Properties of asymmetric nuclear matter

Spectroscopic factors for $N > Z$

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- Symmetry energy
- status of present calculations:
  BHF, variational, RMF, χPT;
  Self-consistent Green Function approach
- constraints: Symmetry energy $\propto$ neutron skin
- nuclear matter: spectral functions
- finite nuclei: use of local density approximation (LDA)

Symmetry Energy

proton fraction $x = \rho_p/(\rho_p + \rho_n) = (1 - \alpha)/2$

$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + ...,$$

$$E(\rho,0) = E_0 + \frac{K}{18\rho_0^2}(\rho - \rho_0)^2 + ...,$$

Symmetry energy expanded around saturation density ρ_0

$$S_2(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\alpha)}{\partial\alpha^2}|_{\alpha=0} = a_4 + \frac{p_0}{\rho_0^2}(\rho - \rho_0) + \frac{\Delta K}{18\rho_0^2}(\rho - \rho_0)^2 + ...$$

Note: Parabolic approximation is accurate (S_4 small)

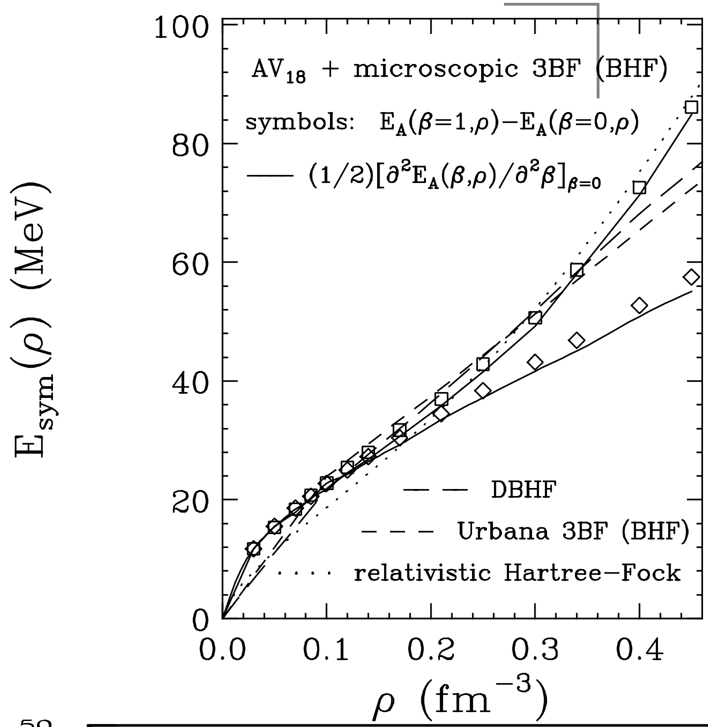
$$a_4 = S_2(\rho_s) = \frac{k_F^2}{6E_F} + S_{\rm pot}$$

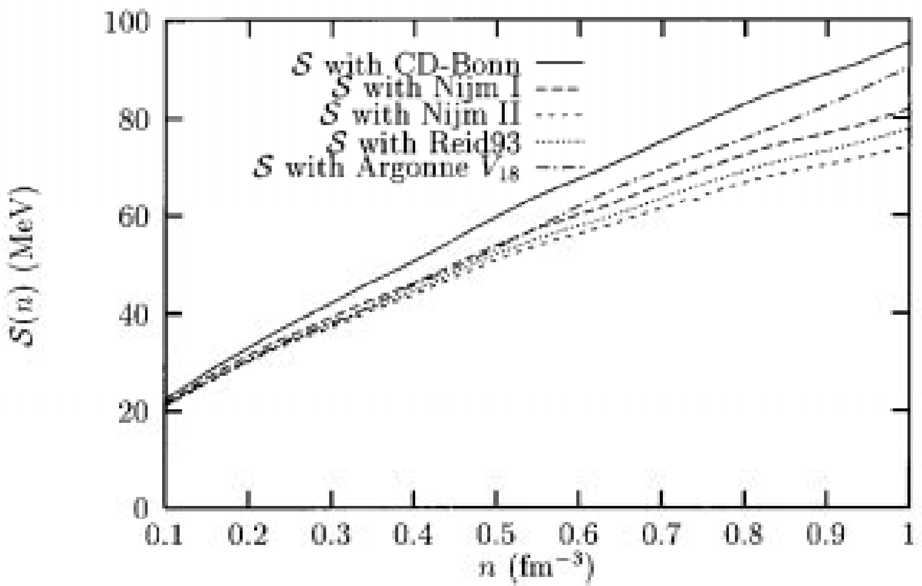
Many-body approaches to EoS

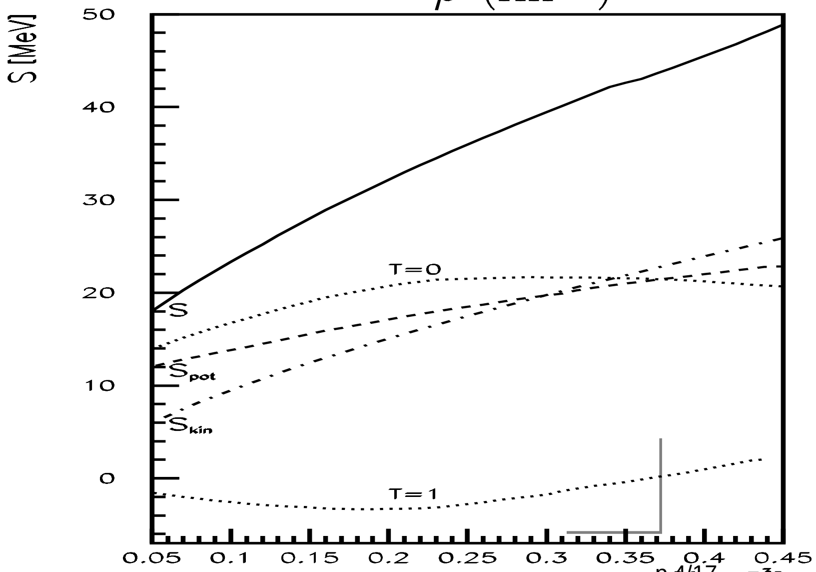
- Realistic NN force as input $\pi + \rho ...$
  Lowest order Brueckner-Hartree-Fock (Dirac-BHF)
  Variational approach
  Self-consistent Green function approach (New)
  EFT
- Effective Lagrangian methods
  Skyrme force
  Relativistic mean field $\sigma, \omega, \rho ...$
- Non-conventional
  hyperons (input NY and YY interaction)
  quark degrees of freedom (QCD)
- Issue: 3-body interactions

(i) Microscopic approaches using NN

- Lowest Order Brueckner-HF
  - S_{pot} Insensitive to details of NN
  - Tensor force important in S_{pot}
  - effect of 3BF for $\rho > 1.5\rho_s$
- Variational+chain summ. Av14,18
  - Lower S than LOBF
  - 3BF increases stiffness
- Self-Consistent Green Function



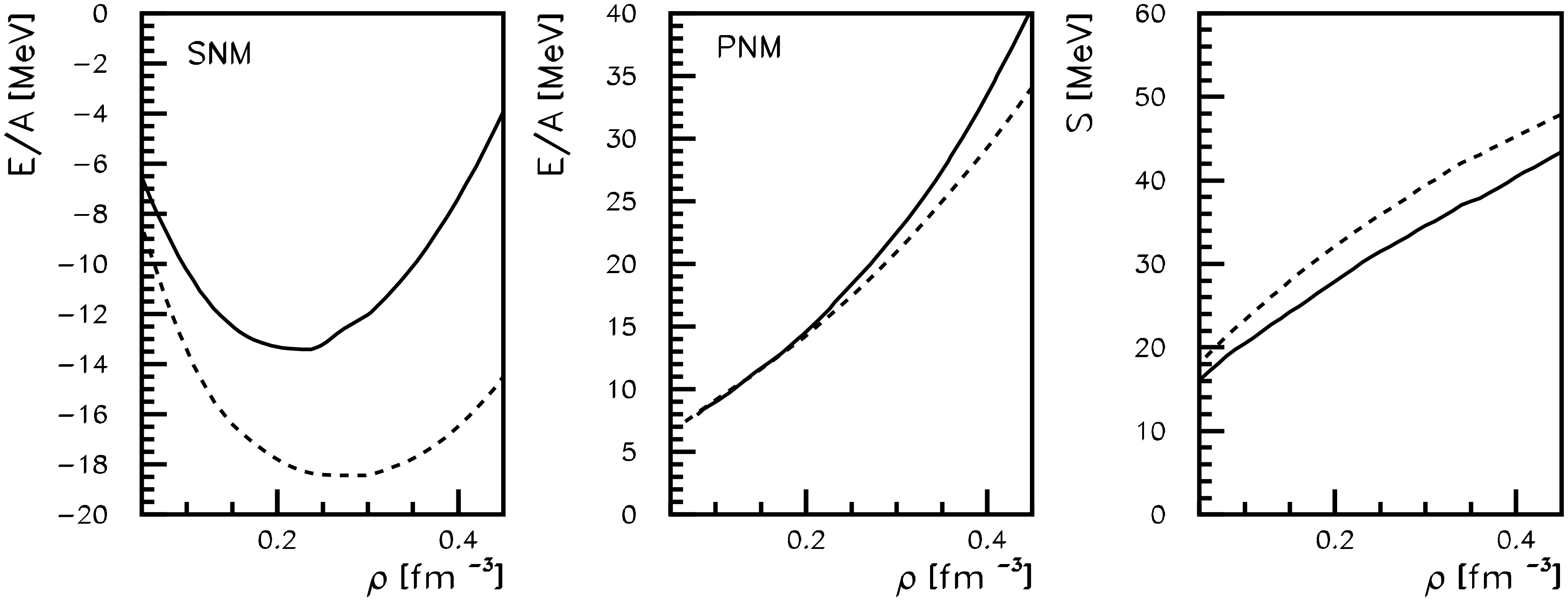




SCGF approach

- $T = V + VGGT$ with dressed s.p. propagator
  $G^{-1} = \omega - \frac{k^2}{2m} - \Sigma$; self-energy $i\Sigma = Tr[TG]$
- Conserving approximation: Hole and particles treated on equal footing; $\epsilon_F = E_B/A$
- Binding energy $\frac{E}{A} = \frac{D}{\rho} \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{\varepsilon_F} d\omega \, \left(\frac{k^2}{2m} + \omega \right) S_h(k,\omega)$,
  Spectral function:
  $S_h(k,\omega) = ImG(k,\omega) =< \psi|a_k^\dagger \delta(\omega - H) a_k|\psi >$
- Inclusion of high-momentum nucleons:
  SNM: doubling of E_{kin} and $E_{pot},$ smaller $E(\rho)$;
  (increasing with density); stiffer EoS, improved ρ_s.
- PNM: absence of T=0 tensor force: effect is smaller
  Symmetry energy is reduced compared to LOBHF
- particle-hole correlations not included

Result SCGF compared to ccBHF



full curve: SCGF; dashed: ccBHF
NN interaction: Reid93, PRC68(2003)

Note: improved saturation density; reduced $S_{\rm pot}$

(ii) Mean field approaches

- Non-relativistic Skyrme forces
- Relativistic Mean field models: Non-linear extension of Walecka model (no pions)
  either (i) σ and ω self interactions
  or (ii) density dependent couplings
  isovector channel: ρ and δ ($I = 1$ scalar) exchange
  Result: $a_4 \approx \frac{k_F^2}{6(M^{*2}+k_F^2)^{1/2}} + \frac{g_\rho^2}{8m_\rho^2}\rho_s - \frac{g_\delta^2}{8m_\delta^2}(\frac{M^*}{\epsilon_F})^2\rho_s;$

  $$p_0 \sim \frac{g_\rho^2}{8m_\rho^2}\rho_s^2$$

Chiral Dynamics

Kaiser et al: Expansion in terms of k_F/m_π up to $O(k_F^5)$
Pion loops with in-medium nucleon propagator

$$(\not p + M)\left(\frac{i}{p^2 - M^2 + i\epsilon} - 2\pi\delta(p^2 - M^2)\theta(p_0)\theta(k_F - |p|)\right)$$

1π exch Hartree+ Fock with 2+3 medium insertions

iterated one-pion irreducible 2π exchange

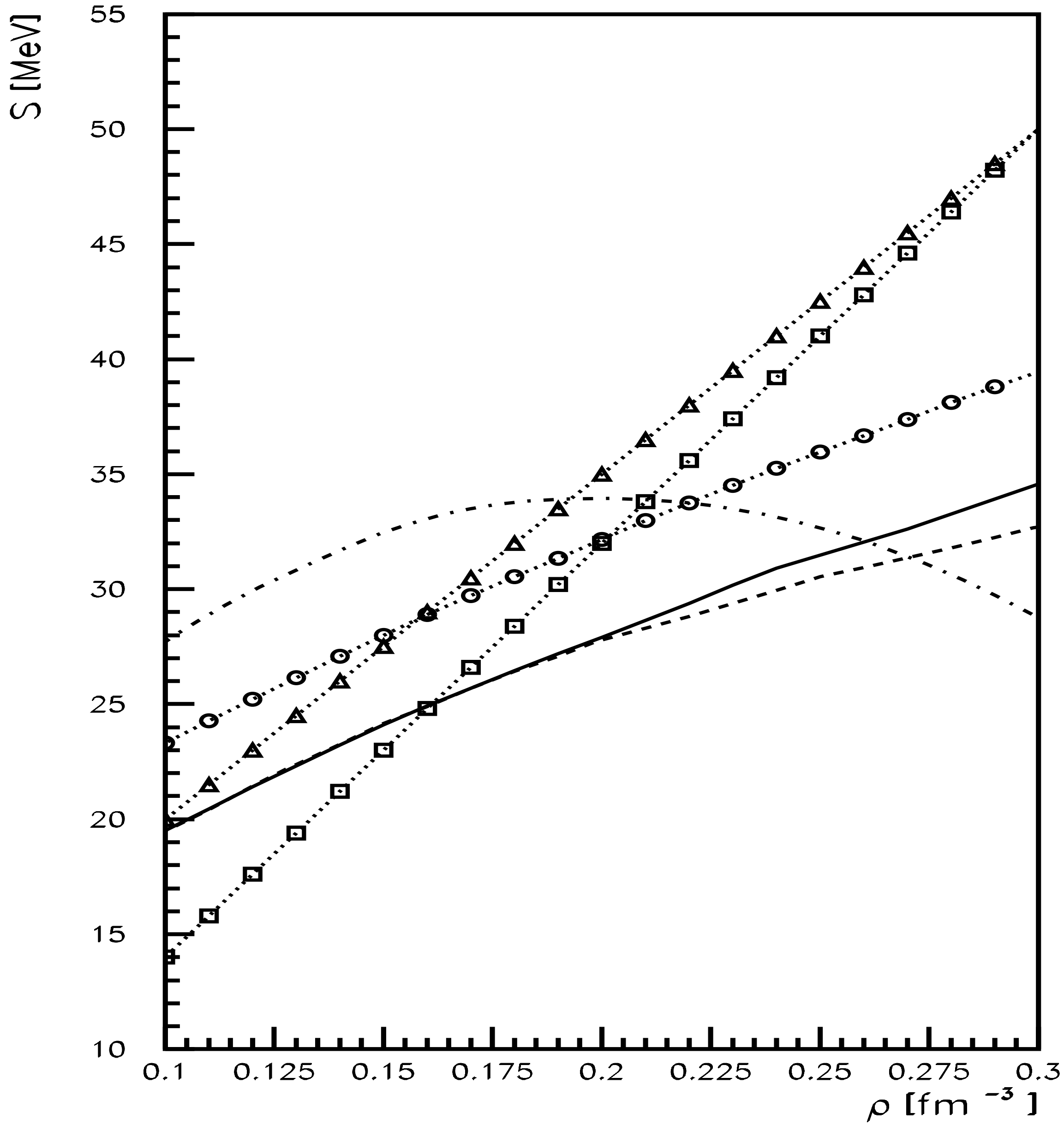
- describes SNM with $\Lambda = 0.65$ GeV
- chiral limit:

$$E^{SNM} = \frac{3k_F^2}{10M} - \alpha(\Lambda)\frac{k_F^3}{M^2} + \beta\frac{k_F^4}{M^3}$$

- SE levels off at higher ρ

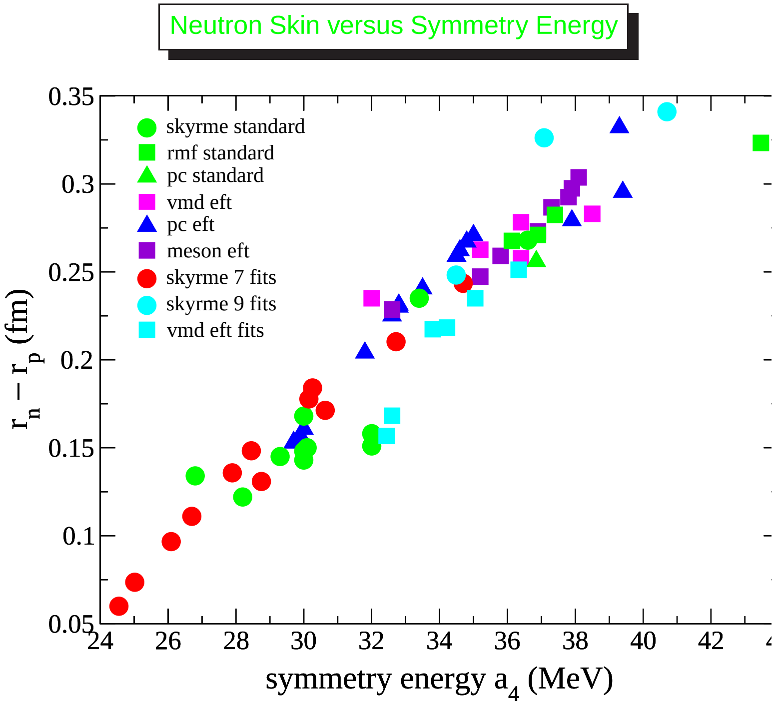
Comparison

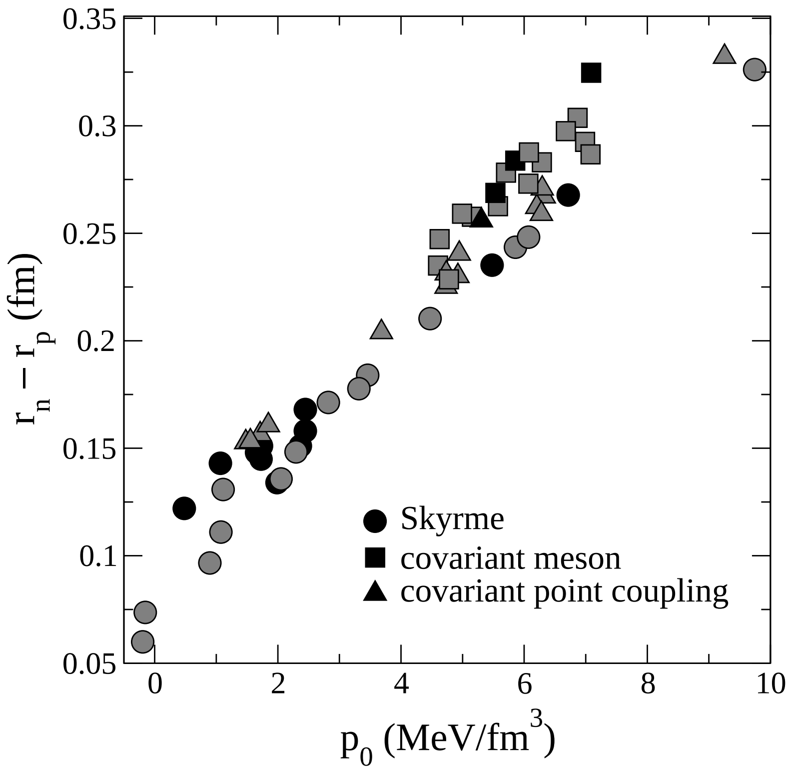
- LOBF
- LOBHF+ 3BF
- Variational
- SCGF
- χPT with pions
- DBHF
- RMF



Symmetry energy and neutron skin

Furnstahl et al pointed out correlation between a_4, $p_0 = \rho_s^2 S'(\rho_s)$, and neutron skin, $\Delta R = R_n - R_p$ in ^{208}Pb





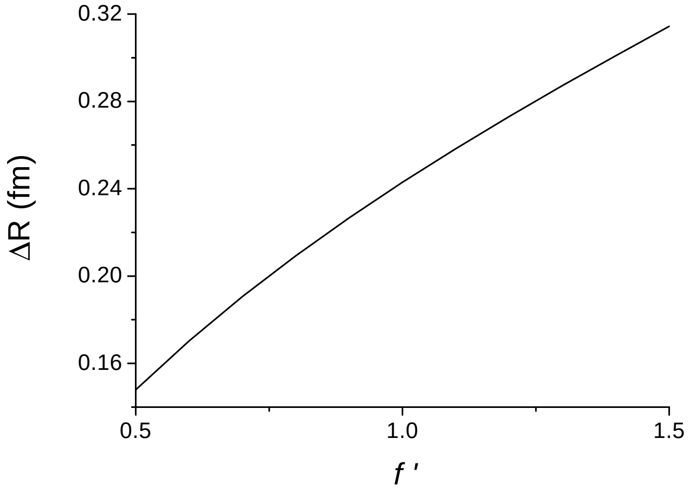
$$E(\rho,\alpha) = E(\rho,0) + S_2(\rho)\alpha^2 + S_4(\rho)\alpha^4 + \cdots \qquad \alpha \equiv \frac{N-Z}{A}$$

$$E(\rho,0) = -a_v + \frac{K_0}{18\rho_0^2}(\rho-\rho_0)^2 + \cdots$$

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_0^2}(\rho-\rho_0) + \frac{\Delta K_0}{18\rho_0^2}(\rho-\rho_0)^2 + \cdots$$

Can one understand the S vs ΔR relation?

- Landau-Fermi liquid theory (thanks to V.Rodin)
  $V_{ph} = F + F'\tau_1.\tau_2 + G\sigma_1.\sigma_2 + G'\sigma_1.\sigma_2\tau_1.\tau_2$
  In mean field approx $H_{MF} = T + V_0 + V_s\tau^3 + V_c$
- self-consistency: $V_s(r) = 2F'(\rho_n(r) - \rho_p(r))$
- Migdal relation: $a_4 = \frac{\epsilon_F}{3}(1 + 2F')$ relates SE with F'



- if F' density independent then $p_0 \sim a_4$
- SR for IVGMR: $r_n^2 - r_p^2 = F' \sum_s \frac{<0|N|s>><s|\sum_i r_i^2 \tau_i^3|0>}{E_0 - E_s}$

Exp approaches to ΔR

- p,p' at 500-1000 MeV
- isovector Giant (spin) dipole resonance, IAS
- anti-protonic atoms
- parity violating electron scattering

Input of ΔR needed for

- weak charge Q_W
- deeply bound pionic atoms

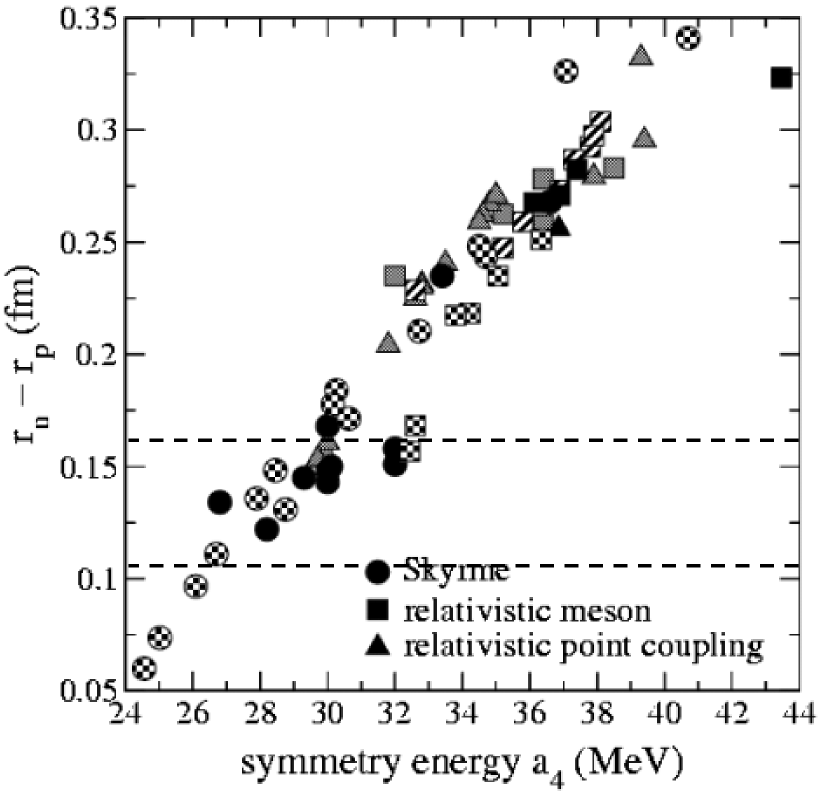
Present status ΔR in ^{208}Pb

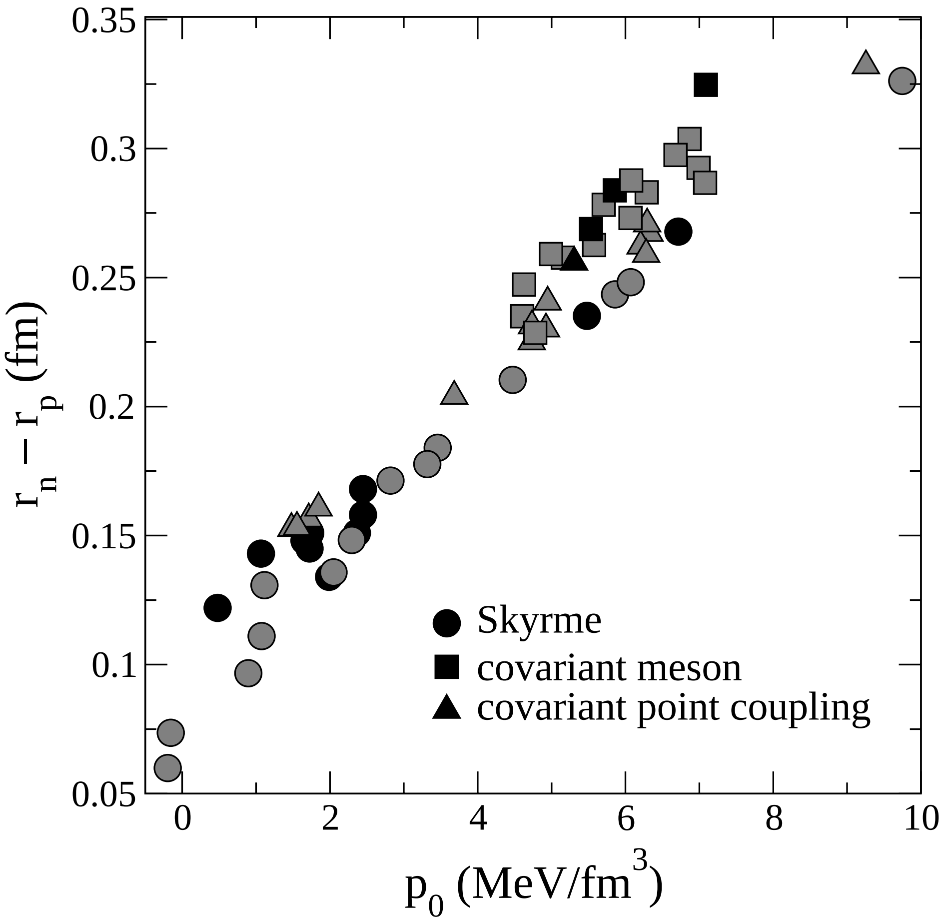
method	ΔR [fm]	Error [fm]	ref
$(\vec{p}, p')$ $(E = 0.5 - 1.0$ GeV)	0.097	0.014	Clark03
nucleon scatt (E= 40-200 MeV)	0.17		Karat02
anti-protonic atoms	0.15	0.02	Trzcin01
isovector giant dipole res.	0.19	0.09	Kras94
giant dip. res. (unpubl.)	0.14	0.05	Kras02
parity violating el sc	planned	1%	Jlab

- Exp consistent with $\Delta R \sim 0.10 - 0.15$ fm
- Clark et al: “Exp. seem to disagree with usual relativistic mean field models"
- New constraints might come from heavy ion reactions which probe higher densities

Symmetry energy and ΔR

In terms of mean field mc





Finite nuclei: LDA

- Effect of s.r.c. appears to be A-independent suggests use of LDA, see Van Neck et al, PRC51,1800 (1994)
- LDA Suppose one knows nm density $n^{nm}(k, k_F)$ as a function of $k_F = (\frac{3\pi^2}{2}\rho)^{1/3}$
- naive LDA $n_A(k) = \int dR n^{nm}(\rho(R), k)$ diverges for $k \to 0$
- improved LDA: apply to correlation part only $n_A(k) = n_A^{MF}(k) + \int dR \delta n_<(\rho(R), k) + \int dR \delta n_>(\rho(R), k)$, where $n_A^{MF}(k) = \int dR \int dr \exp{(ikr)} \sum_h \phi_h^*(r_1)\phi_h(r_2)$

spectral function in LDA: N=Z

I refer to Van Neck et al, PRC 51 1800 (1994)

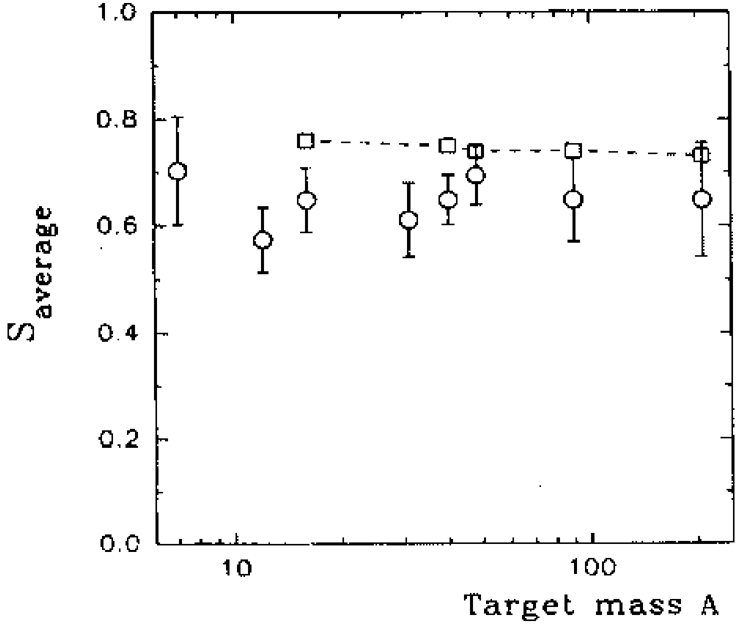
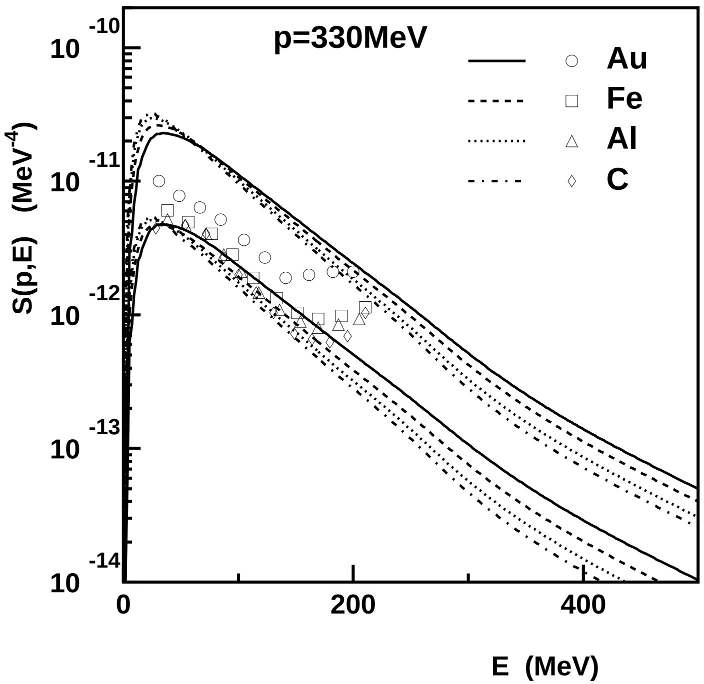
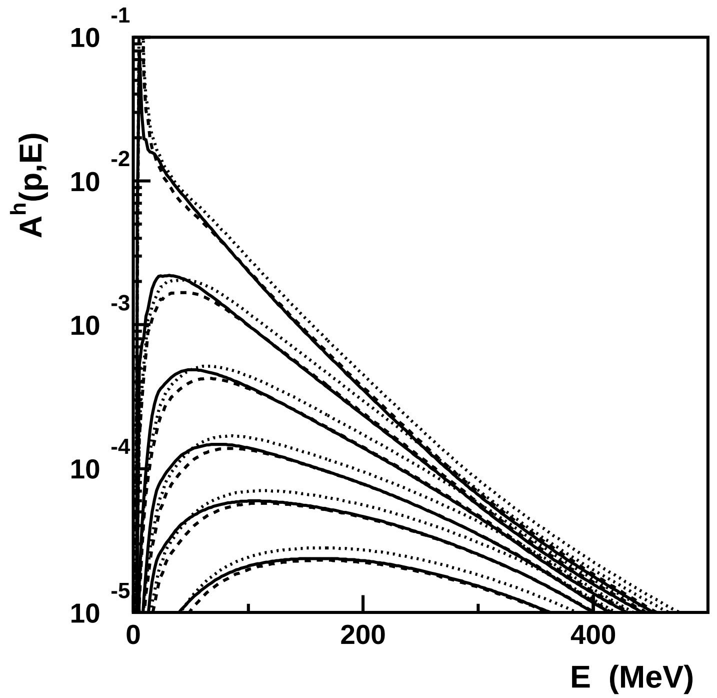


FIG. 5. Average spectroscopic factor for the valence hole

spectral functions in N>Z finite nuclei





(from Bozek, nucl-th/0311046)