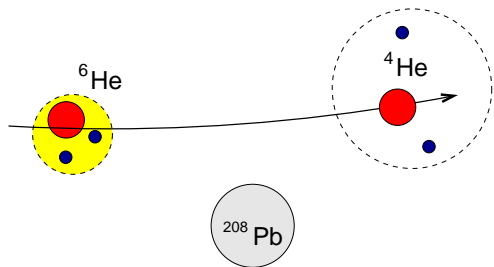


${}^6\text{He} + {}^{208}\text{Pb}$: breakup

What is the mechanism responsible for the production of α 's ?



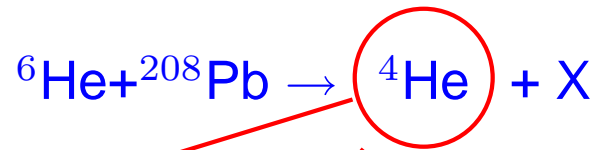
Direct Breakup
(BU)



$$E_{\alpha} \approx \frac{4}{6} E_{\text{beam}}$$

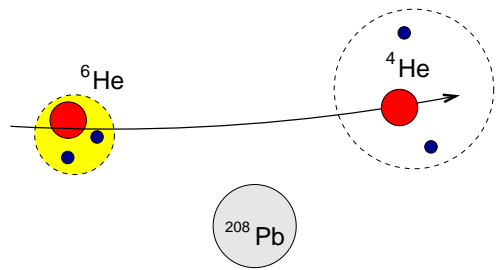
${}^6\text{He} + {}^{208}\text{Pb}$: breakup

What is the mechanism responsible for the production of α 's ?

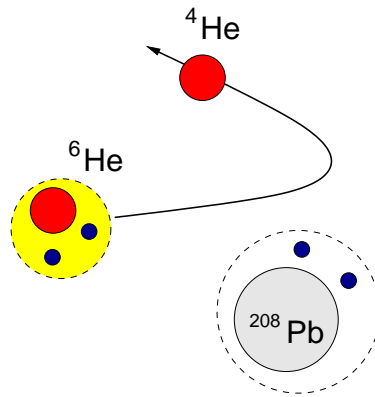


Direct Breakup
(BU)

Transfer to the Continuum
(TC)



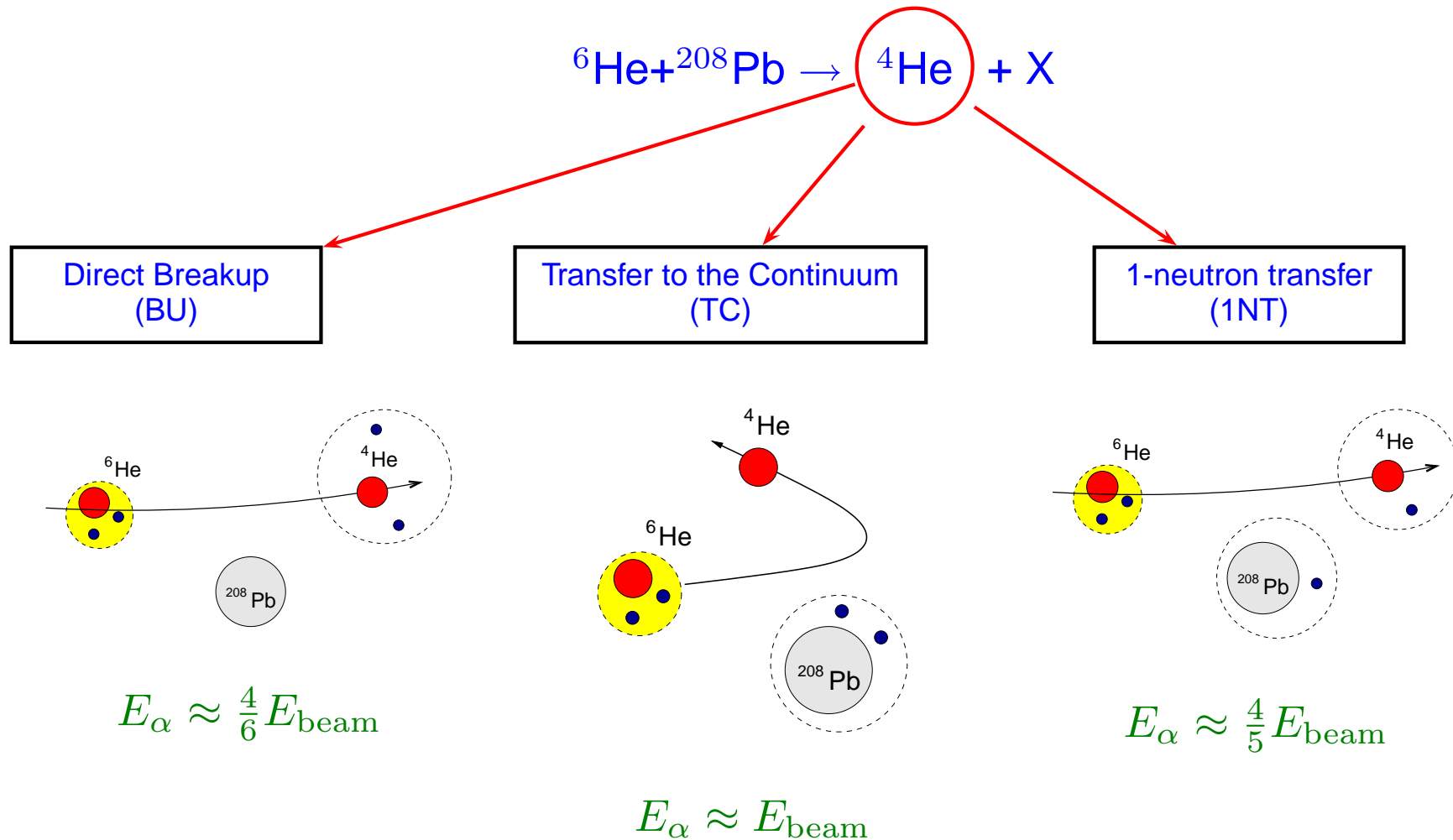
$$E_\alpha \approx \frac{4}{6} E_{\text{beam}}$$



$$E_\alpha \approx E_{\text{beam}}$$

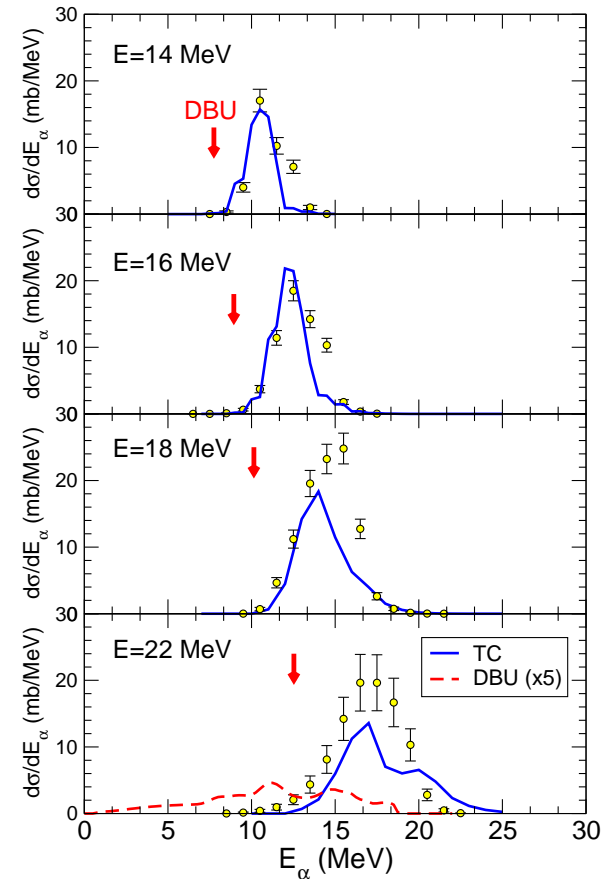
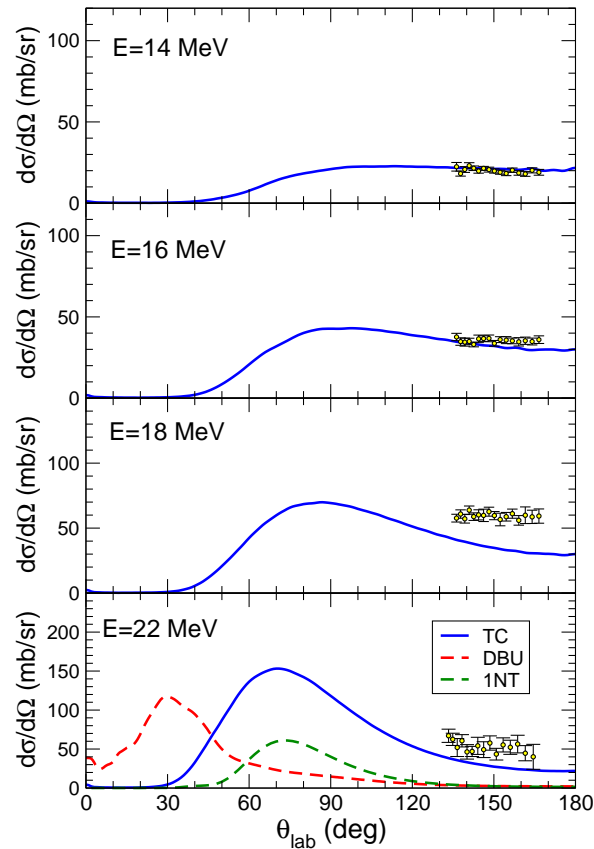
${}^6\text{He} + {}^{208}\text{Pb}$: breakup

What is the mechanism responsible for the production of α 's ?



${}^6\text{He} + {}^{208}\text{Pb}$: breakup

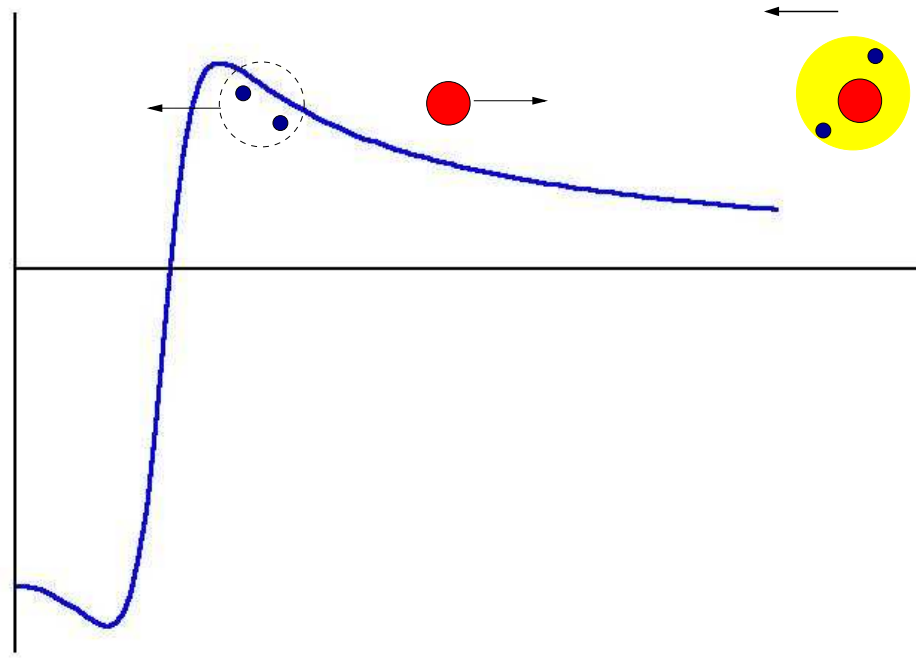
D. Escrig et al, Nucl. Phys. A792 (2007) 2



- The TC model explains satisfactory the magnitude and energy distribution of the measured α 's
- DBU fails to explain the yield and energy of α 's

${}^6\text{He} + {}^{208}\text{Pb}$: breakup

Our calculations suggest a picture in which the α particles are repelled by the Coulomb field, while the neutrons are transferred to highly excited states of the target (transfer to the continuum)



List of collaborators

- J. Gómez-Camacho (Univ. de Sevilla) (DINEX collaboration).
- H. Jeppesen, K. Riisager (Univ. Aarhus) (REX-ISOLDE collaboration)
- F. Nunes (MSU)
- R. Johnson (Univ. Surrey).
- L. Acosta, I. Martel, F. Pérez-Bernal, A. Sánchez-Benítez (Univ. de Huelva)
- D. Escrig, M.J. Borge (CSIC, Madrid)

Thanks!

A hand-drawn illustration of a smiling person with their arms raised, positioned below the word 'Thanks!'. The person has a round face with a wide smile, two dots for eyes, and a few strands of hair. Their arms are raised in a gesture of joy or gratitude. A horizontal line is drawn above the person's head, separating them from the text above. A small '©' symbol is visible at the bottom right of the drawing.