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SEZIONE DI PISA

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Pisa: Torre Pendente

## Unbound nuclei studied via transfer to the continuum reactions

**D. M. Brink, A. Bonaccorso, N.Vinh Mau**

- Semi-classical theory of transfer to the continuum
- Example of  $n+{}^9\text{Li}$
- Example of  $n+{}^{12}\text{Be}$
- Conclusions

# Semiclassical treatment of core-target relative motion, **BUT** full QM treatment of n-target interaction

AB and DM Brink, PRC38, 1776 (1988), PRC43, 299 (1991), PRC44, 1559 (1991).

$$\frac{d\sigma}{d\varepsilon_f} = C^2 S \int_0^\infty d\mathbf{b}_c \frac{dP(b_c)}{d\varepsilon_f} P_{el}(b_c), \quad \text{where} \quad P_{el}(b_c) = |S_{cT}|^2$$

$$P_{ct}(b_c) = e^{(-\ln 2 \exp[(R_s - b_c)/a])}$$

$$R_s \approx 1.4(A_p^{1/3} + A_t^{1/3})$$

$$A_{if} = \frac{1}{i\hbar} \int dt \langle \psi_f(t) | V(r) | \psi_i(t) \rangle$$

$$V(r) = U(r) + iW(r)$$

**A study of semi-classical approximations for heavy ion transfer reactions,**

**H. Hasan and D.M. Brink, J Phys G4, 1573 (1978).**

**Perturbation approach to nucleon transfer in heavy ion reactions,**

**L. Lo Monaco and D.M. Brink, J.Phys. G, 935 (1985).**

## Understanding the transfer and breakup mechanisms

$$\begin{aligned} \frac{dP_t(b_c)}{d\varepsilon_f} &= \frac{1}{8\pi^3} \frac{mk_f}{\hbar^2} \frac{1}{2l_i + 1} \sum_{m_i} |A_{fi}|^2 \\ &\approx \frac{4\pi}{2k_f^2} \sum_{j_f} (2j_f + 1) (|1 - \bar{S}_{j_f}|^2 + 1 - |\bar{S}_{j_f}|^2) (1 + F_{l \rightarrow j}) B_{l_f, l_i} \\ &= \sigma_{nN}(\varepsilon_f) \mathcal{F} \end{aligned}$$

elastic      absorption  
diffraction      stripping

enhancement factor of final state interaction theory

$$B_{l_f, l_i} = \frac{1}{4\pi} \left[ \frac{k_f}{mv^2} \right] |C_i|^2 \frac{e^{-2\eta b_c}}{2\eta b_c} M_{l_f l_i}$$

k<sub>f</sub> ≡ (iη, k<sub>z</sub>)      |ψ̃<sub>i</sub>|<sup>2</sup> Fourier transform of initial w. f.      angular parts of ψ<sub>i, f</sub>

## If both initial and final state have $l=0$

Bound to bound

$$\sigma(\varepsilon_f) = \frac{\pi}{2} |C_i C_f|^2 \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty db_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{(-\ln 2 \exp[(R_s - b_c)/a])}$$

Bound to continuum

$$\frac{d\sigma}{d\varepsilon_f} = \left( \frac{\sin \delta_0}{k_f} \right)^2 |C_i|^2 \frac{mk_f}{\hbar^2} \left[ \frac{\hbar}{mv} \right]^2 \int_0^\infty db_c \frac{e^{-2\eta b_c}}{\eta b_c} e^{(-\ln 2 \exp[(R_s - b_c)/a])}$$

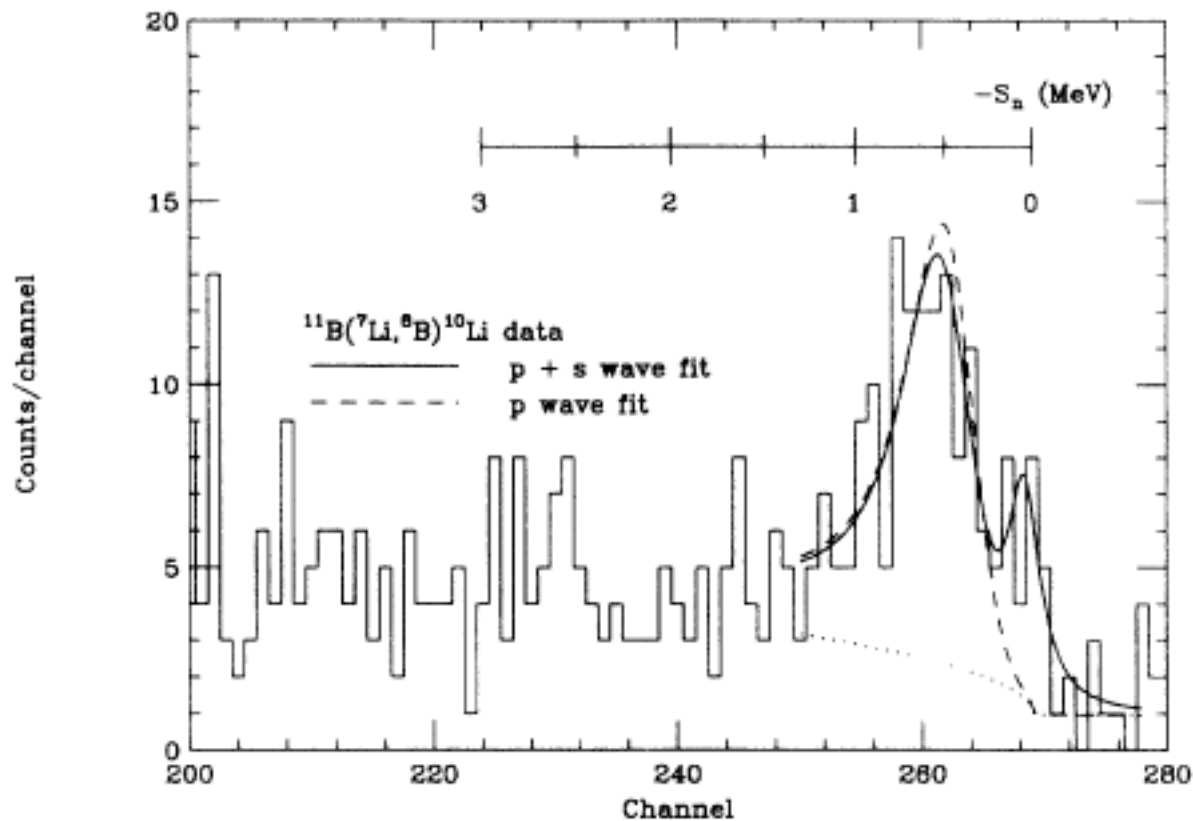
scattering length

$$a_s = - \lim_{k \rightarrow 0} \frac{\tan \delta_0}{k}$$

Low-lying structure of  $^{10}\text{Li}$  in the reaction  $^{11}\text{B}(^7\text{Li},^8\text{B})^{10}\text{Li}$ 

B. M. Young, W. Benenson, J. H. Kelley, N. A. Orr,\* R. Pfaff, B. M. Sherrill, M. Steiner, M. Thoennessen,  
J. S. Winfield, J. A. Winger,<sup>†</sup> S. J. Yennello,<sup>†</sup> and A. Zeller  
*National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University,  
East Lansing, Michigan 48824*

(Received 22 June 1992)



The first excited state of  ${}^9\text{Li}$  is at  $E^*=2.7$  MeV

→  $p_{1/2}$  state is at 0.595 MeV

→ s-state is virtual

↳ No inelastic channel

↳ No use of the stripping part  
of the transfer cross section

# Potential correction which originates from particle-vibration couplings

N. Vinh Mau and J. C. Pacheco, Nucl. Phys. **A607** (1996) 163.

$$U(r) = V_{WS} + \delta V$$

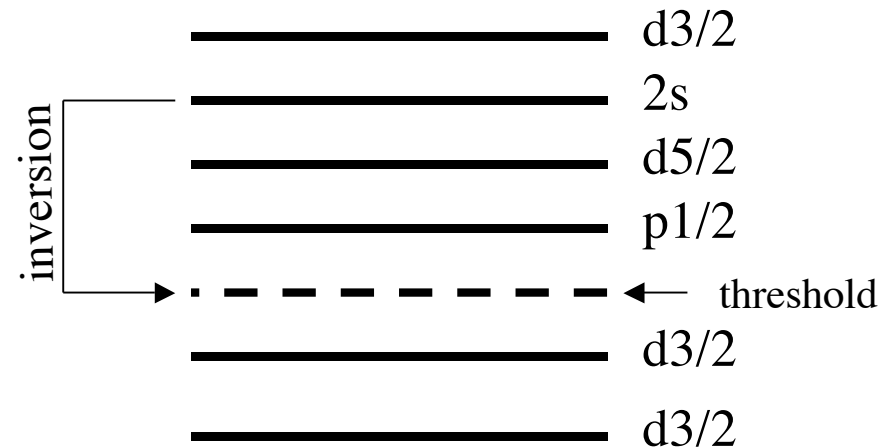
$$\delta V(r) = 16\alpha e^{2(r-R)/a} / (1 + e^{(r-R)/a})^4$$

with  $R \approx r_0 A^{1/3}$

For  $n+{}^9\text{Li}$ :

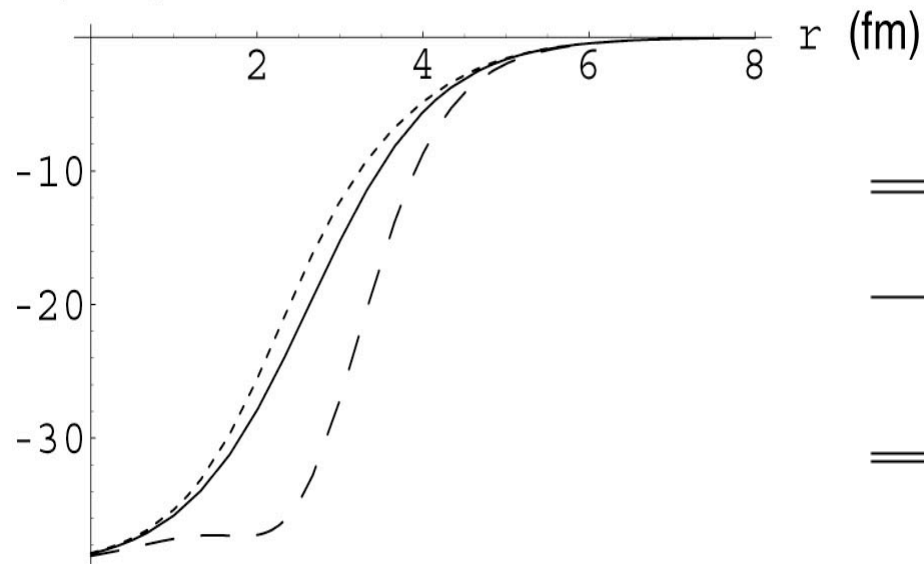
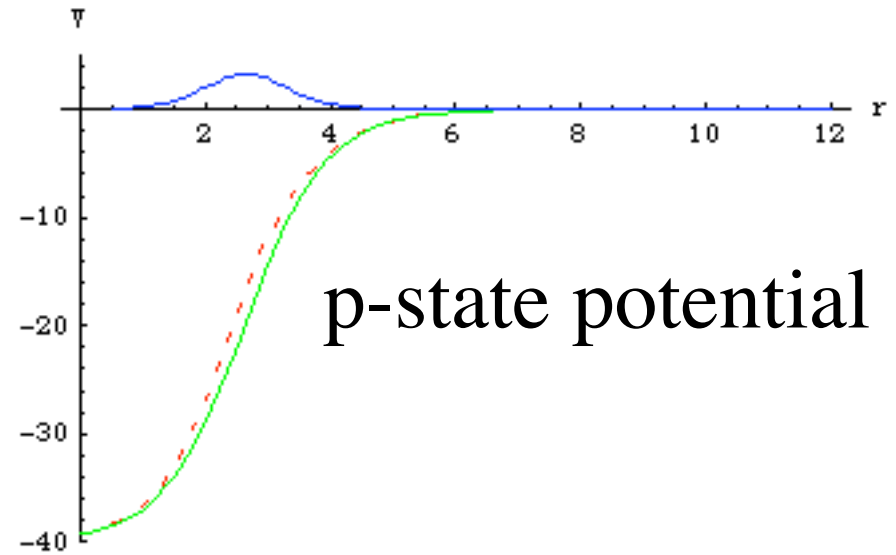
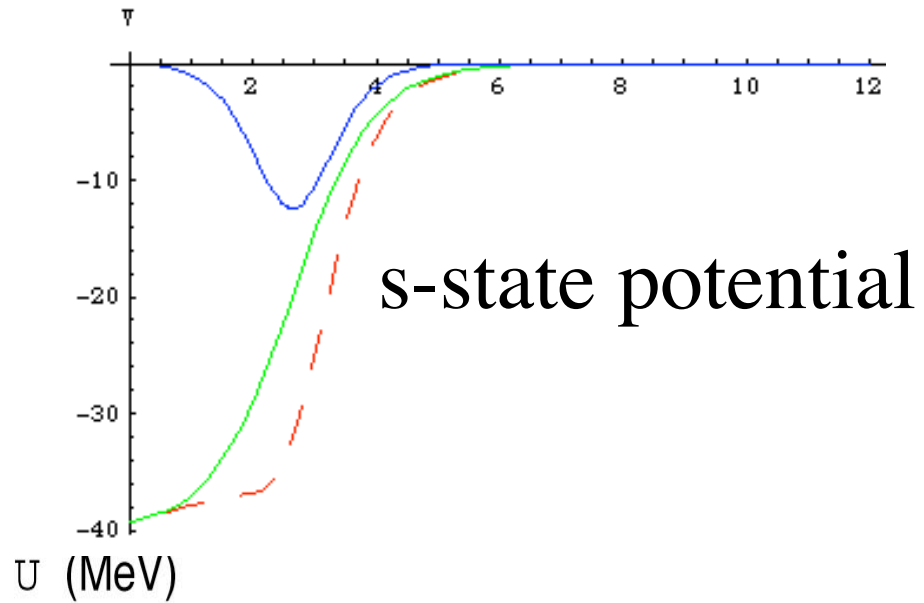
$\alpha = -12.5$  for the s-state

$\alpha = 3.33$  for the p-state



# Potential model for n+<sup>9</sup>Li continuum

$$U(r) = V_{WS} + \delta V$$



## Resonance states in <sup>10</sup>Li

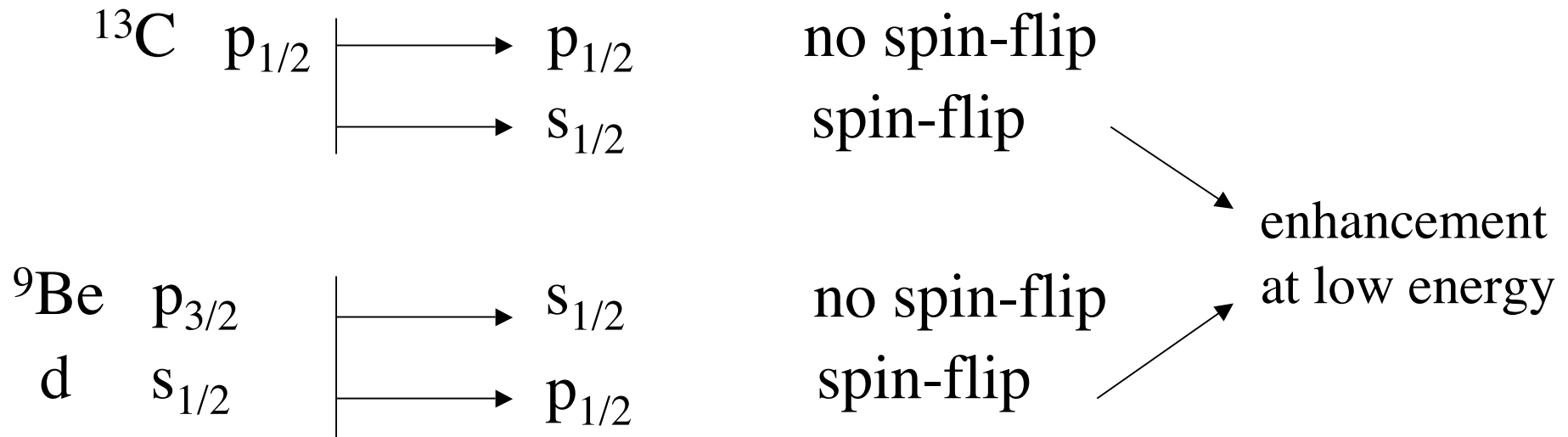
	$\epsilon_{res}$ (MeV)	$\Gamma$ (MeV)	$a_s$ (fm)	$\alpha$ (MeV)
$2s_{1/2}$			323	-12.5
			-17.20	-10.0
$1p_{1/2}$	0.595	0.48		3.3



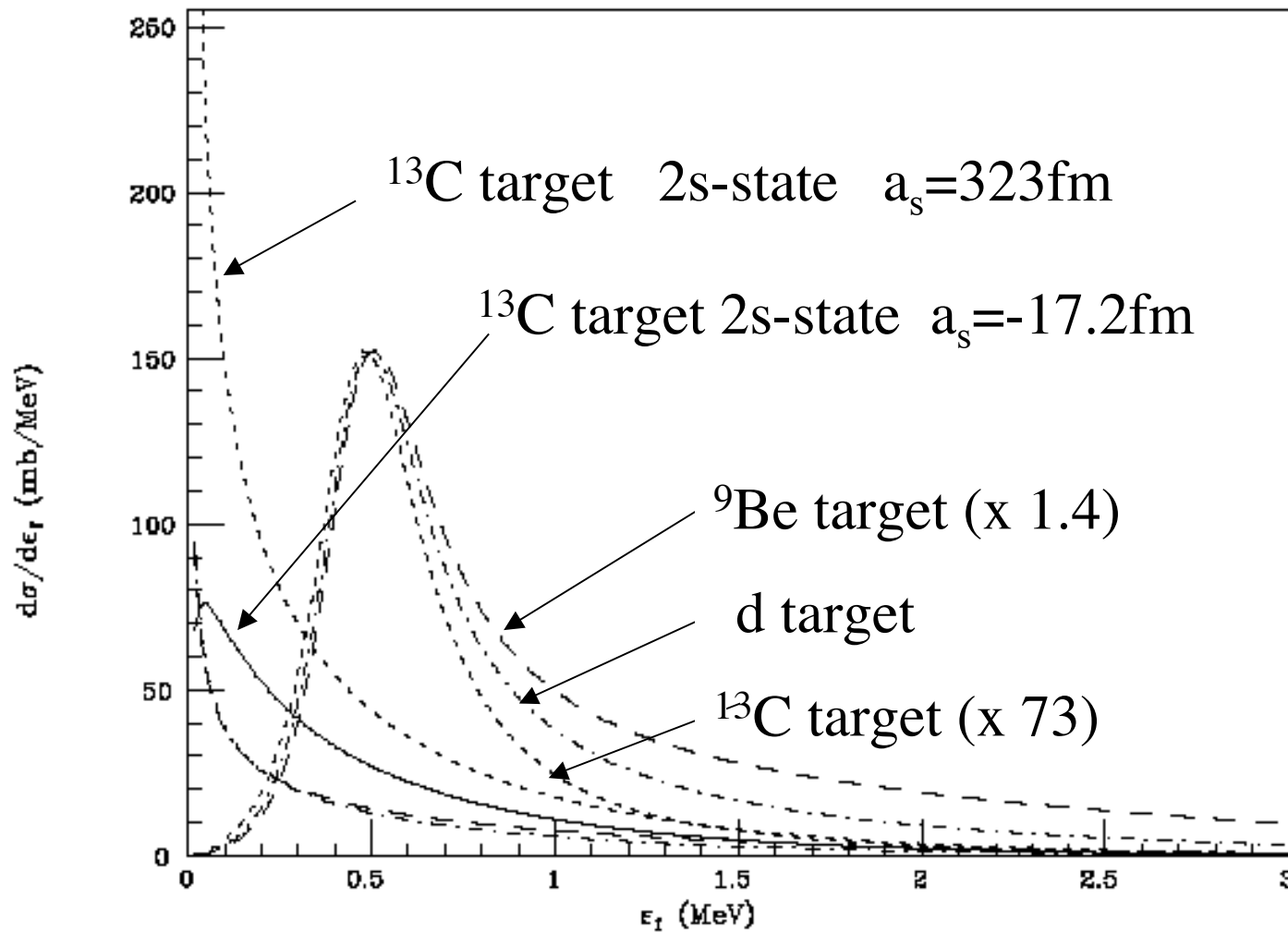
# Targets and neutron bound initial state parameters:

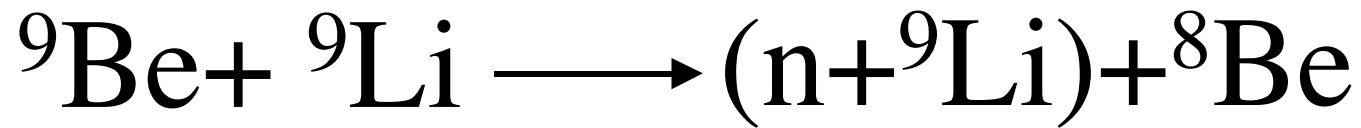
Target	$d$	${}^9\text{Be}$	${}^{13}\text{C}$
$\varepsilon_i(\text{MeV})$	-2.22	-1.66	-4.95
$l_i$	0	1	1
$j_i$	1/2	3/2	1/2
$C_i(\text{fm}^{-\frac{1}{2}})$	0.95	0.68	1.88

at 2 A.MeV

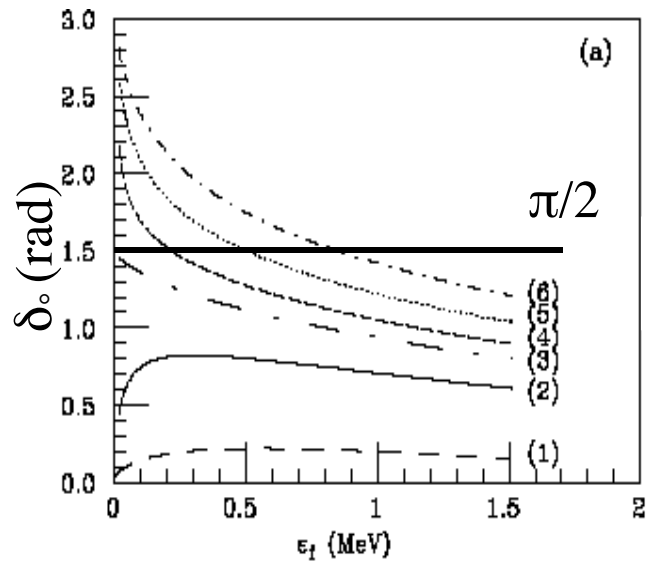


# ${}^9\text{Li}(X, X-1) {}^{10}\text{Li}$

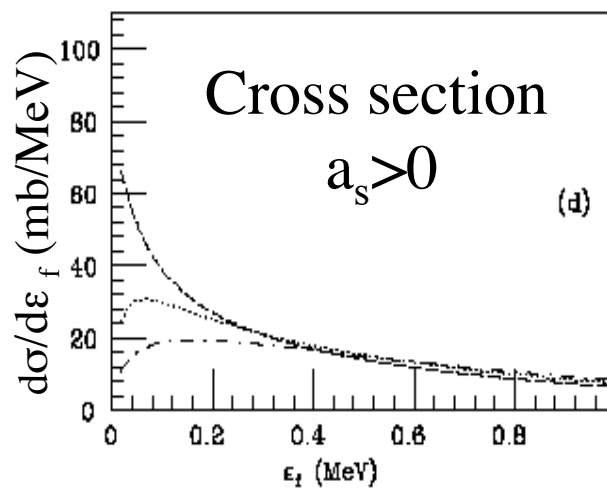
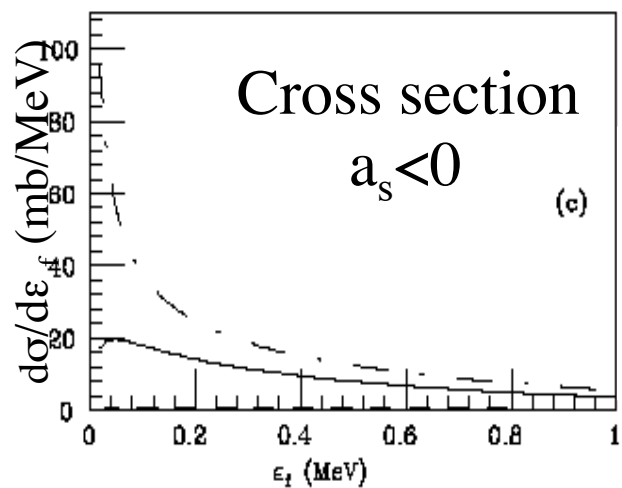
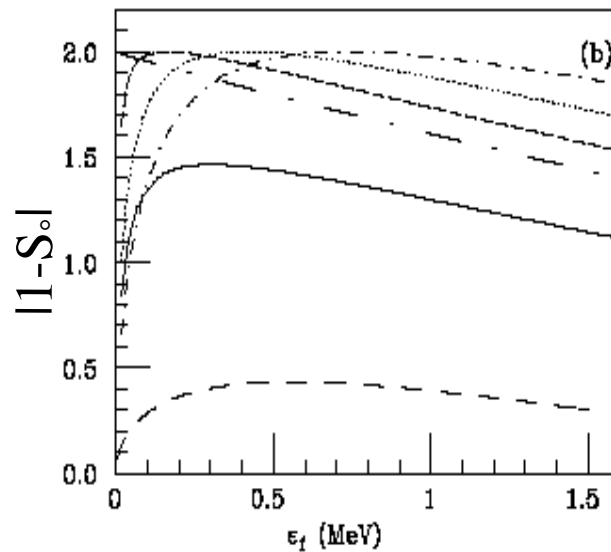




Phase shift



Shape elastic factor



	$V_0$ (MeV)	$\alpha$ (MeV)	$a_s$ (fm)
(1)	-39.83	-4.0	-2.4
(2)	-	-10.0	-17.2
(3)	-	-12.2	-318
(4)	-	-12.4	955
(-)	-	-13.3	53.9
(5)	-	-15.0	21.4
(6)	-42.80	-13.3	12.9

scattering length

$$a_s = - \lim_{k \rightarrow 0} \frac{\tan \delta_0}{k}$$

# effective range theory (1<sup>st</sup> order)

$$\rightarrow k \cotan \delta = -\frac{1}{a_s} + \frac{1}{2} r_o k^2$$

$$\alpha = -10.915 \quad a_s = -26.2 \text{ fm} \quad r_o = 4.3 \text{ fm}$$

$$\text{Nakamura } r_o = 5.6 \text{ fm}$$

# $^{12}\text{Be-n}$ relative energy spectrum $\text{C}(^{14}\text{B}, ^{12}\text{Be+n})\text{X}$ (these J.L. Lecouey 2002)

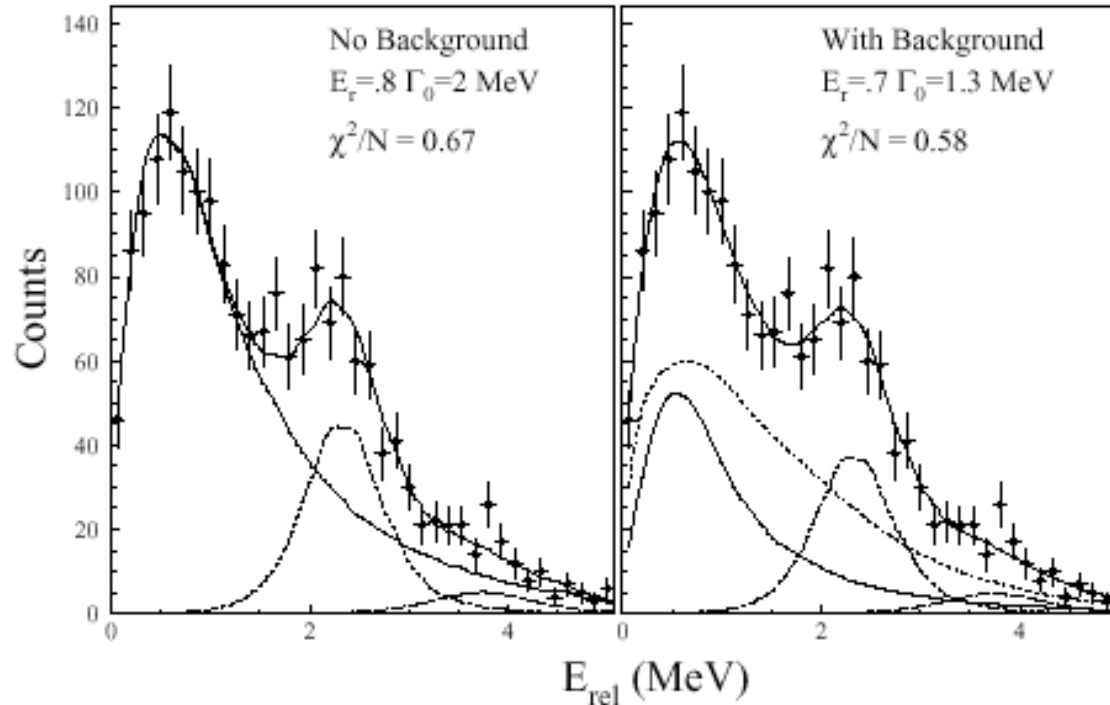
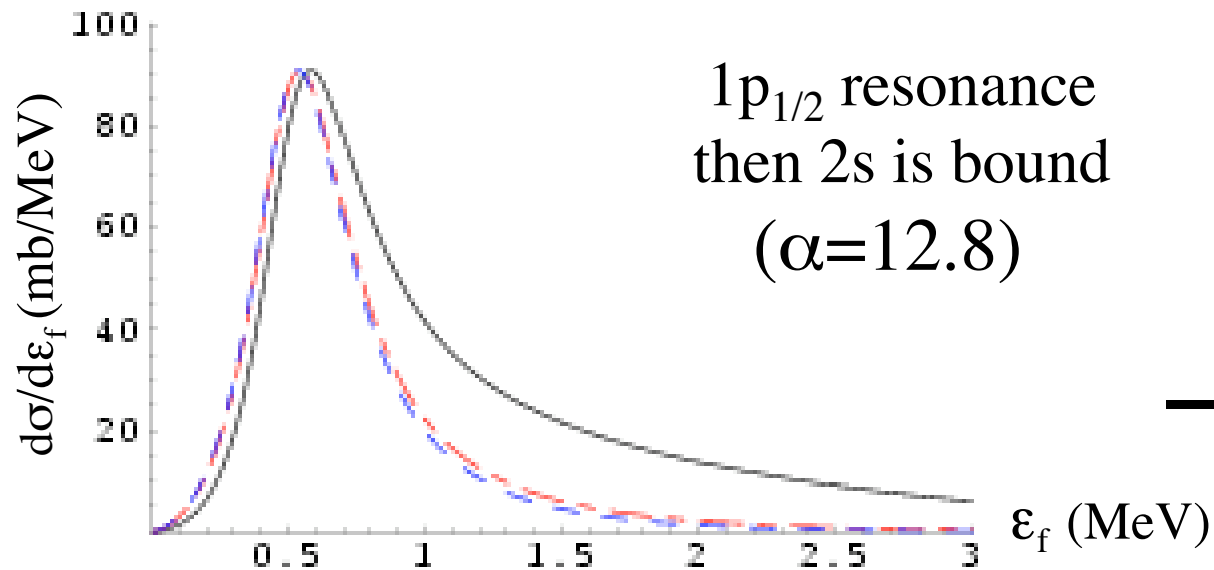
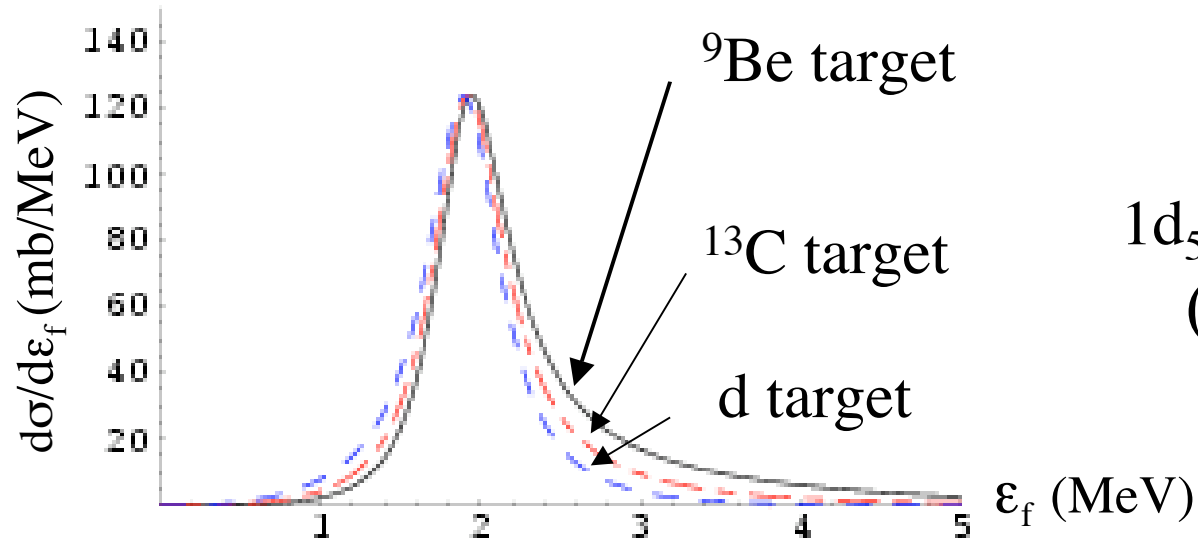


FIG. 3.  $^{12}\text{Be-n}$  relative energy spectrum. The points are the data, the thick solid line the result of a fit including an  $s$ -wave resonance (thin solid line) and a  $d$ -wave resonance (dashed line) and in the right panel, an event-mixing "background" (dotted-dashed line). The parameters shown are those of the  $s$ -wave resonance Breit-Wigner lineshape. Note : a third resonance was tentatively introduced near 4 MeV (dotted line) but is not statistically significant. Its presence does not modify the fit in the region of interest.

# resonance states in $n+^{12}\text{Be}$

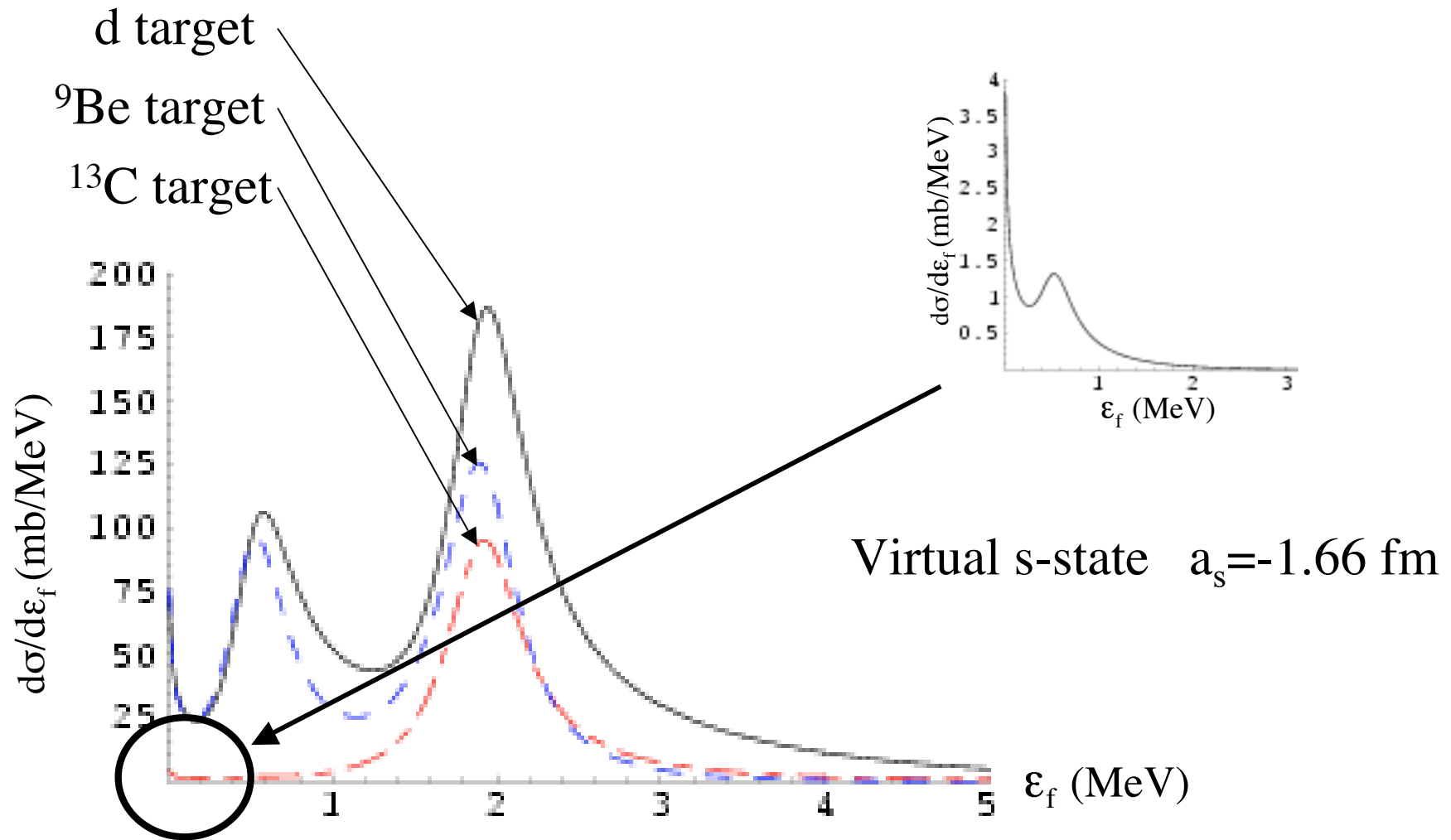


$1d_{5/2}$  resonance  
( $\alpha = -2.5$ )

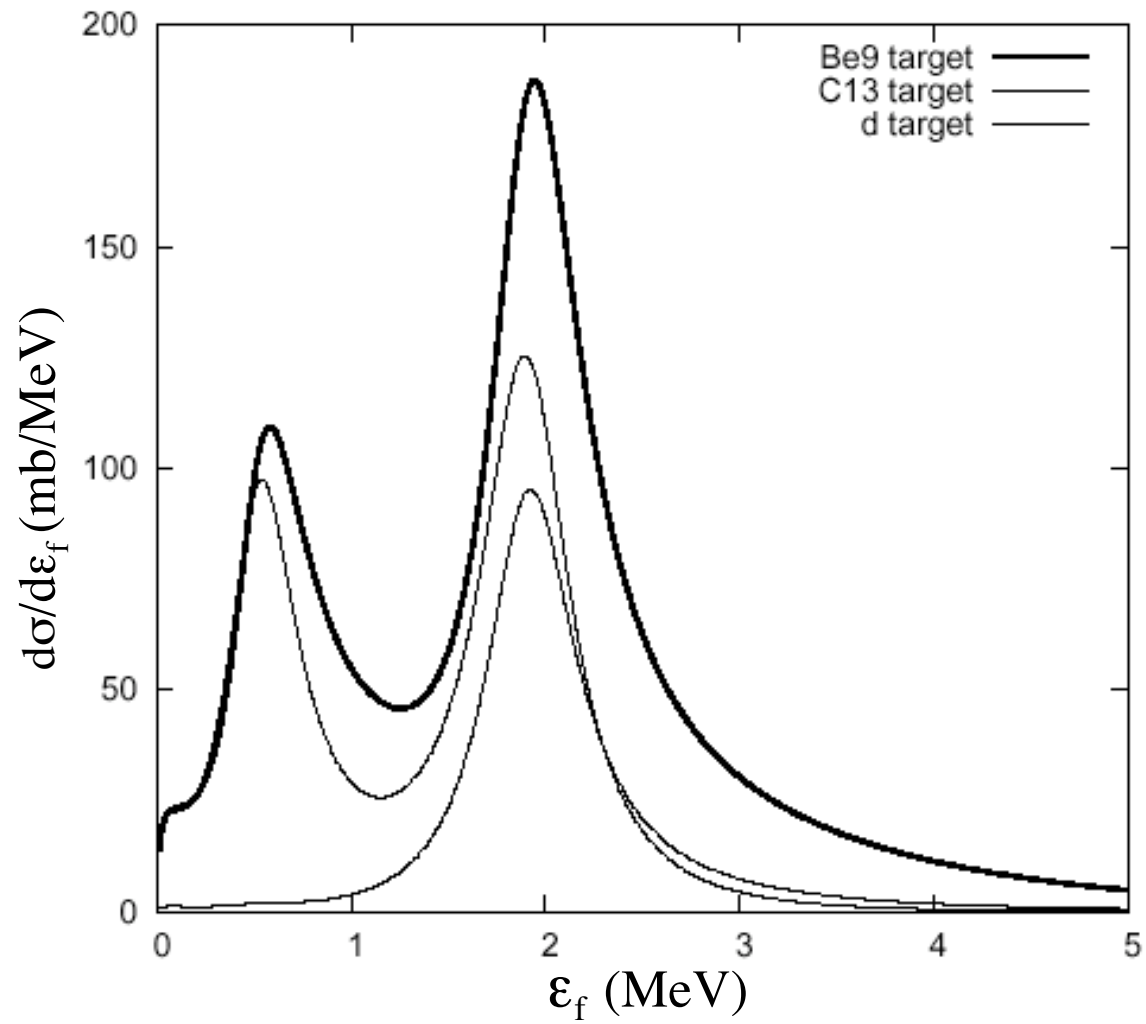
$1p_{1/2}$  resonance  
then  $2s$  is bound  
( $\alpha = 12.8$ )

→ No dependence  
on the target

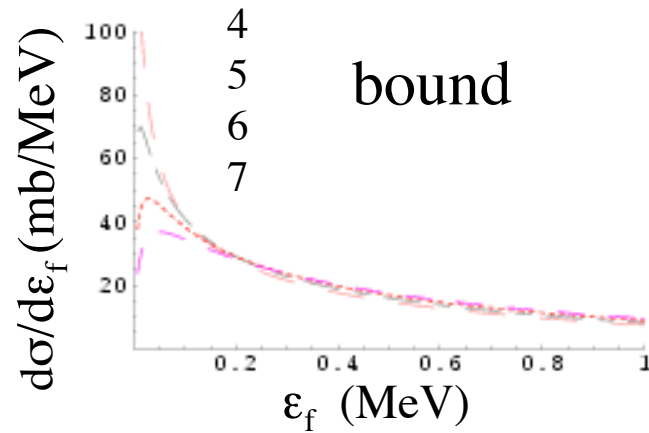
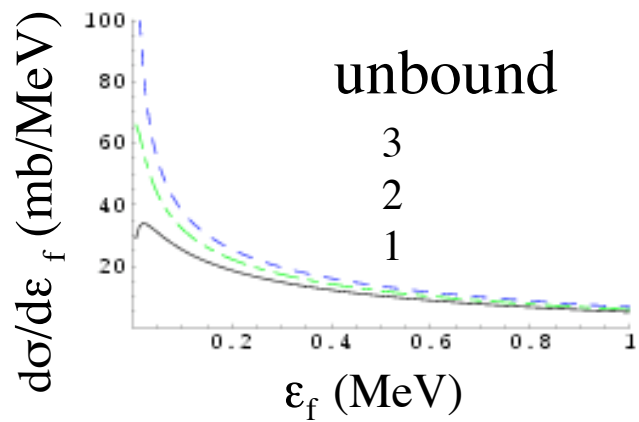
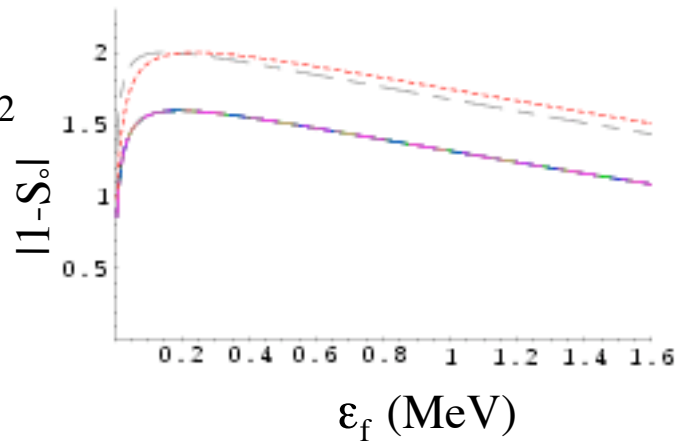
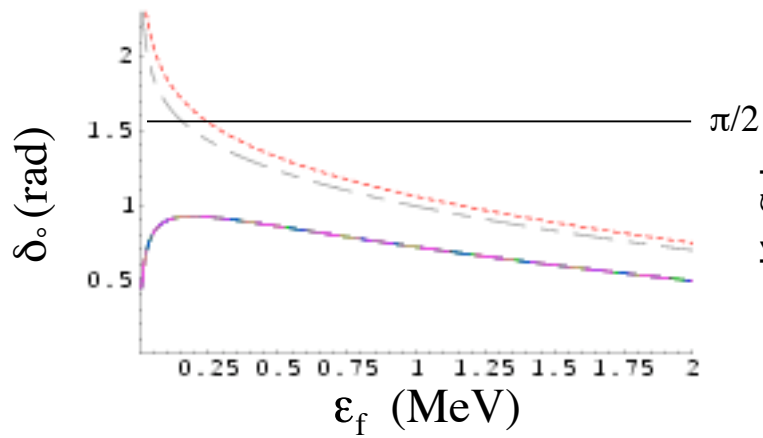
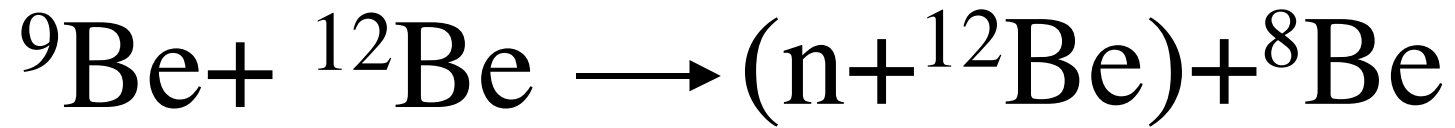
# Overview for $n+^{12}\text{Be}$



$$a_s = 11.5$$







	$\alpha$	$a_s$
1	-0.6	-28
2	-1.2	-51
3	-1.9	-206
4	-2.5	118
5	-3.1	48
6	-3.8	31
7	-4.4	23

# Conclusions

- transfer to the continuum method is well suited to study unbound systems such as  $^{10}\text{Li}$  which are building blocks of borromean nuclei.
- we are able to determine  $a_s$  and the resonance of unbound single particle states