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Shell structure of one-neutron resonances and their decay width in deformed nuclei (with real energy formalism)

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Change of shell structure in weakly-bound and resonant neutron levels

$\longrightarrow$ new region of deformed/spherical nuclei,
due to the unique behavior of weakly-bound small-ℓ neutron levels compared with larger-ℓ neutron levels.

height of centrifugal barrier $\propto \dfrac{\ell(\ell+1)}{R_h^2}$ where $R_h > r_0 A^{1/3}$

$$= \begin{cases} \text{higher for larger } \ell \text{ orbits} \\ \text{higher for smaller nuclei} \end{cases}$$

This change of neutron shell-structure is independent of the proton number of nuclei.
(cf. The change of neutron shell-structure due to tensor force depends systematically on the proton number.)

Whether a given nucleus with weakly-bound neutrons will deform or not depends, of course, also on the proton number.

ex. Some proton numbers (ex. Z=8, oxygen isotopes) do not want to be deformed.

In nuclei at (or just outside) the neutron drip line

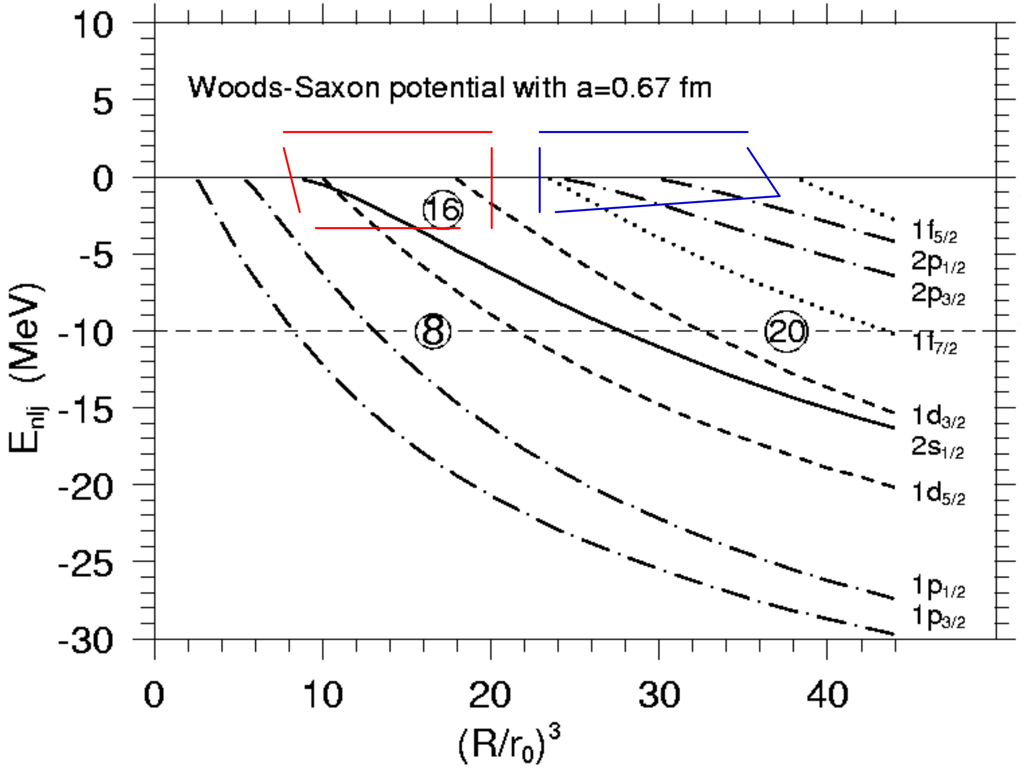
low-lying resonances can have one-particle character in the mean field approximation.

cf. Neutron resonances in stable nuclei have such high excitation energies
with very complicated wave-functions.

Spherical nuclei

Unique behavior of low-ℓ orbits, as $E_{n\ell j}$ $(<0) \rightarrow 0$

Energies of neutron orbits in Woods-Saxon potentials as a function of potential radius



Fermi level of neutron drip line nuclei

Fermi level of stable nuclei

R = radius, r_0 = 1.27 fm,

V_{WS} = – 51 MeV

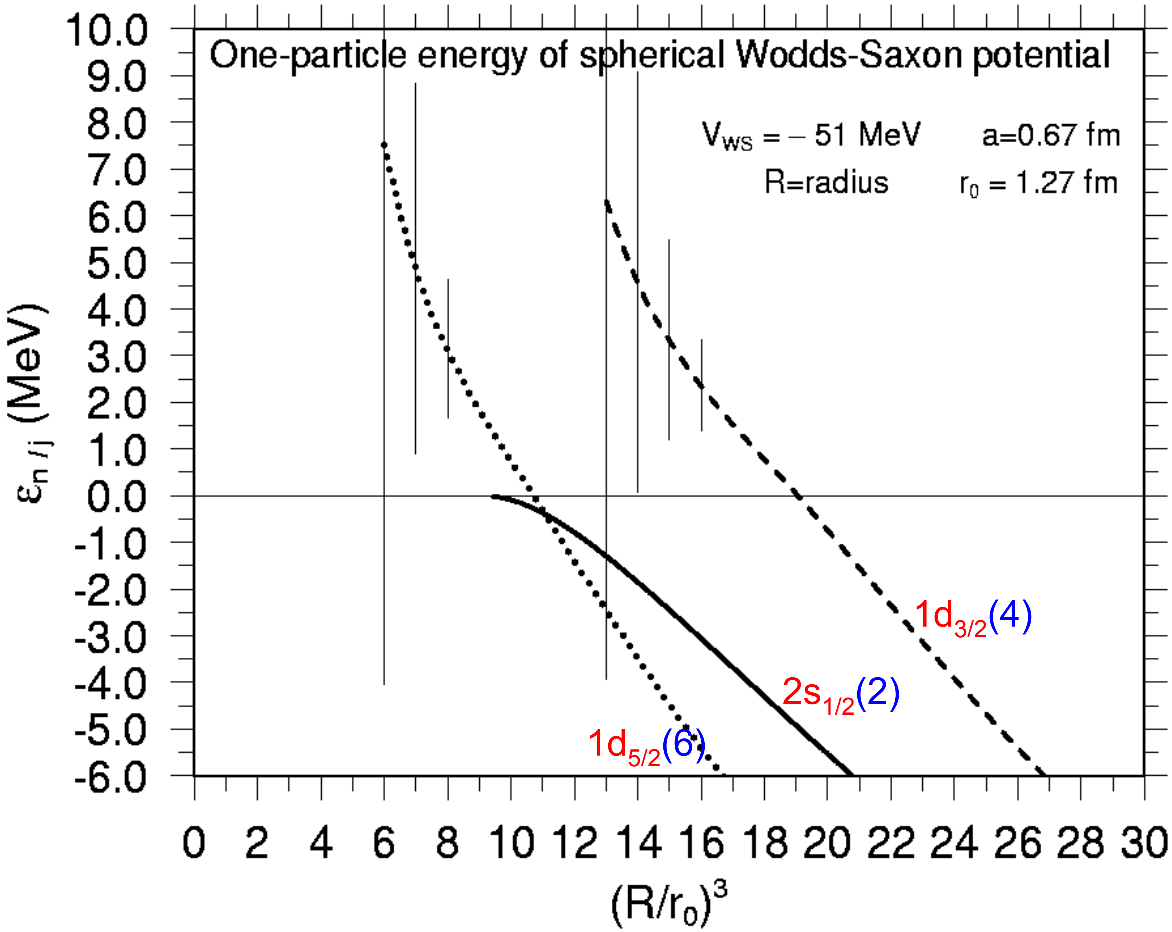
Strength of the potential

For stable nuclei

$(R/r_0)^3 \rightarrow$ A : mass number

Neutron one-particle resonant and bound levels in spherical Woods-Saxon potentials

Unique behavior of $\ell=0$ orbits, both for $\varepsilon_{n\ell j} < 0$ and $\varepsilon_{n\ell j} > 0$



Strength of the potential

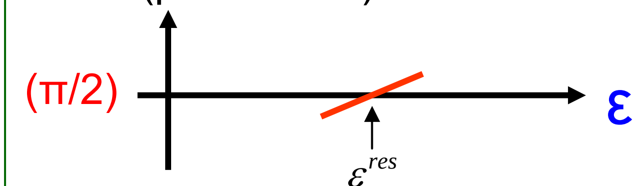
One-particle resonant levels with width

$$R_{\ell j}(r) \propto \sin\left(kr + \delta_{\ell j} - \ell\frac{\pi}{2}\right)$$

for $r \to \infty$ and $kr \equiv r\sqrt{\frac{2m\varepsilon}{\hbar^2}}$

width $\Gamma \equiv \frac{2}{\left.\frac{d\delta}{d\varepsilon}\right|_{\varepsilon=\varepsilon^{res}}}$

δ (phase shift)



In deformed nuclei

not only shell structure of one-particle resonant levels are different from spherical potentials,

but also the decay widths can be very different from those of $l=(\ell j)$ levels in spherical potentials.

Study of one-particle resonant levels

Complex energy formalism

$$\left\{ \begin{array}{l} \text{wave functions} \\ \text{transition probabilities} \\ \text{- - - - - - -} \end{array} \right\} \quad \text{become complex values}$$

Real energy formalism

Proton emission (from deformed nuclei outside the proton drip line)

has been extensively studied using complex energy variable.

E.Maglione, L.S.Ferreira and R.J.Liotta, PRL **81** (1998) 538 ; PRC **59** (1999) R589.

Marked differences from neutron decay in neutron drip line nuclei are ;

High Coulomb barrier especially for medium-heavy nuclei

$\Longrightarrow$ (1) Almost no change in the shell structure of weakly-bound and resonant one-particle levels, compared with the shell structure of stable nuclei ;

(2) Very small decay width even for $s_{1/2}$ protons ;

(3) Electromagnetic transitions within a band are much faster than the proton decays $\rightarrow$ proton emission only from band-head levels

One-particle resonance in a deformed potential – eigenphase formalism

$$H\Psi_\Omega = \varepsilon_\Omega \Psi_\Omega$$

where one-particle wave function $\Psi_\Omega(\vec{r}) = \frac{1}{r}\sum_{\ell j} R_{\ell j\Omega}(r) Y_{\ell j\Omega}(\hat{r})$

one-particle energy ε_Ω

and $Y_{\ell j\Omega}(\hat{r}) \equiv \sum_{m_\ell, m_s} C(\ell, \frac{1}{2}, j; m_\ell, m_s, \Omega) Y_{\ell m_\ell}(\hat{r}) \chi_{m_s}$

Solving the coupled differential equations derived from the Schrödinger equation with the boundary conditions,

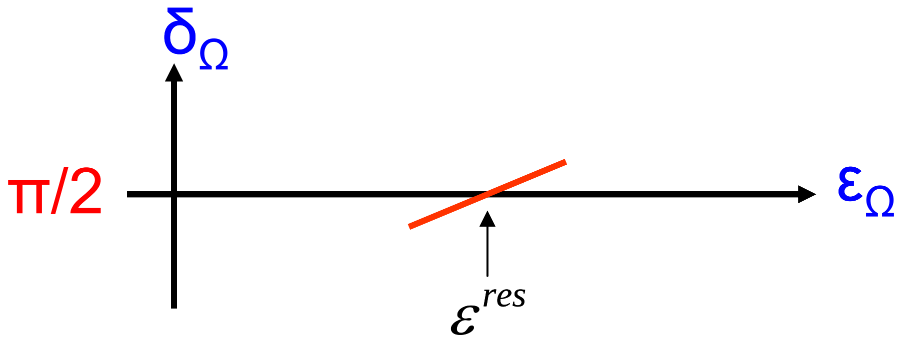
$$\begin{cases} R_{\ell j\Omega}(r) = 0 & \text{for} \quad r = 0 \\ R_{\ell j\Omega}(r) \propto \cos(\delta_\Omega) F_\ell(r) - \sin(\delta_\Omega) G_\ell(r) & \text{for} \quad r \to \infty \end{cases}$$

where $F_\ell(r) = kr j_\ell(kr)$, $G_\ell(r) = kr n_\ell(kr)$ $\qquad k^2 = \frac{2m}{\hbar^2}\varepsilon_\Omega$

δ_Ω : eigenphase common to all ℓj channels

A given eigenchannel : asymptotic radial wave-functions behave in the same way for all ($\ell j\Omega$) components.

A one-particle resonant level with ε_Ω is defined so that one eigenphase δ_Ω increases through $\pi/2$ as ε_Ω increases.



When one-particle resonant level in terms of one eigenphase is obtained, the width Γ_Ω of the resonance in the intrinsic system is calculated by

$$\Gamma_\Omega \equiv \frac{2}{\left[\dfrac{d\delta_\Omega}{d\varepsilon_\Omega}\right]_{\varepsilon_\Omega=\varepsilon_\Omega^{res}}}$$

: intrinsic width

I.H., Phys. Rev. C **73**, 064308 (2006)

Some comments on eigenphase ;

1) For a given potential and a given ε_Ω
there are several (in principle, an infinite number of) solutions of eigenphase δ_Ω .

2) The number of eigenphases for a given potential and a given ε_Ω is equal to that of wave function components with different (ℓ,j) values.

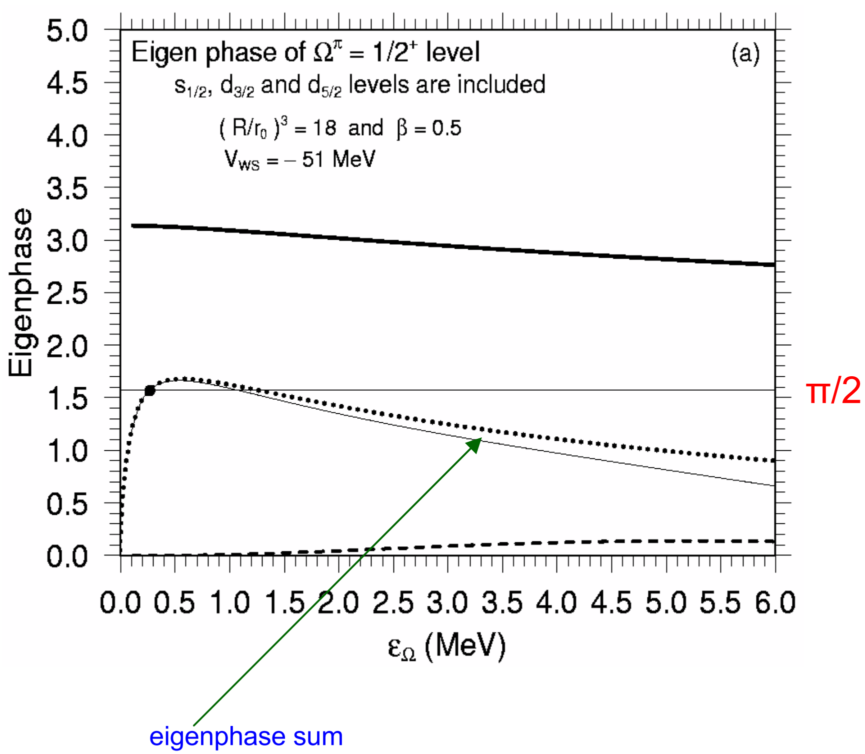
3) The value of δ_Ω determines the relative amplitudes of different (ℓ,j) components.

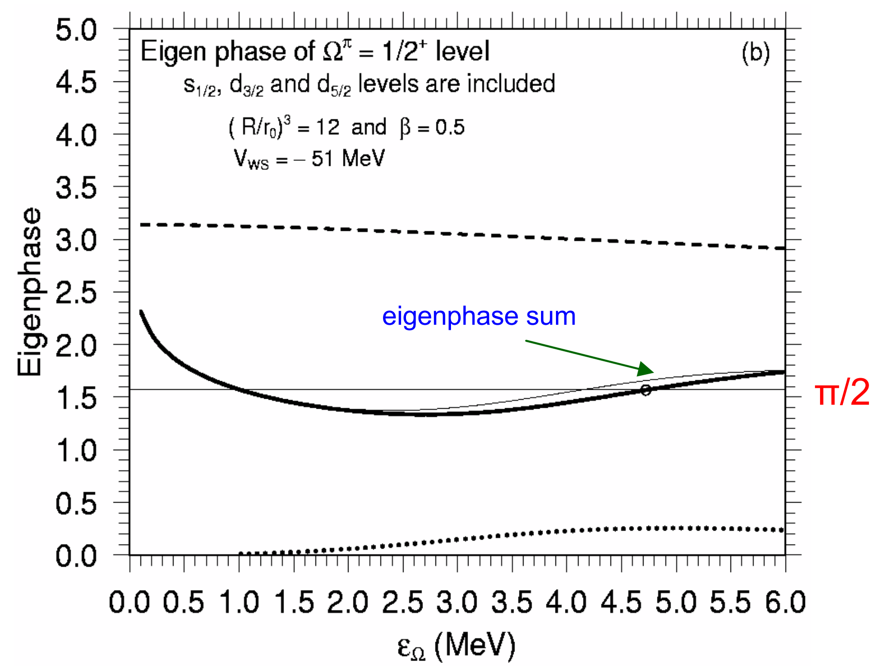
4) In the region of small values of ε_Ω (> 0), only one of eigenphases varies strongly as a function of ε_Ω , while other eigenphases remain close to the values of $n\pi$.

In the limit of $\beta \rightarrow 0$, the definition of one-particle resonance in eigenphase formalism

$\longrightarrow$ the definition in spherical potentials, in terms of phase shift.

Ex. Variation of all three eigenphases of $\Omega^{\pi} = 1/2^{+}$
($s_{1/2}$, $d_{3/2}$ and $d_{5/2}$ levels are included in the coupled channels.)



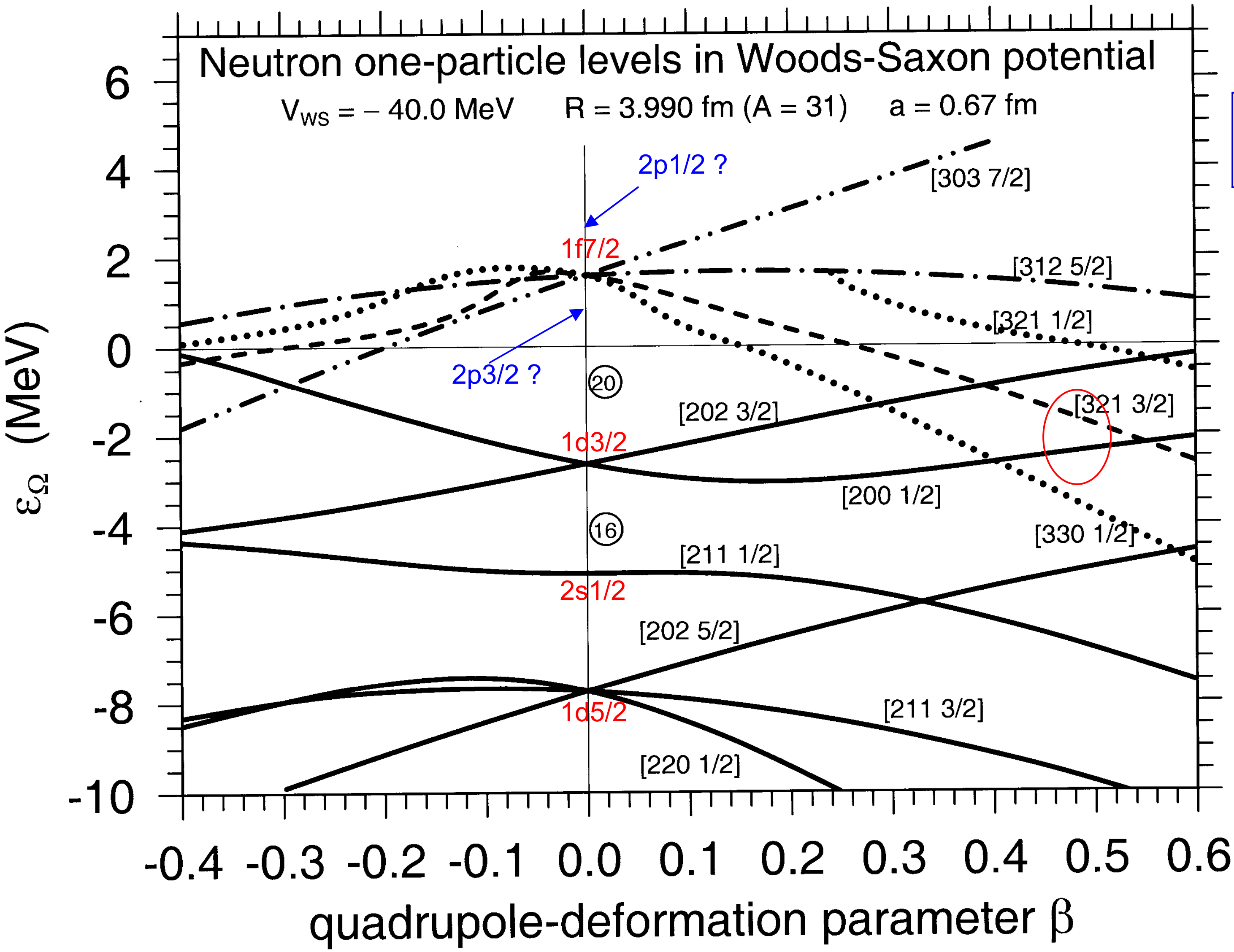


No weakly-bound Nilsson level with $\Omega^{\pi}=1/2^{+}$ is present for this potential.

A weakly-bound Nilsson level with $\Omega^{\pi}=1/2^{+}$ is present for this potential.

Near degeneracy of one-particle resonant levels, { $2p_{1/2}$, $1f_{7/2}$, $2p_{3/2}$ }, at β=0 is unexpected.

- an origin of deformation (Jahn-Teller effect) of "island of inversion" !?



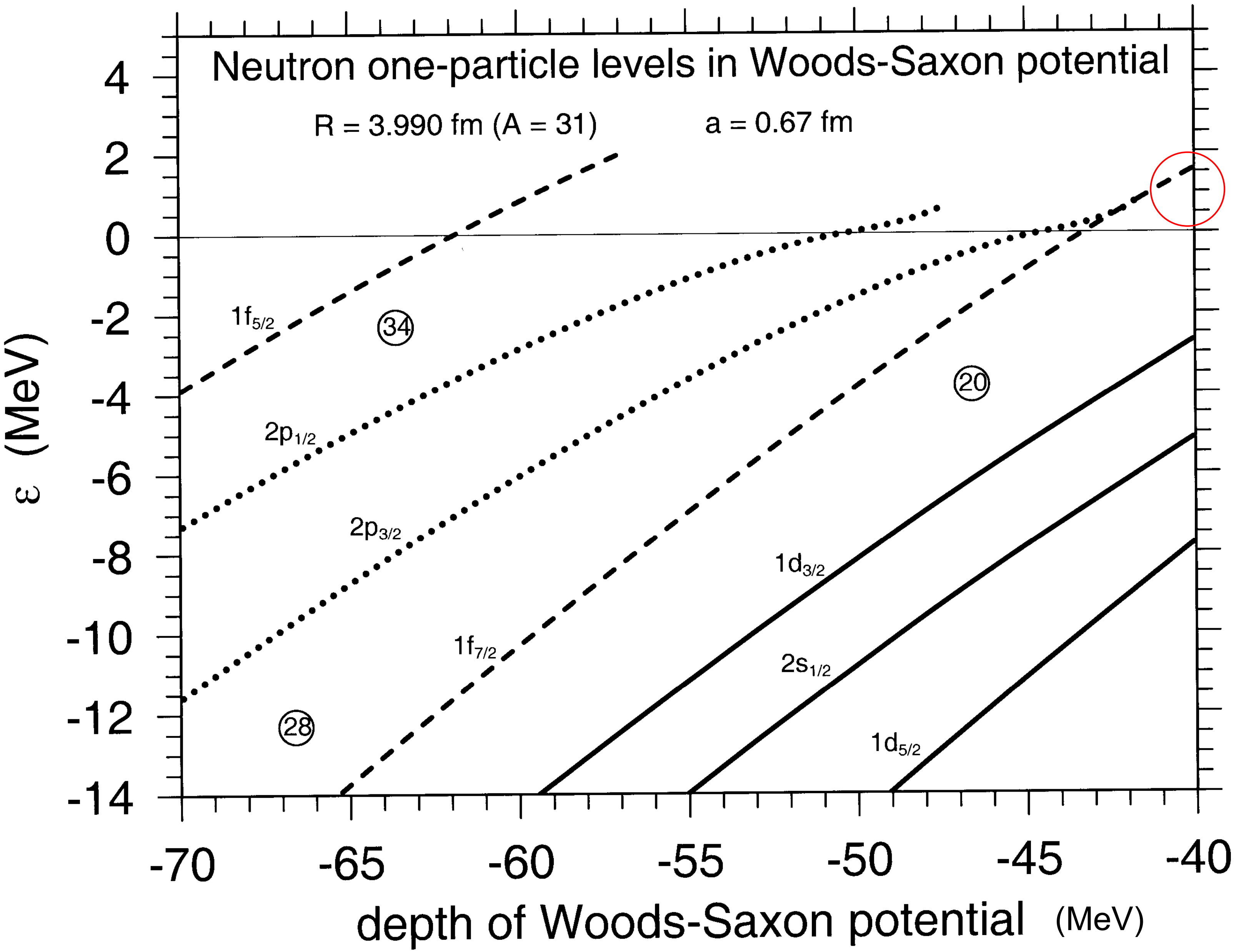
At β=0 ;
ε(2p3/2) < ε(1f7/2)

$\varepsilon(1f_{5/2})$ = +8.96 MeV

$^{33}Mg_{21}$ (3/2–)
S(n) = 2.22 MeV

$^{31}Mg_{19}$ (1/2+)
S(n) = 2.38 MeV

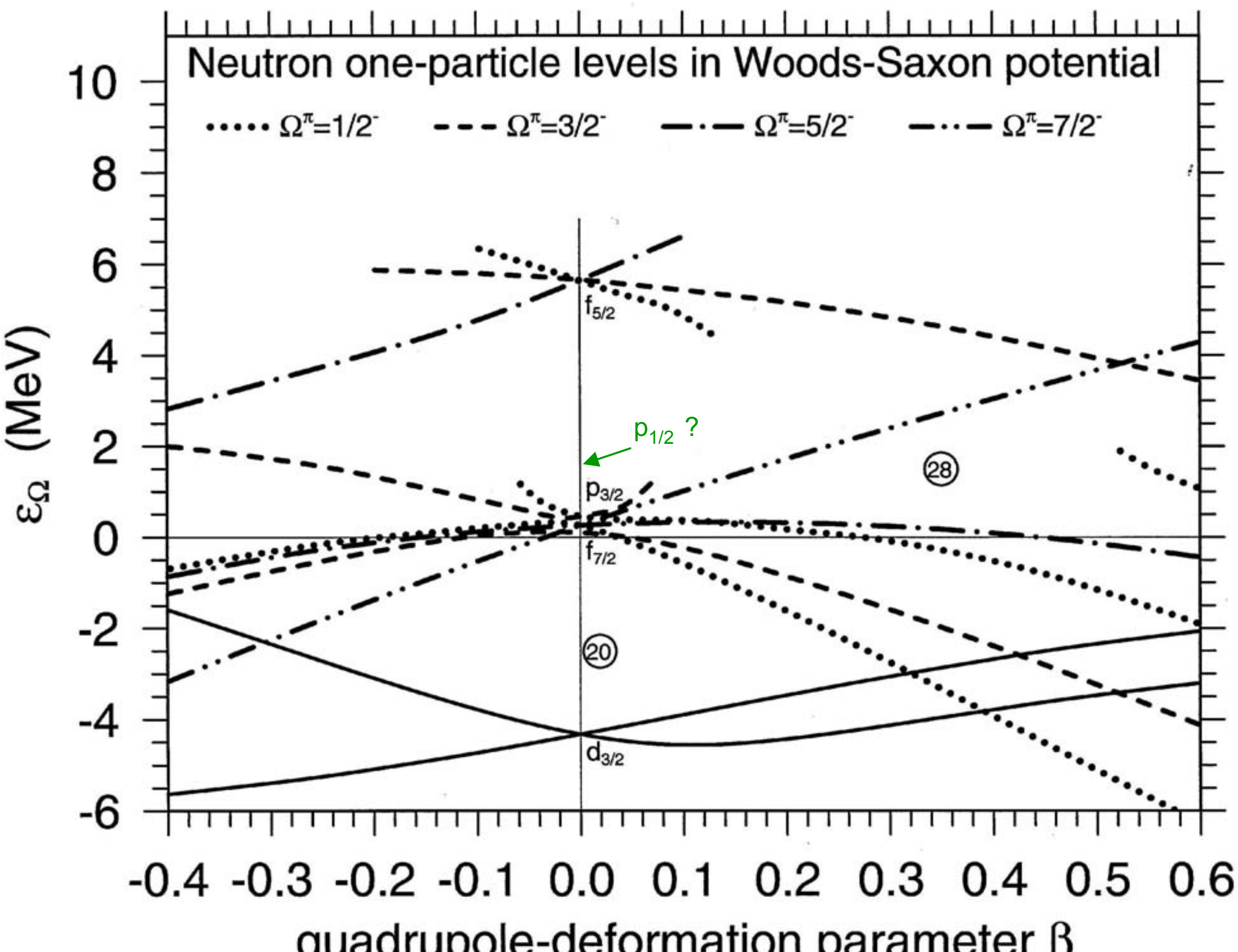
I.H., PRC**76**, 054319 (2007)



Neutron one-particle levels in Woods-Saxon potential
R = 3.990 fm (A = 31)
a = 0.67 fm
ε (MeV)
depth of Woods-Saxon potential (MeV)
4
2
0
-2
-4
-6
-8
-10
-12
-14
-70
-65
-60
-55
-50
-45
-40
$1f_{5/2}$
$2p_{1/2}$
$2p_{3/2}$
$1f_{7/2}$
$1d_{3/2}$
$2s_{1/2}$
$1d_{5/2}$
34
20
28

The parameters of the Woods-Saxon potential are designed approximately for the unbound nucleus $\boxed{{}^{39}_{12}Mg_{27}}$ (a = 0.67 fm, R = 4.31 fm, V_{WS} = –37 MeV)

The one-particle level which the N=27th neutron will occupy is unbound.



At β=0 ;
ε(2p3/2) – ε(1f7/2) = 180 keV

For ε(2p3/2) ≈ ε(1f7/2)

the N=27th neutron has

$$\Omega^{\pi} = 5/2^{-}$$

for moderate prolate-deformed shape.

If N=28 energy gap > 2 MeV, the N=27th neutron may have

$$\Omega^{\pi} = 7/2^{-} \text{ or } ½^{-}$$

for moderate prolate-deformed shape.

I.H., PRC **77** (2008) 054311

Decay ; a band-head resonant state in odd-N nuclei

$\rightarrow$ 0^+ ground state of neighboring even-even nuclei

excited configurations of nuclei inside the drip line or
lowest-lying states of nuclei just outside the neutron drip line

Partial decay width using real-energy formalism

For large values of r, where the potential is negligible,

$$R_{\ell j\Omega}(r) \propto \cos(\delta_\Omega)F_\ell(r)+\sin(\delta_\Omega)G_\ell(r) \propto G_\ell(r) \quad \text{for} \quad \delta_\Omega = \frac{\pi}{2}$$

$$F_\ell(r) \equiv krj_\ell(kr)$$
$$G_\ell(r) \equiv krn_\ell(kr)$$

Considering the normalization condition of continuum radial wave functions

$$\int_0^\infty r^2 dr \phi_{\varepsilon_1}^\ell(r)\phi_{\varepsilon_2}^\ell(r) = \delta(\varepsilon_1-\varepsilon_2)$$

the relative probability of the (ℓj) component for a given energy is given by

$$\left(\alpha_{\ell j\Omega}(r)\right)^2 \equiv \underline{\frac{2}{2j+1}}\left(\frac{R_{\ell j\Omega}(r)}{G_\ell(r)}\right)^2 \quad \text{where} \quad \alpha_{\ell j\Omega}(\cancel{r}) \quad \text{for} \quad r \gg R_0 \quad \text{(: potential radius)}$$

kinematical factor in [the intrinsic system $\rightarrow$ the lab system]

Probability of finding neutron in the (ℓj) channel for $r \gg R_0$

$$P_{\ell j\Omega} = \frac{(\alpha_{\ell j\Omega})^2}{\sum_{\ell' j'}(\alpha_{\ell' j'\Omega})^2}$$

Partial neutron decay width for a band-head $I=j=\Omega$ in odd-N nuclei $\rightarrow$ 0^+ ground state of e-e nuclei

$$\Gamma_{\ell j\Omega} = \Gamma_\Omega P_{\ell j\Omega}$$

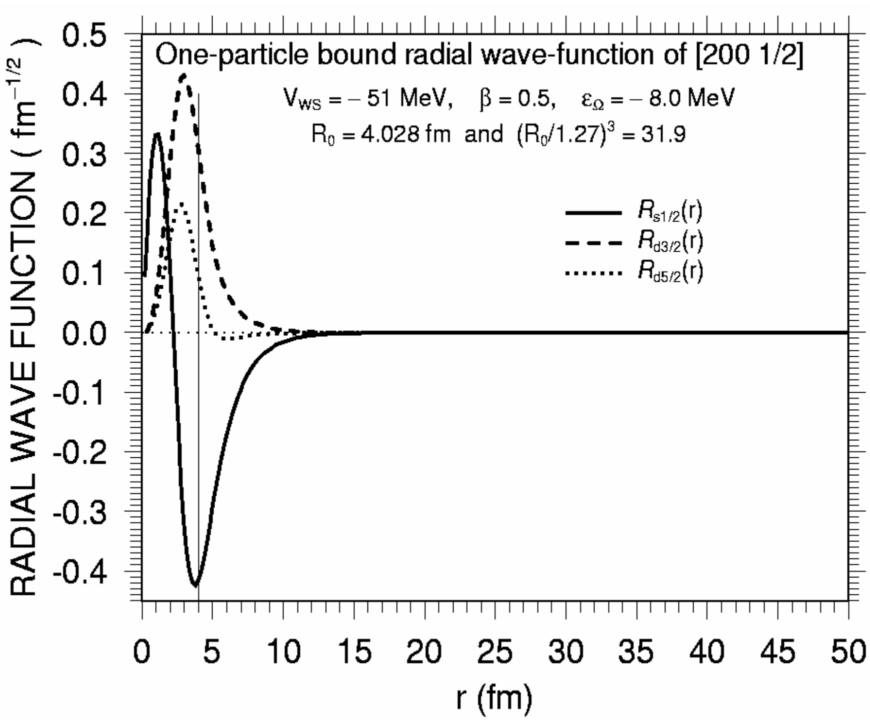
$P_{\ell j\Omega}$: often very different from the probability estimated using wave functions inside the potential.

Radial wave functions of the [200 ½] level in Woods-Saxon potentials

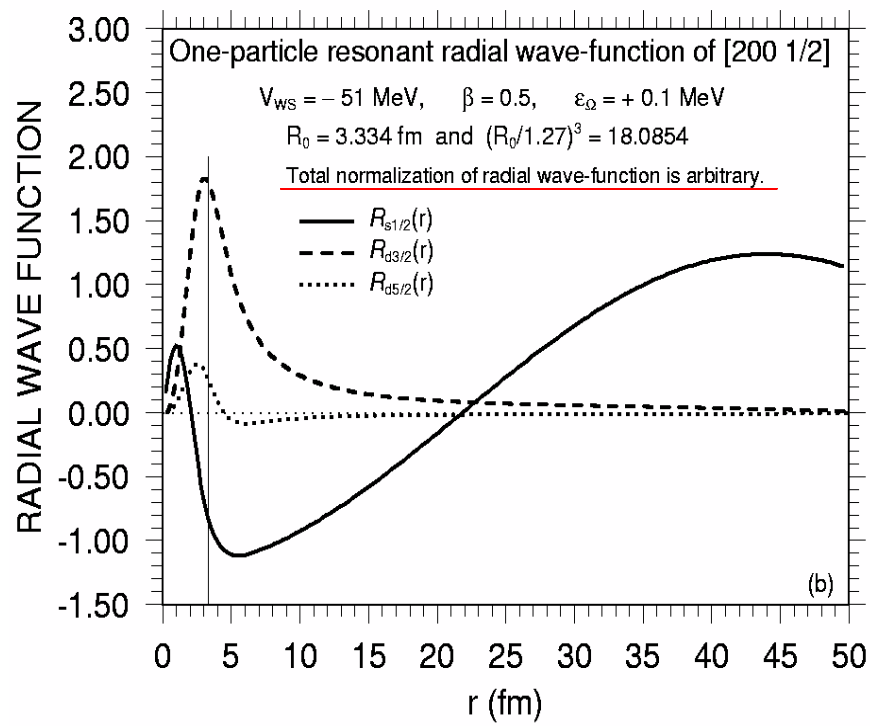
The potential radius is adjusted to obtain respective eigenvalue ($\varepsilon_\Omega < 0$) and resonance ($\varepsilon_\Omega > 0$).

$s_{1/2}$ —— $d_{3/2}$ - - - - $d_{5/2}$ ·········

Bound state with ε_Ω = -8.0 MeV.



Resonant state with ε_Ω = +100 keV



Similar behavior to wave functions in harmonic osc. potentials.

Existence of resonance ← d component
Width of resonance ← s component

ex. $\boxed{{}^{13}_{4}Be_9}$ (100 – 200 keV outside the neutron drip line)

Measured spectroscopic properties of ${}^{12}_{4}Be_8$ $\Rightarrow$ strongly deformed

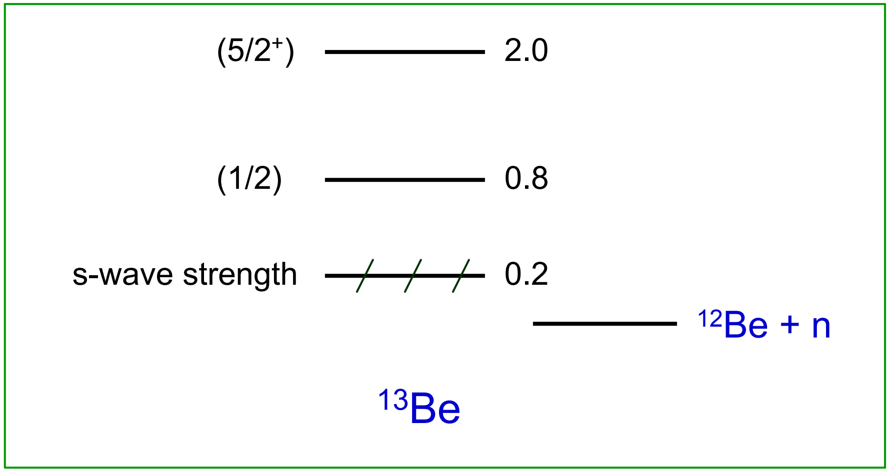
Theoretical interpretation of available spectroscopic data on ^{11}Be and ^{12}Be

$\Rightarrow$ Both ^{11}Be and ^{12}Be are strongly deformed

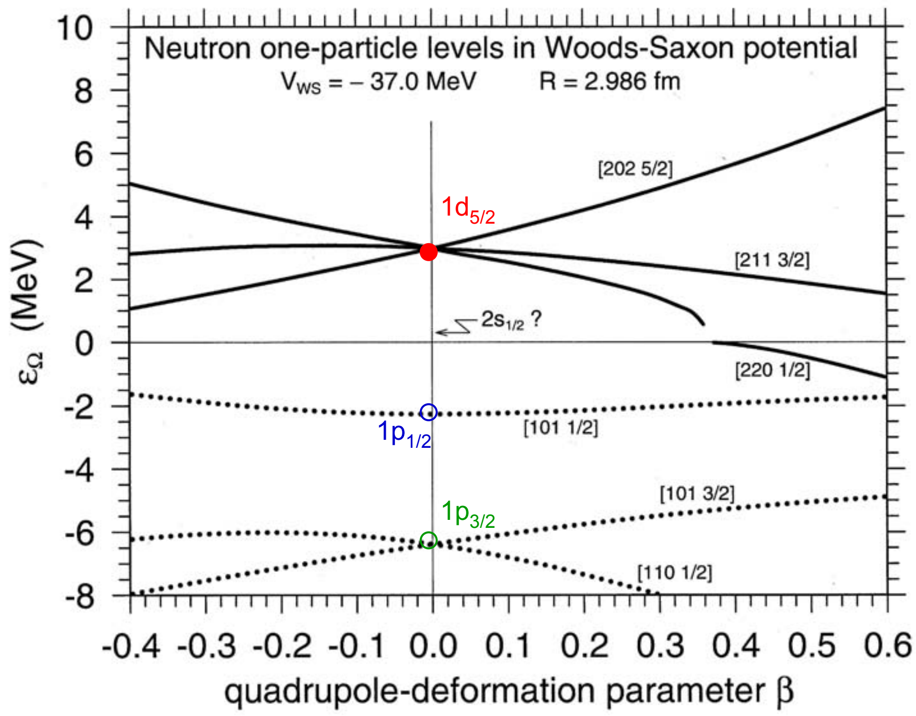
Experimental information on low-lying levels of ^{13}Be is not clear !

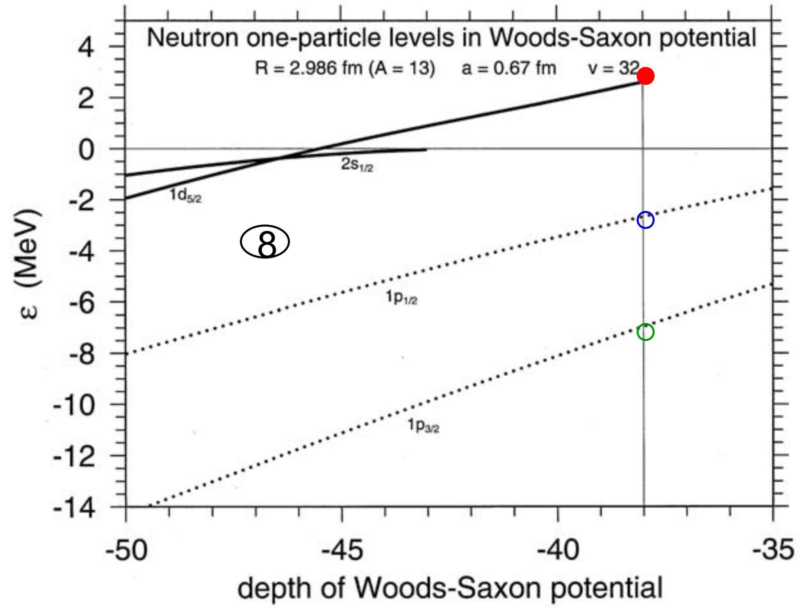
M.Thoennessen et al., PRC **63** (2000) 014308

Sequential neutron decay spectroscopy



$^{13}_{4}Be_{9}$?





Properties of the [220 1/2] resonance on prolate side

deformation(β)	ε_{res} (MeV)	Γ_Ω (MeV)	$P_{s1/2}$	$\Gamma_{s1/2}$ (MeV)
0	2.99	1.74	0	0
0.20	2.06	1.36	0.656	0.89
0.30	1.41	2.05	0.914	1.88
0.36	0.52	∞	0.994	∞

partial neutron decay width

The lowest state of ^{13}Be would be a broad resonance-like $I^{\pi} = 1/2^{+}$ state,
irrespective of whether ^{13}Be is spherical or deformed.

If ^{13}Be is moderately prolate-deformed,
the 2^{nd} lowest one-particle resonant level may have again $I^{\pi} = 1/2^{+}$,
of which

$\Gamma_{1/2^{+}} << \Gamma_{s_{1/2}}$ for spherical shape at the same ε_{res}