Influence of the Dirac Sea on Nucleon Electromagnetic Knockout

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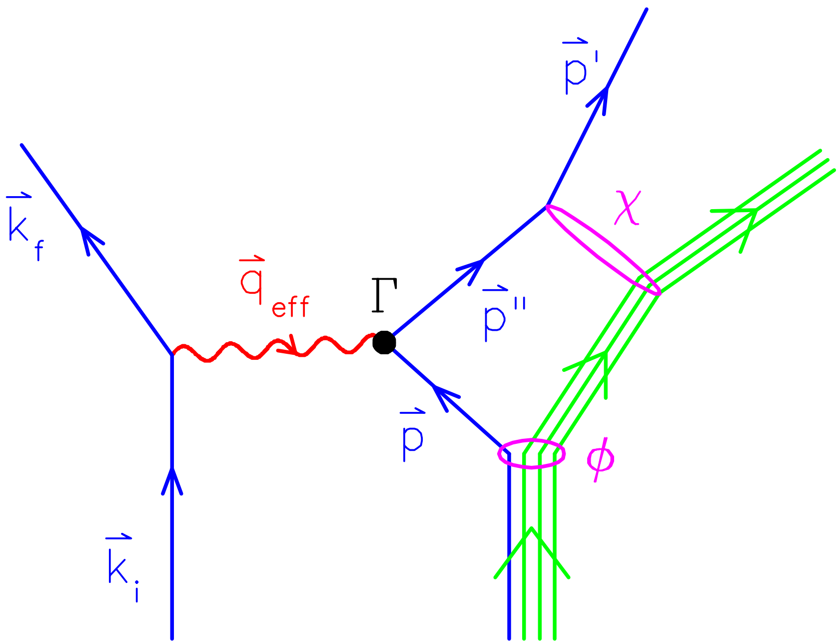
Outline

- RDWIA model
  - spinor distortion, projection on positive-energy states
  - effective momentum approximation (EMA)
- ^{16}O$(e, e'p)$
  - sensitivity of A_{LT} to spinor distortion and sea
  - model dependence
  - spectroscopic factors
- ^{12}C$(e, e'p)$ for $Q^2 > 0.6$
  - sensitivity of A_{LT} to spinor distortion and sea
  - model dependence
  - transparency and spectroscopic factors
- Conclusions

Model

- ϕ: overlap with single-nucleon knockout state represented by Dirac bound state
- χ: FSI represented by Dirac distorted waves
- Γ: electromagnetic current represented by one-body operator with energies placed on shell



RDWIA one-body current

$$\mathcal{J}^{\mu} = \int d^3r \;\exp(i\boldsymbol{t}\cdot\boldsymbol{r})\langle\bar{\Psi}(\boldsymbol{p}',\boldsymbol{r})|\Gamma^{\mu}(\boldsymbol{p}',\boldsymbol{p}'-\boldsymbol{q})|\phi(\boldsymbol{r})\rangle$$

$\boldsymbol{t} = \frac{E_B}{W}\boldsymbol{q}$ includes recoil correction

$\boldsymbol{p}_m = \boldsymbol{p}' - \boldsymbol{q}$ using external kinematics

$$\phi_{\kappa m}(\boldsymbol{r}) = \begin{pmatrix} f_{\kappa}(r)\mathcal{Y}_{\kappa m}(\hat{r}) \\ ig_{-\kappa}(r)\mathcal{Y}_{-\kappa m}(\hat{r}) \end{pmatrix}$$

$$\Psi(\boldsymbol{p},\boldsymbol{r}) = \begin{pmatrix} \psi(\boldsymbol{p},\boldsymbol{r}) \\ \zeta(\boldsymbol{p},\boldsymbol{r}) \end{pmatrix}$$

$$\left[\boldsymbol{\alpha}\cdot\boldsymbol{p} + \beta(m+S) + (V-E)\right]\Psi = 0$$

2^{nd}-order Dirac equation

$$\left[\nabla^2 + k^2 - 2\mu\left(U^C + U^{LS}\boldsymbol{L}\cdot\boldsymbol{\sigma}\right)\right]\xi = 0$$

$$U^C = \frac{E}{\mu}\left[V + \frac{m}{E}S + \frac{S^2 - V^2}{2E}\right] + U^D$$

$$U^D = \frac{1}{2\mu}\left[-\frac{1}{2r^2 D}\frac{d}{dr}(r^2 D') + \frac{3}{4}\left(\frac{D'}{D}\right)^2\right]$$

$$U^{LS} = -\frac{1}{2\mu}\frac{D'}{rD}, \qquad D = 1 + \frac{S - V}{E + m}$$

$$\psi = D^{1/2}\xi, \qquad \zeta = \frac{\boldsymbol{\sigma}\cdot\mathbf{p}\psi}{E + m + S - V}$$

Spinor Distortion

$$
\begin{aligned}
J^\mu &= \tilde{\Omega}_c(\boldsymbol{p}', r)\gamma^0\Gamma^\mu\Omega_b(\boldsymbol{p}_m, r) \\
\Omega_\alpha(\boldsymbol{p}, r) &= \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{(E_\alpha+m)D_\alpha(r)} \end{pmatrix} D_\alpha^{1/2}(r) \\
D(r) &= 1 + \frac{S(r) - V(r)}{E + M} \sim 0.6
\end{aligned}
$$

- Darwin nonlocality factor reduces amplitude $\Longrightarrow$ increases fitted spectroscopic factor. Effective at low p_m.
- Dynamical enhancement of lower component breaks factorization, alters A_{LT} and recoil polarization. Increases with p_m.

Unraveling effects

- Positive-energy projection:

$$J^\mu \longrightarrow \Lambda_+^\dagger(\boldsymbol{p}')J^\mu\Lambda_+(\boldsymbol{p}) \ , \qquad \Lambda_+(\boldsymbol{p}) = \frac{m+\not{p}}{2m}$$

  eliminates contributions from Dirac sea (Udías *et al*.)

- noSV

$$\Omega(\boldsymbol{p},r) \longrightarrow \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E+m} \end{pmatrix} D_\alpha^{1/2}(r)$$

- EMA: $\boldsymbol{p}$ in spinors uses kinematics instead of operator
- EMA-noSV: eliminates dynamical enhancement of lower component but retains suppression of upper component

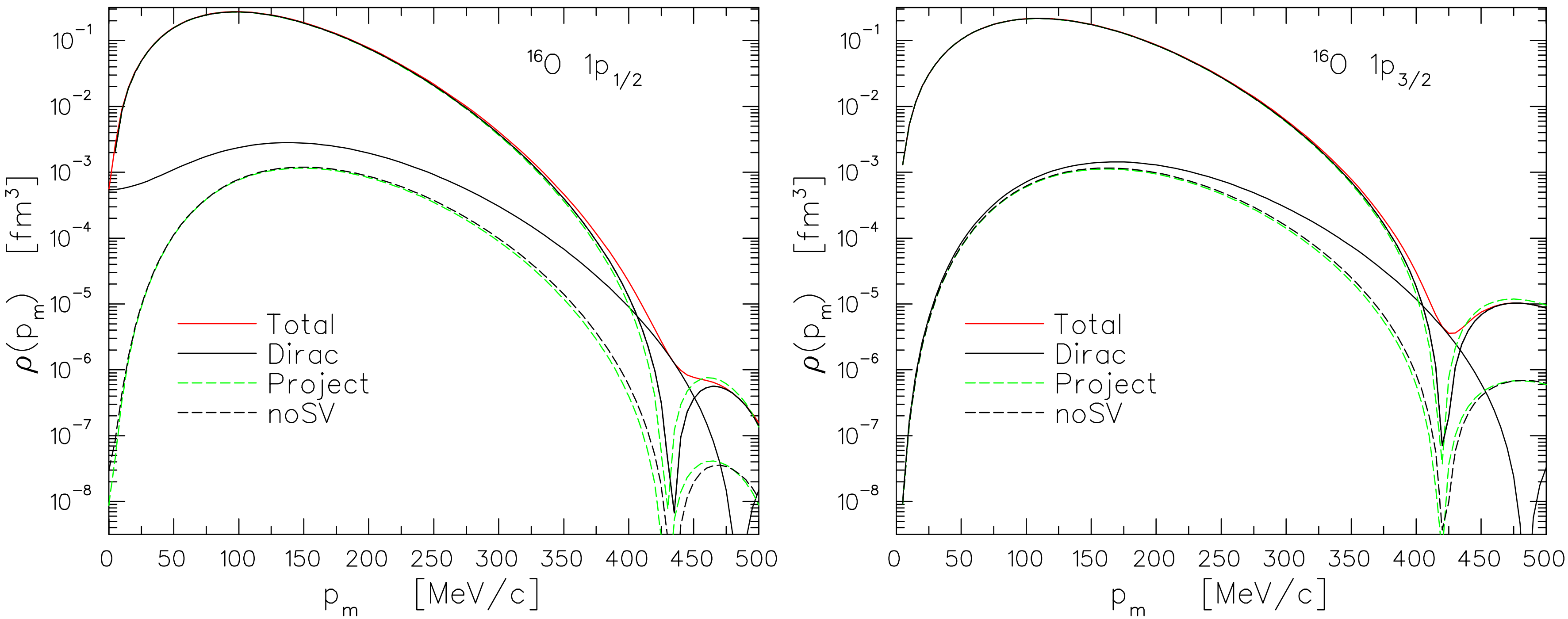
J^{μ}_{eff} variations

J^{μ}_{eff} is a 2×2 operator on Pauli spinors from relativized Schrödinger equation that incorporates relativistic dynamics without p/m expansions.

- EMA-noSV
  - includes Darwin suppression
  - no Gordon ambiguity
  - factorizes except for spin-orbit FSI
- EMA-SV
  - includes dynamical enhancement of lower components
  - breaks factorization even without FSI (Donnelly *et al*.)

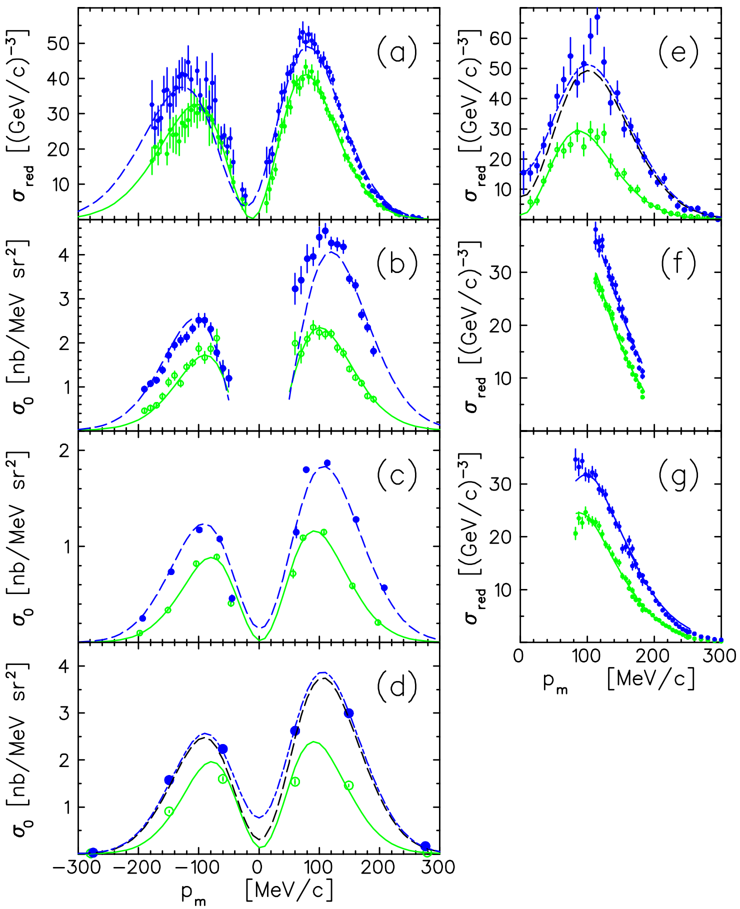
Without EMA, 2×2 J^{μ}_{eff} completely equivalent to 4×4 J^{μ}.

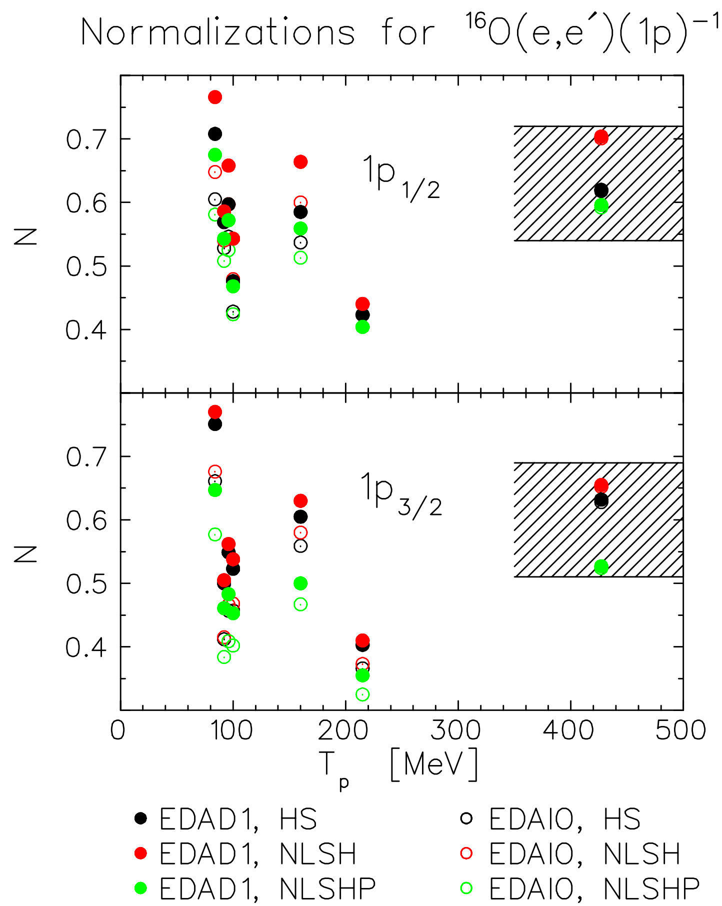
Momentum distribution



- $j = \ell - 1/2$ more sensitive to Dirac sea (Caballero *et al*.)
- sensitivity increases with p_m
- interference between upper and lower components emphasizes sea
- EMA-noSV similar to positive-energy projection

Spectroscopic factors: $^{16}O(e,e'p)(1p)^{-1}$





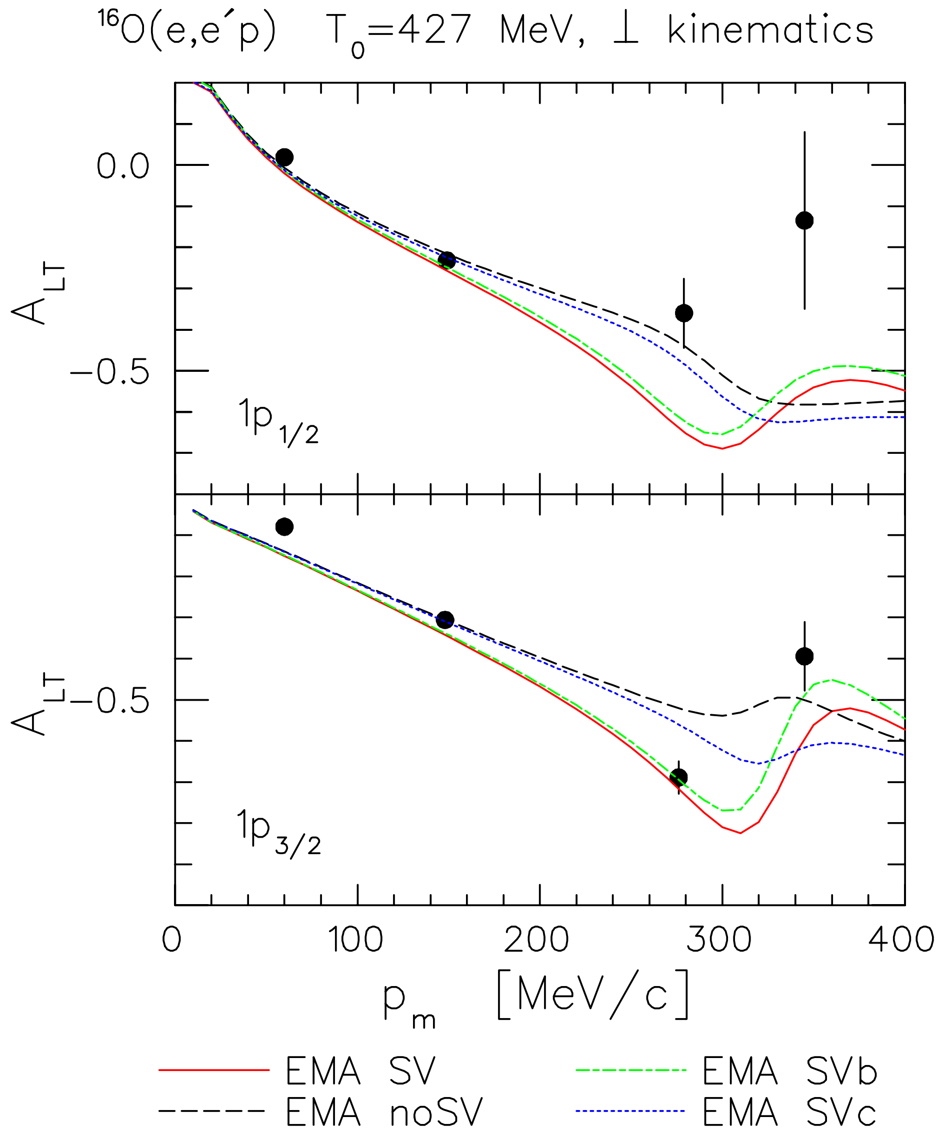
- Model dependence smaller at high Q^2, but still $\sim \pm 15\%$.
- No clear trend with Q^2.

EMA Spinor Distortion

Gao *et al.*, JLab $Q^2 = 0.8$

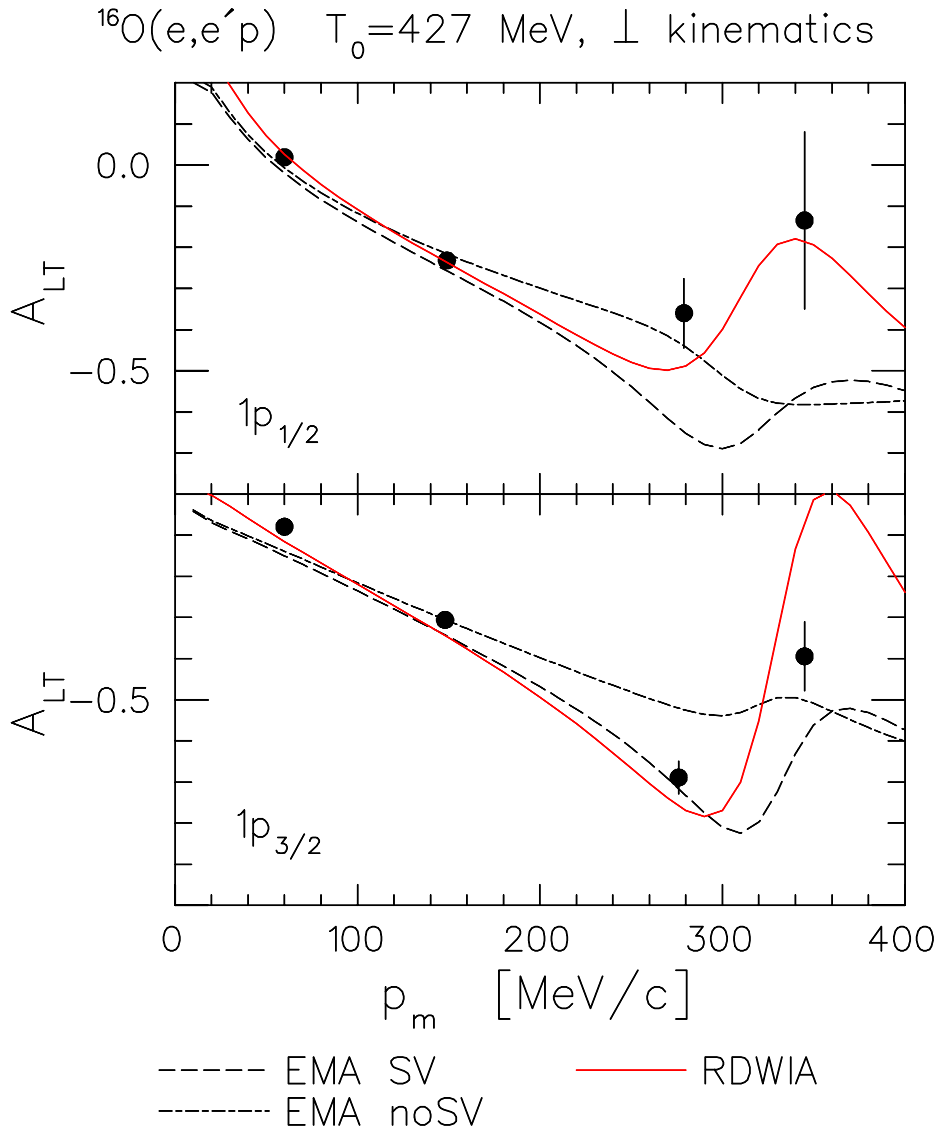
$$A_{LT} = \frac{\sigma(\phi=0) - \sigma(\phi=\pi)}{\sigma(\phi=0) + \sigma(\phi=\pi)}$$

- A_{LT} most sensitive to bound-state spinor distortion
- variations due to bswf, om, gauge/Gordon ambiguities smaller



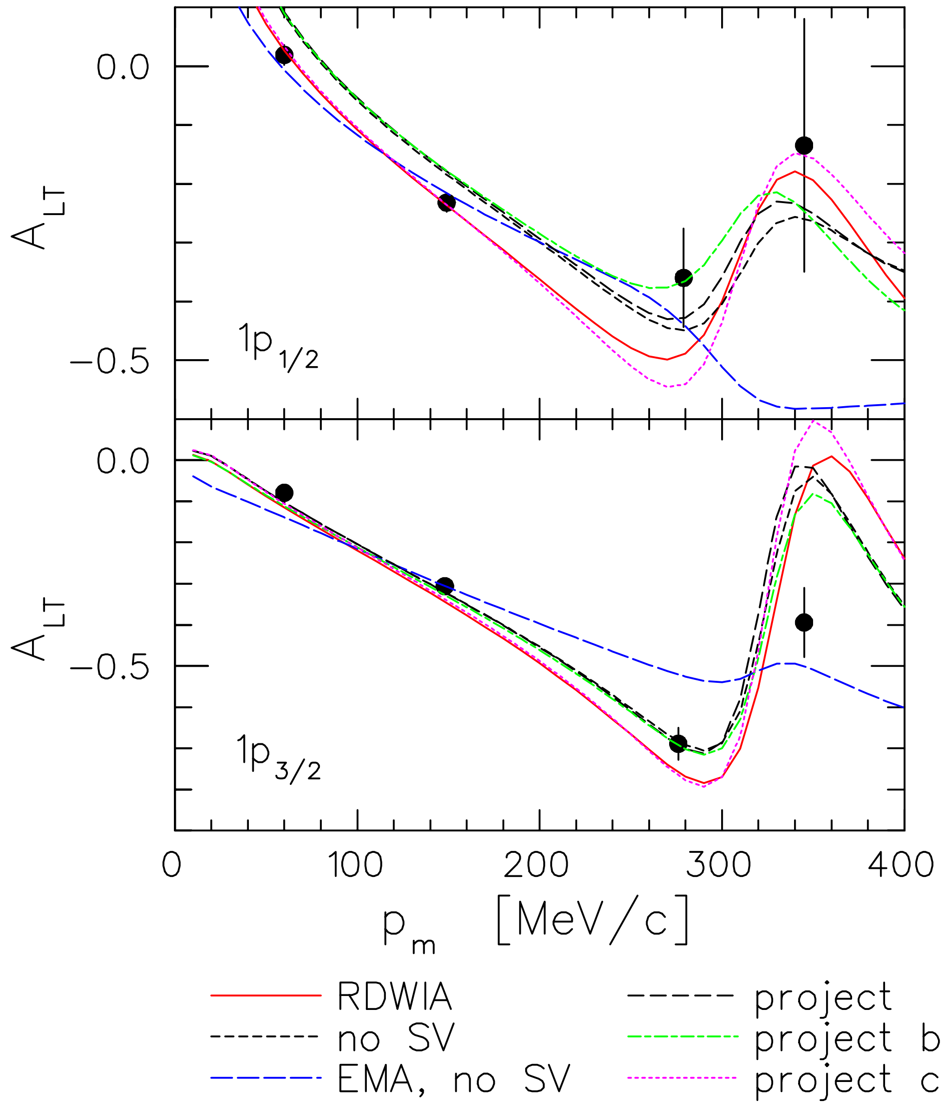
Accuracy of EMA

- EMA offers qualitative description of A_{LT} enhancement for modest p_m
- breaks down for large p_m in oscillatory region
- more accurate for $1p_{3/2}$ than for $1p_{1/2}$ where there is a discrepancy at low p_m



Contribution of Dirac Sea

- no SV $\approx$ projection
- bound > continuum projection
- sea contribution greater for $1p_{1/2}$, especially at small p_m
- effect at low p_m is subtle, but data support sea contribution



Gordon Ambiguity

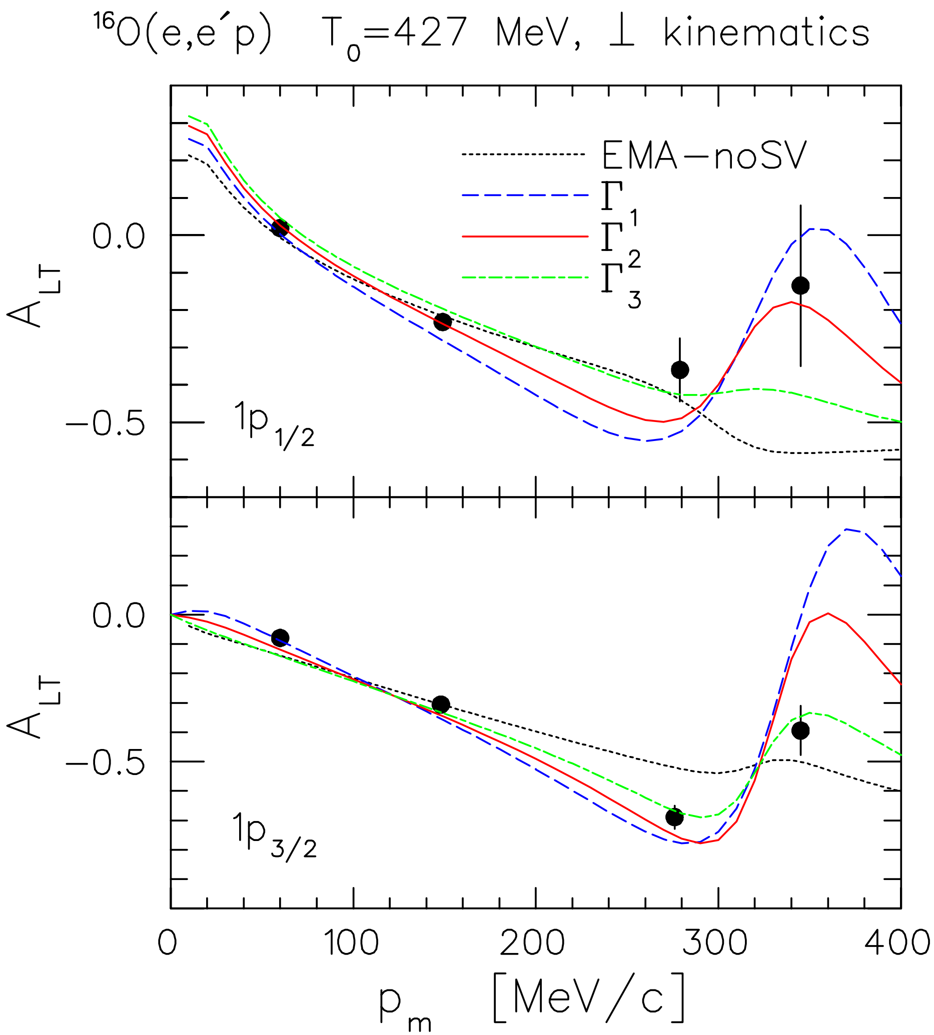
$$\Gamma_1^\mu = \gamma^\mu G_M - \frac{P^\mu}{2m} F_2$$

$$\Gamma_2^\mu = \gamma^\mu F_1 + i\sigma^{\mu\nu}\frac{q_\nu}{2m} F_2$$

$$\Gamma_3^\mu = \frac{P^\mu}{2m} F_1 + i\sigma^{\mu\nu}\frac{q_\nu}{2m} G_M$$

- equivalent for free spinors
- on-shell prescription:

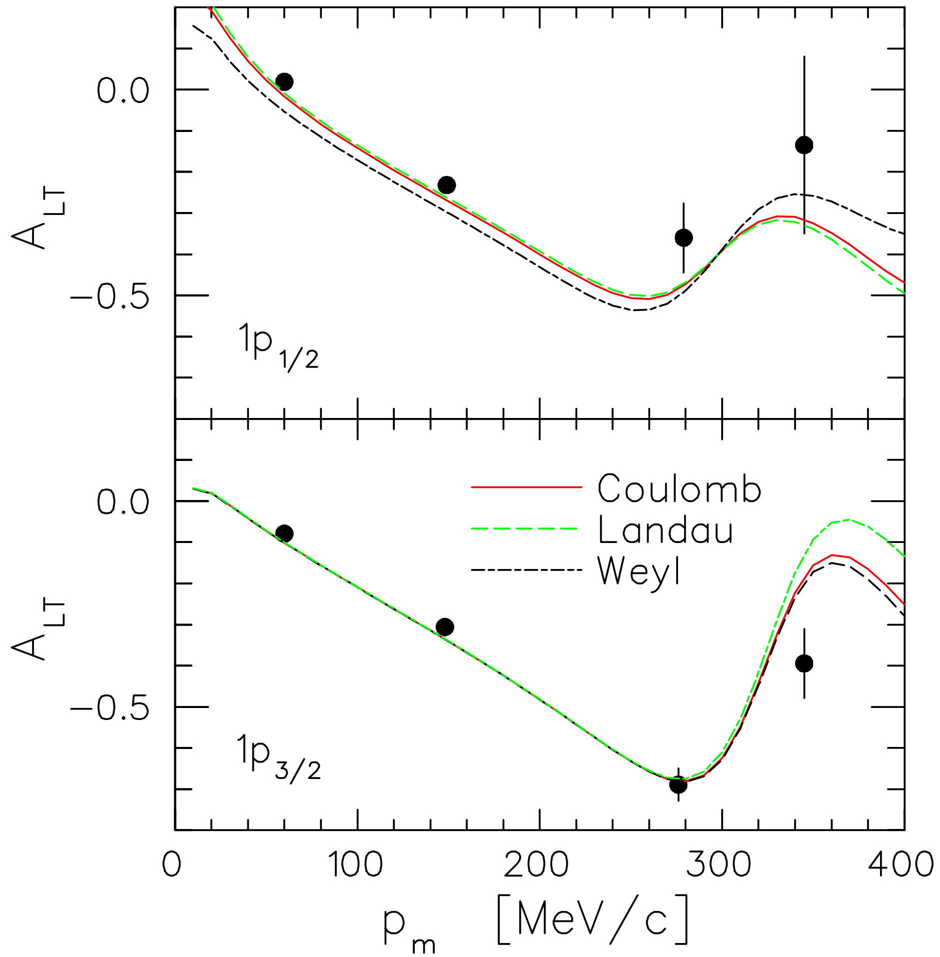
$$E \rightarrow \bar{E} = \sqrt{m^2 + p^2}$$
$$\omega \rightarrow \bar{\omega} = \bar{E}' - \bar{E}$$



Γ_1 most, Γ_3 least sensitive to spinor distortion

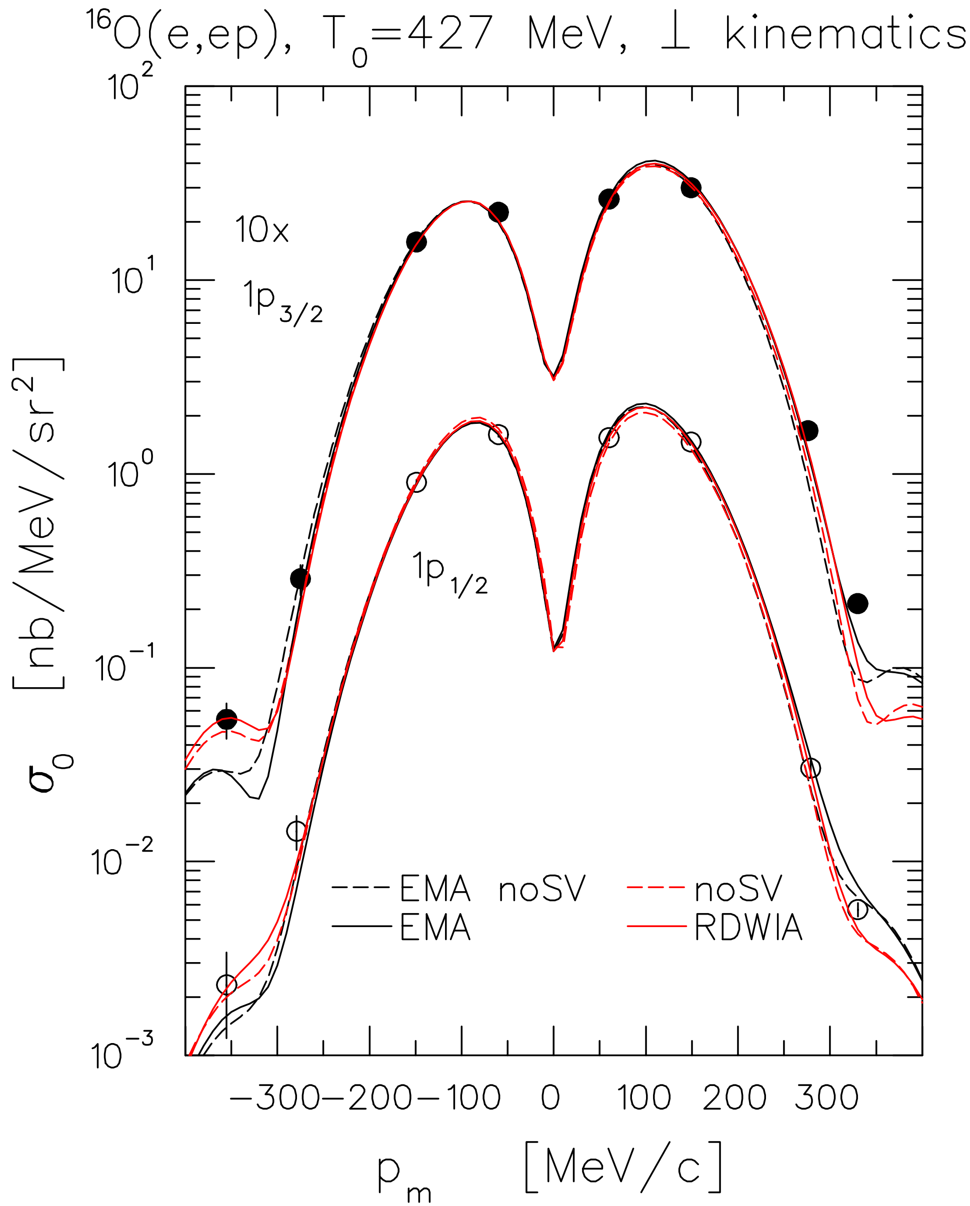
Gauge Ambiguity

- smaller than in NRDWIA
- small for $p_m \lesssim 300$ MeV/c
- Weyl disfavored



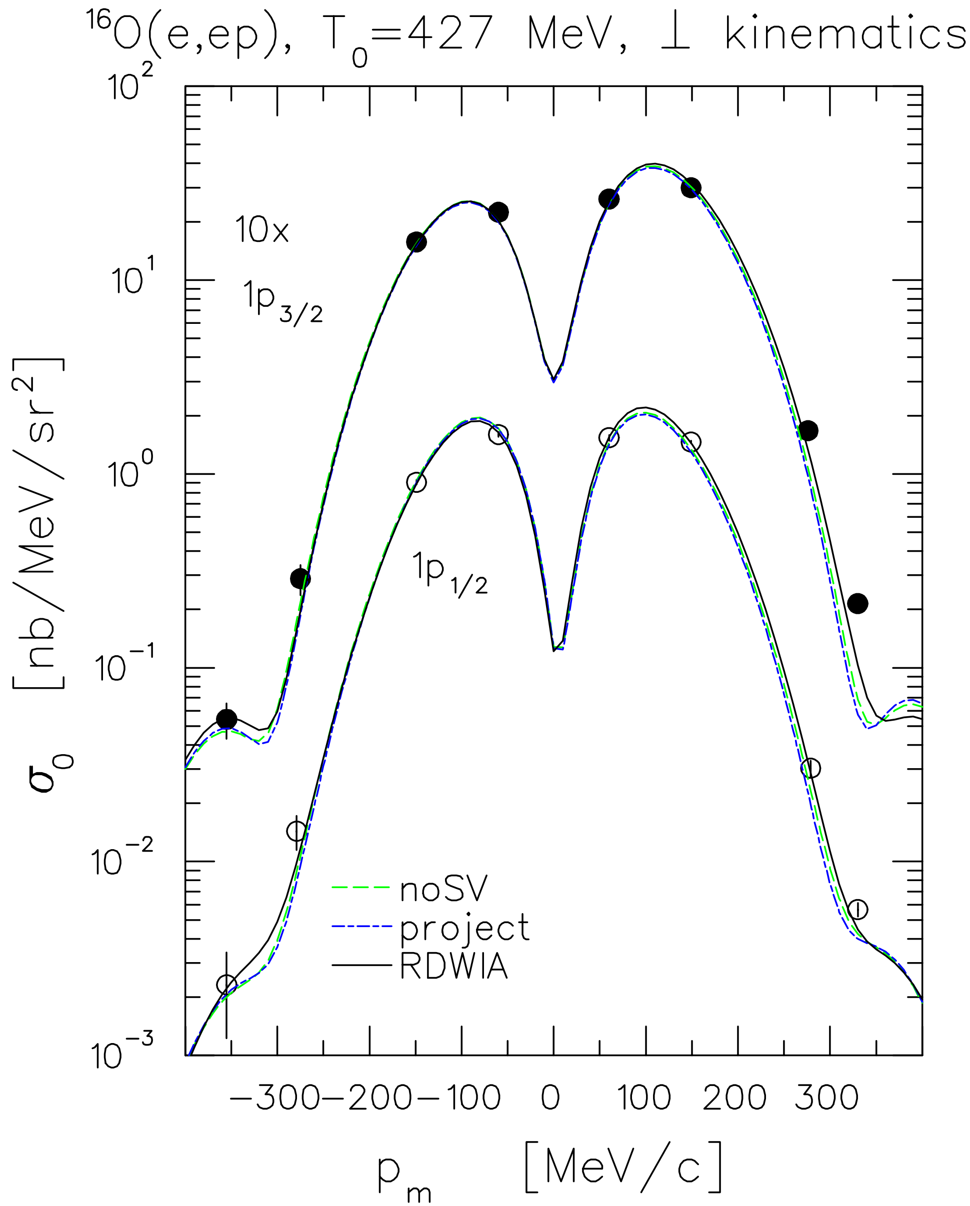
$^{16}O(e,e'p)$ at $Q^2 = 0.8$ (GeV/c)^2

- both models provide slight improvement to cross section
- neglect of dispersion in momentum of lower component most important for large p_m

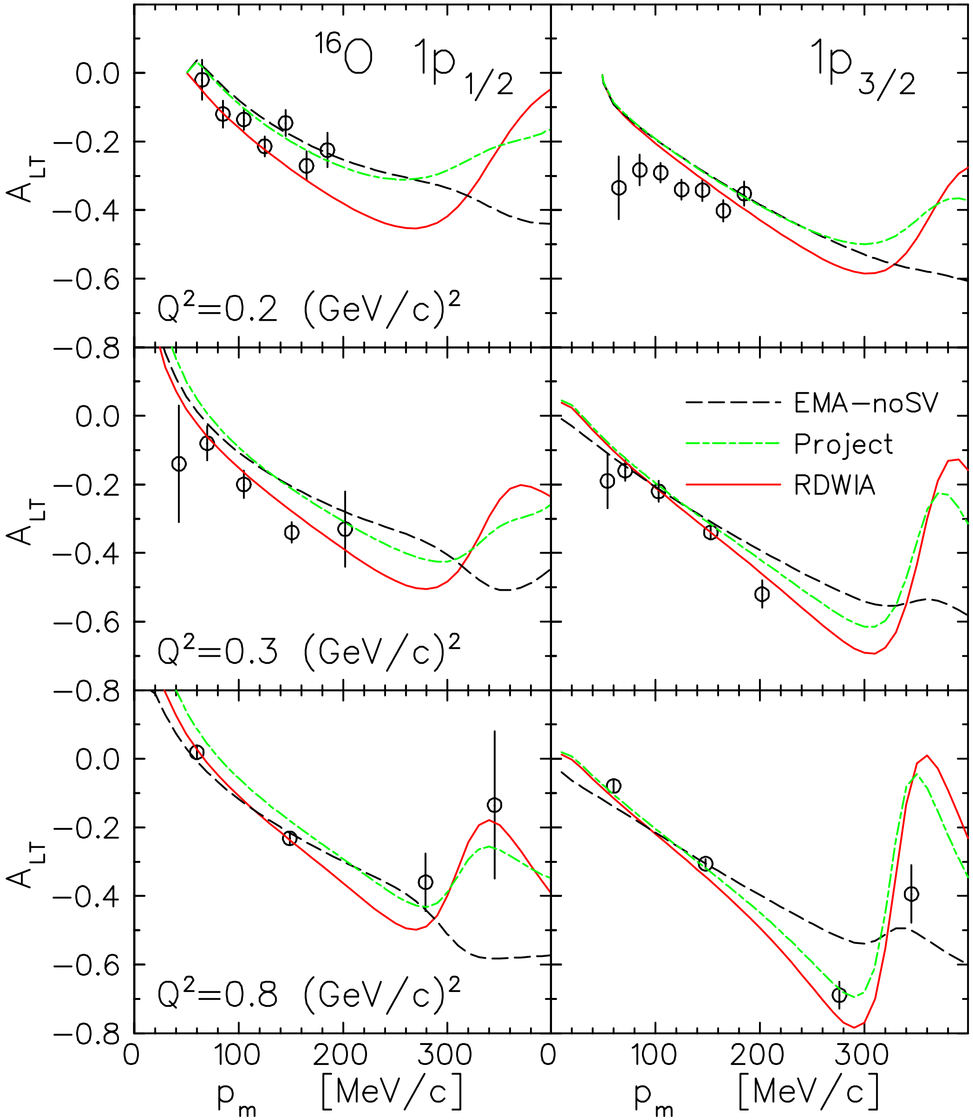


$^{16}O(e,e'p)$ **at** $Q^2 = 0.8$ **(GeV/**c**)**^{2}

- noSV similar to positive-energy projection
- contribution of sea small for cross section, but amplified in A_{LT}



Q^2 dependence of A_{LT} for $^{16}O(e, e'p)$



- Systematics favor spinor distortion
- visible in $1p_{1/2}$ for modest p_m at low Q^2
- large p_m needed for $1p_{3/2}$
- need more complete data

$^{12}C(e,e'p)$ at JLab

- Motivation: measure nuclear transparency, looking for color transparency
- Sufficient resolution to almost separate $1p$ and $1s$ shells for ^{12}C $\Longrightarrow$ fit

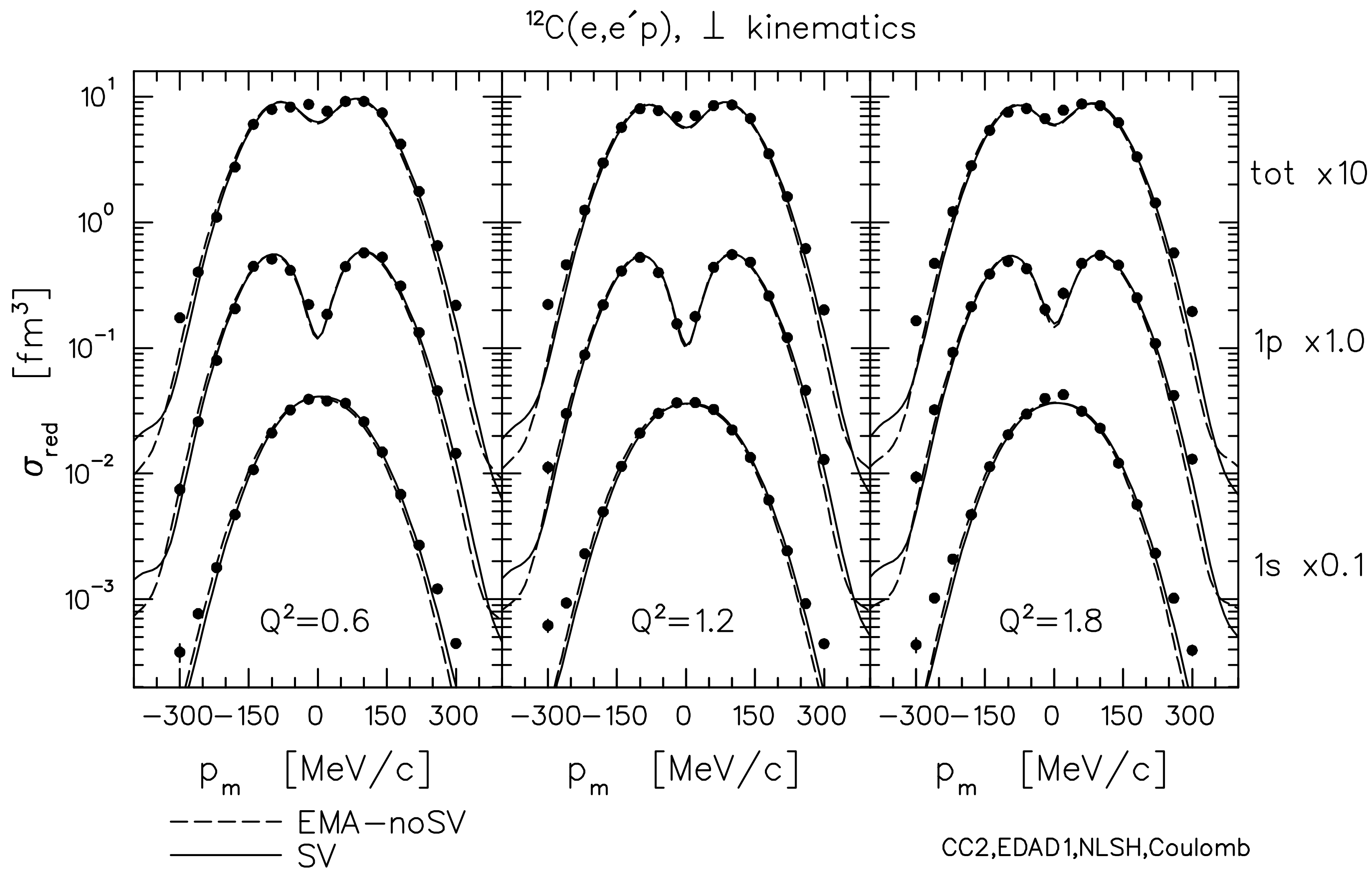
$$\sigma_{\text{red}} = S_{1p}\sigma_{\text{red}}(1p_{3/2}) + S_{1s}\sigma_{\text{red}}(1s_{1/2})$$

  to bins centered on shells and to semi-inclusive yield
- Sufficient p_m coverage to determine (Dutta *et al*.)

$$a_{LT} = \frac{\sigma_{\text{red}}(\phi=0) - \sigma_{\text{red}}(\phi=\pi)}{\sigma_{\text{red}}(\phi=0) + \sigma_{\text{red}}(\phi=\pi)}$$

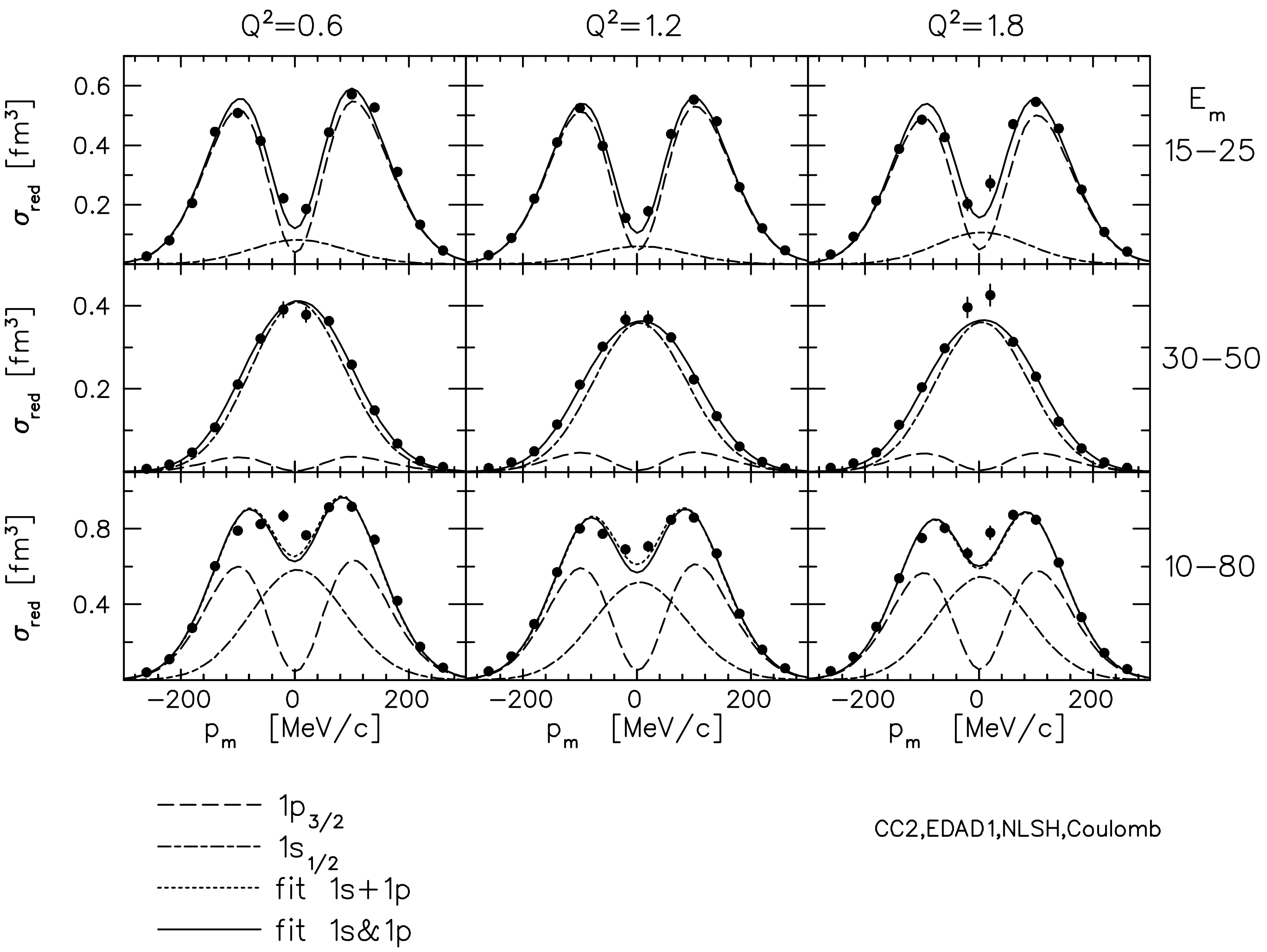
- a_{LT} vanishes for factorized calculation

Reduced cross section $^{12}C(e, e'p)$



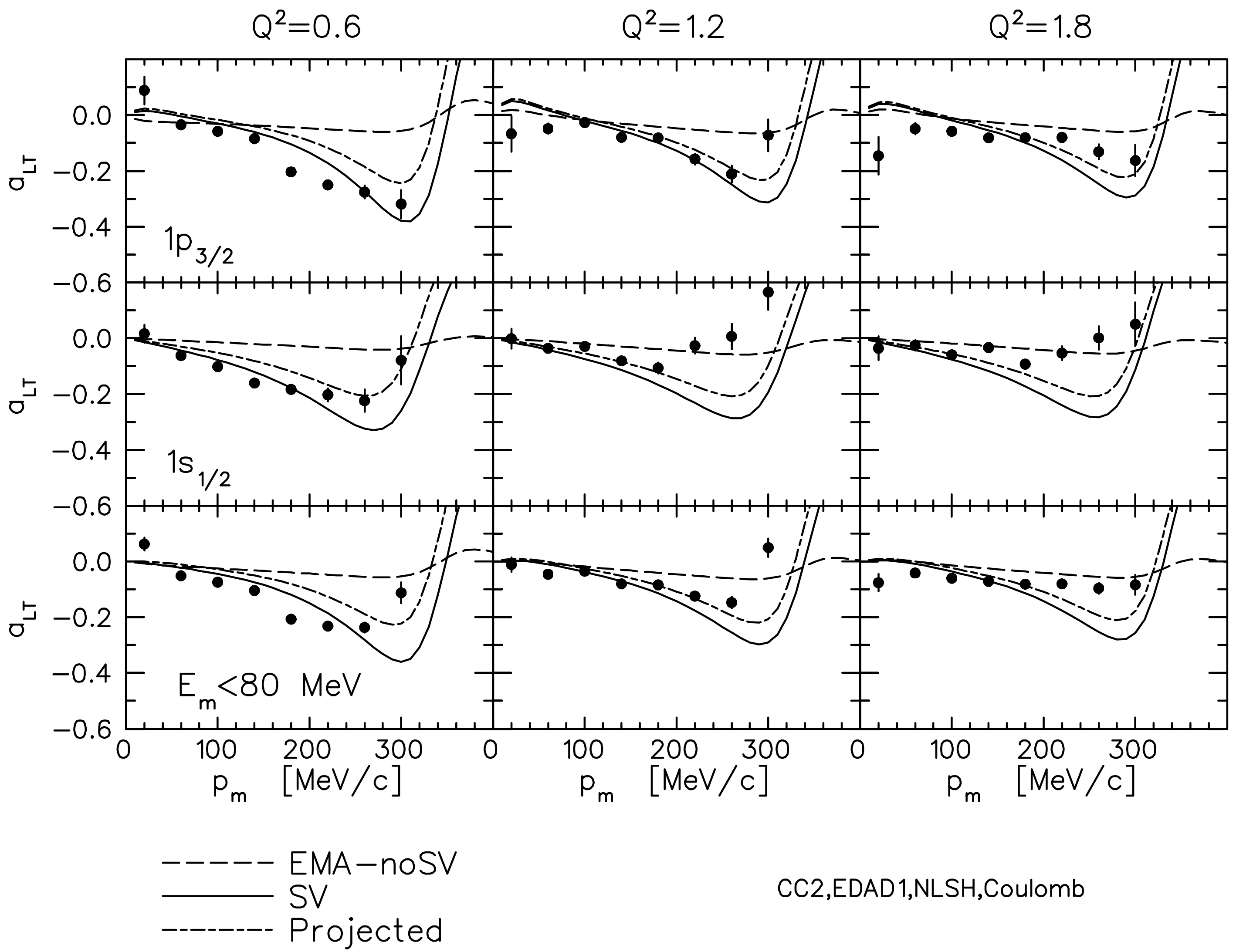
Effect on cross section subtle!

$1p$,$1s$ contributions to $^{12}C(e,e'p)$



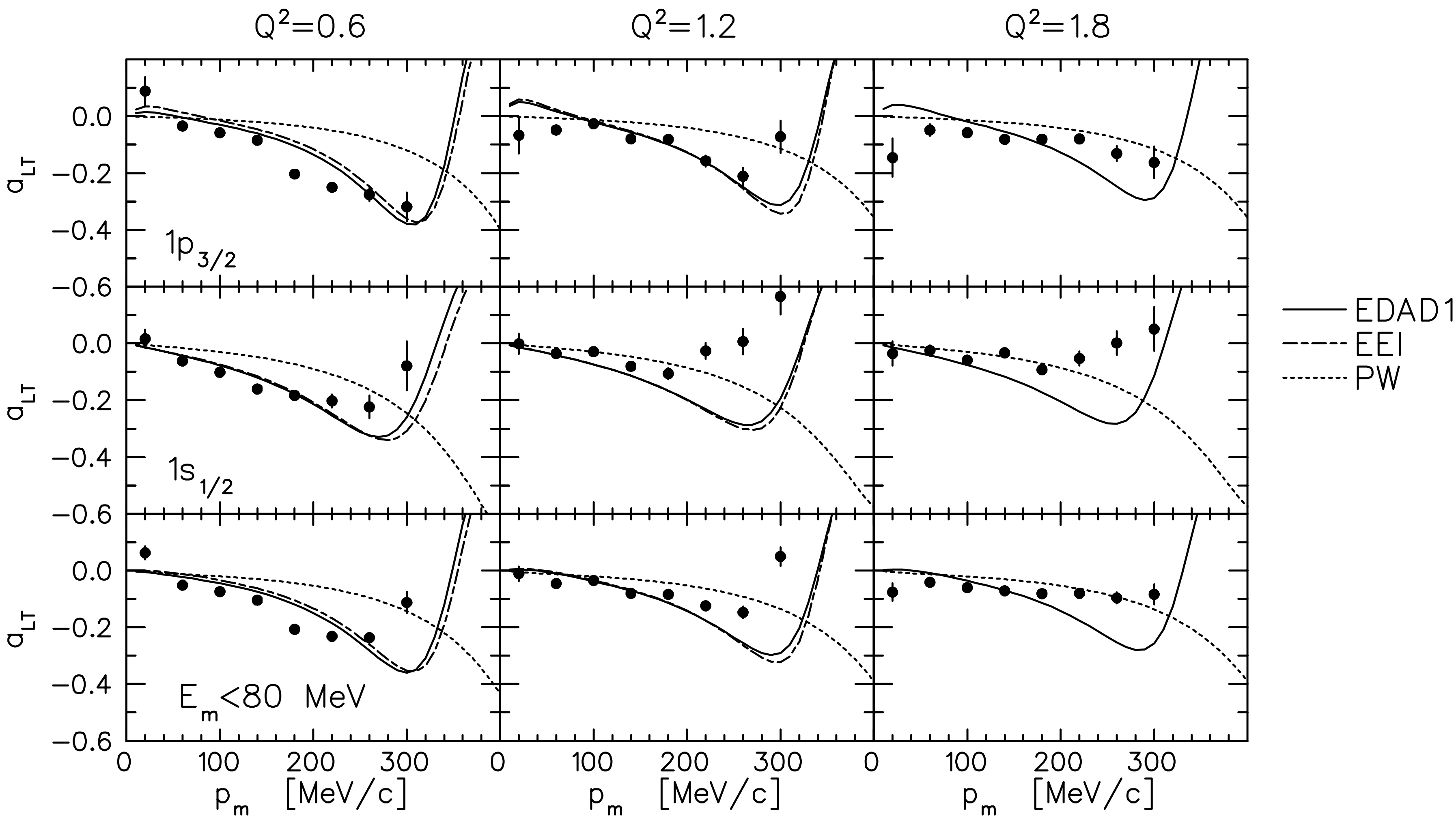
$1p$ contrib. to s-bin may simulate continuum

Q^2 dependence of a_{LT} for $^{12}C(e,e'p)$



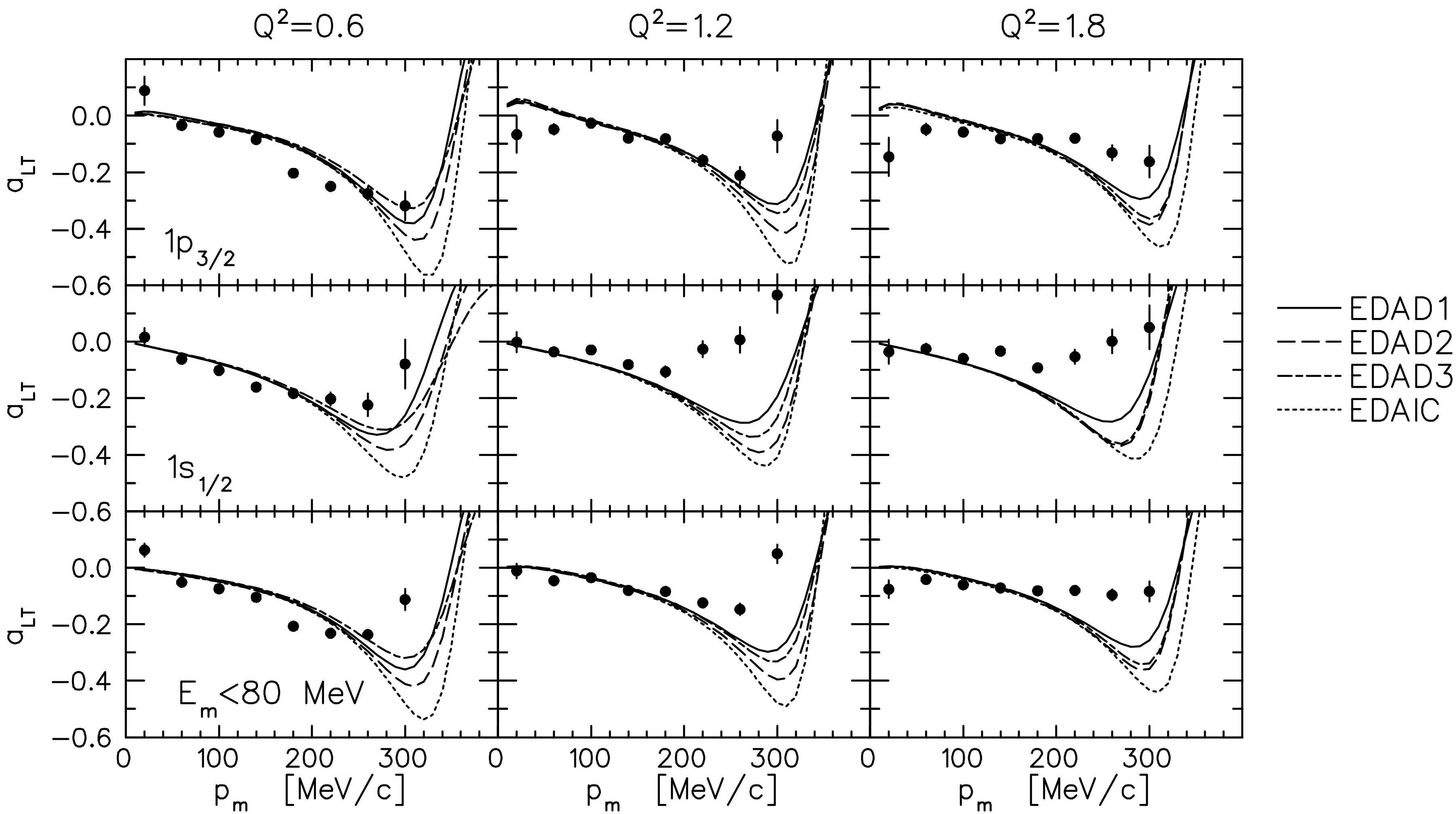
SV effect reduced at high Q^2. Sea small but visible.

FSI



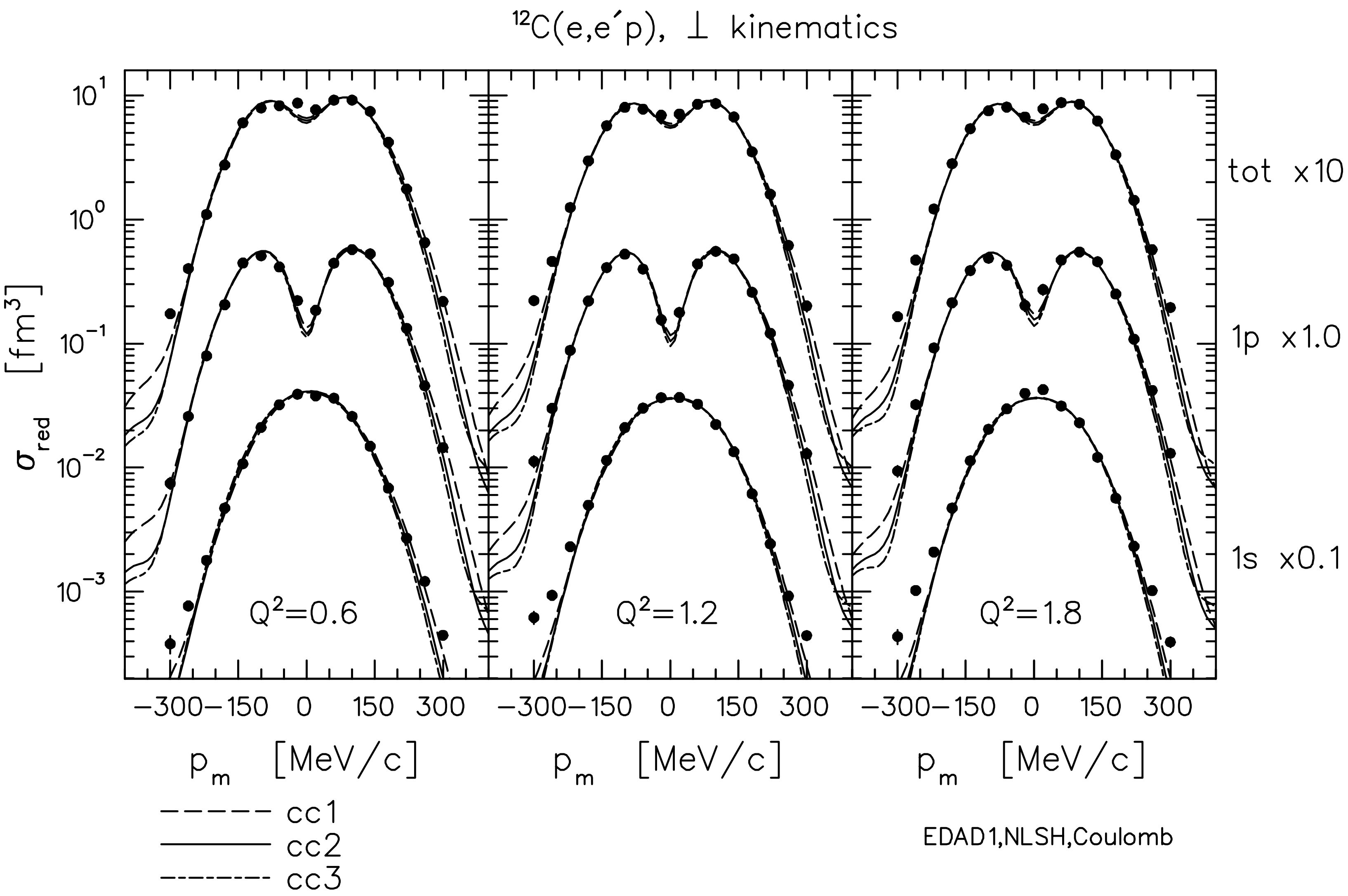
a_{LT} small without FSI. U^{LS} may be too strong for $Q^2 = 1.8$.

Dirac phenomenology

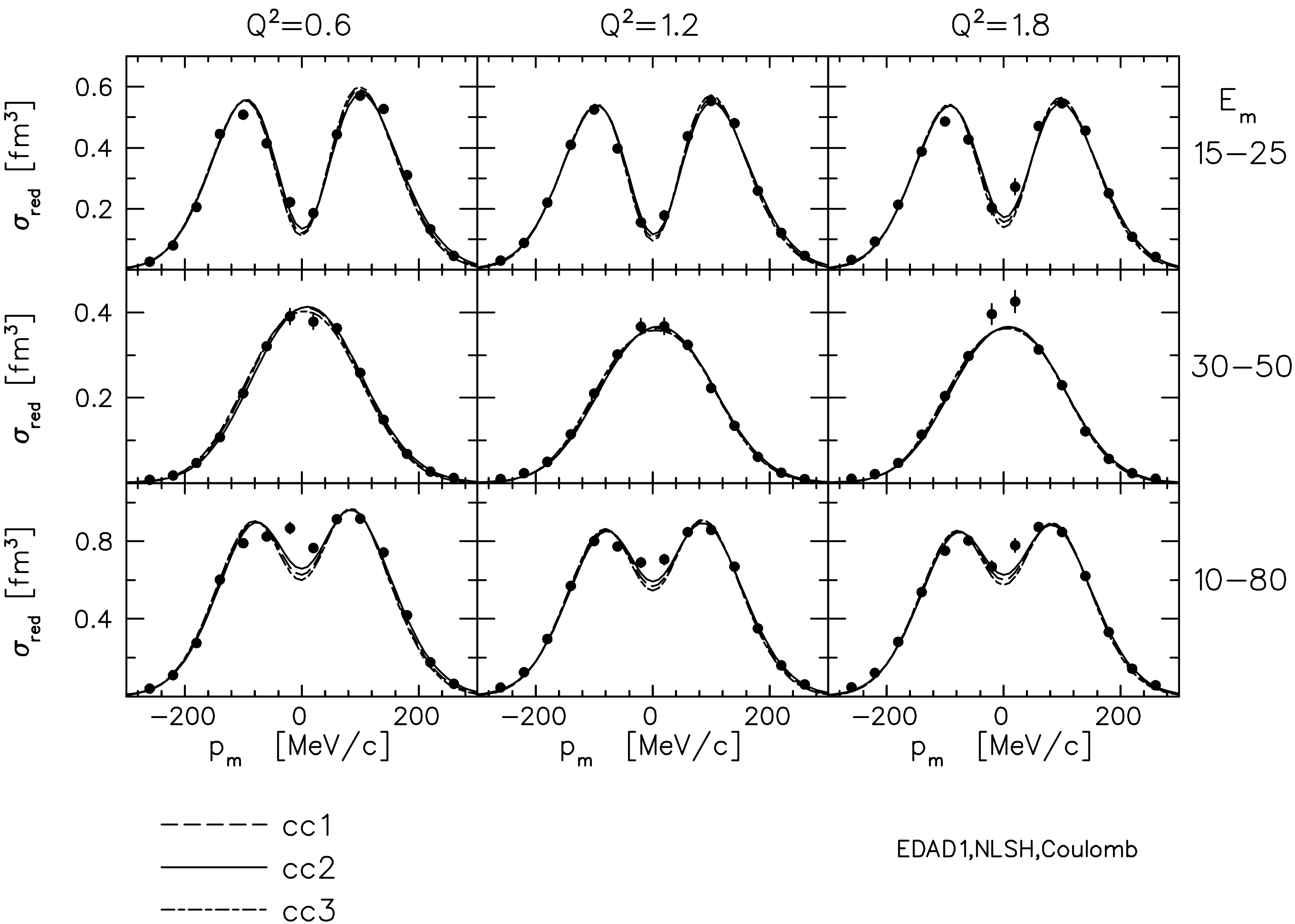


Little ambiguity for $p_m \lesssim 250$ MeV/c.

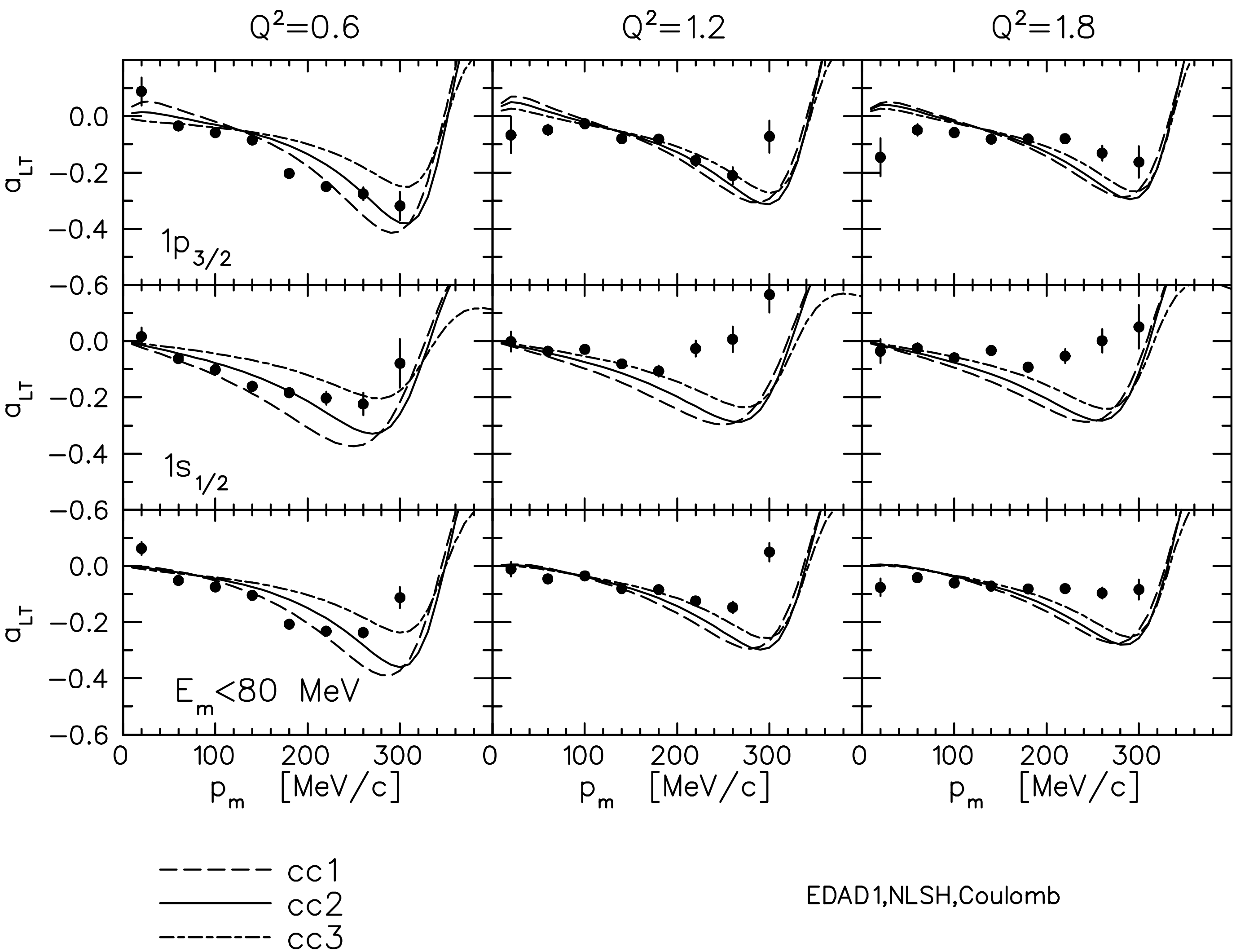
Gordon ambiguity for $^{12}C(e,e'p)$



Gordon ambiguity for $^{12}C(e, e'p)$



Gordon ambiguity for $^{12}C(e, e'p)$



Reduced at large Q^2, favors $\bar{\Gamma}_2$.

Transparency for $(e, e'p)$

Experimental definition:

$$\mathcal{T}_{\text{exp}} = \frac{\int_V d^3p_m dE_m \; \sigma_{e,e'p}}{f \sum_\alpha \int_V d^3p_m dE_m \; K\rho_\alpha(p_m) S(E_m) \sigma_{ep}}$$

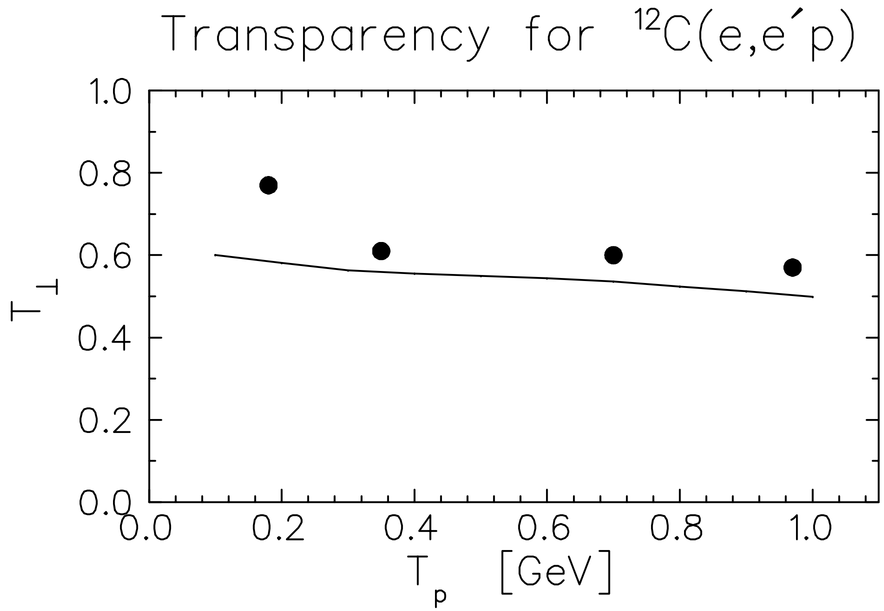
- depletion by short-range correlations $\Longrightarrow f \approx 0.90$
- simulated yield assumes factorization for both $S_\alpha(p_m, E_m)$ and reaction mechanism

RDWIA calculation:

$$\mathcal{T}_\perp = \frac{\sum_\alpha \int dp_m p_m \; \sigma_{\text{RDWIA}}(p_m, E_\alpha)}{\sum_\alpha \int dp_m p_m \; \sigma_{\text{RPWIA}}(p_m, E_\alpha)}$$

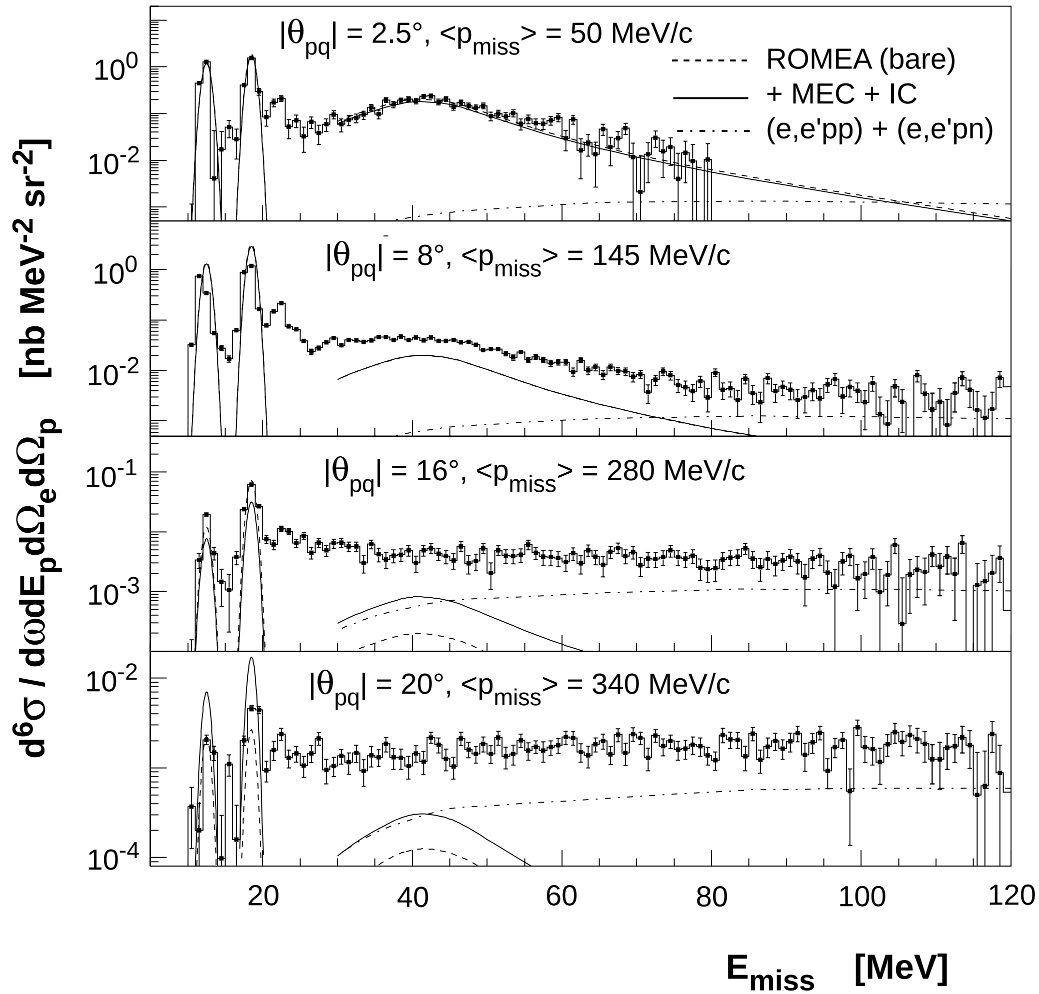
Transparency for $^{12}C(e,e'p)$

- fixed E_0, T_p
- EDAD1, NLSH, CC2
- spinor distortion small
- Darwin factor reduces $T_\perp$ by 11–6% (relative) for $0.6 < Q^2 < 1.8$



- fitted $S_{1s+1p} \approx 1 \Longrightarrow f\mathcal{T}_{\text{exp}} \approx \mathcal{T}_\perp$.
- $\approx 10\%$ deficit consistent with neglect of continuum

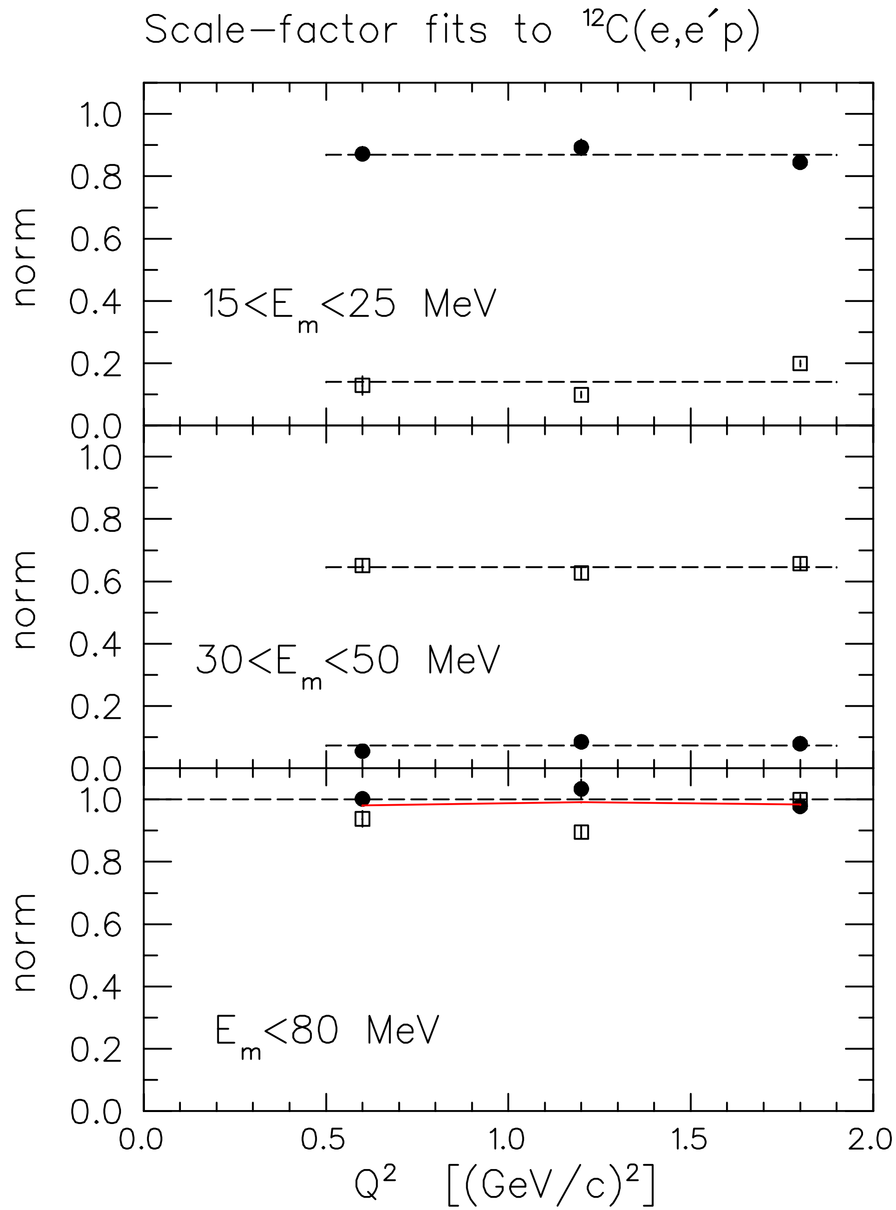
$^{16}O(e,e'p)$ **continuum at** $Q^2 = 0.8$



- Importance of continuum increases with p_m.
- Nearly full s-shell occupancy (Ryckebusch *et al.*)

Scale factors for $^{12}C(e,e'p)$

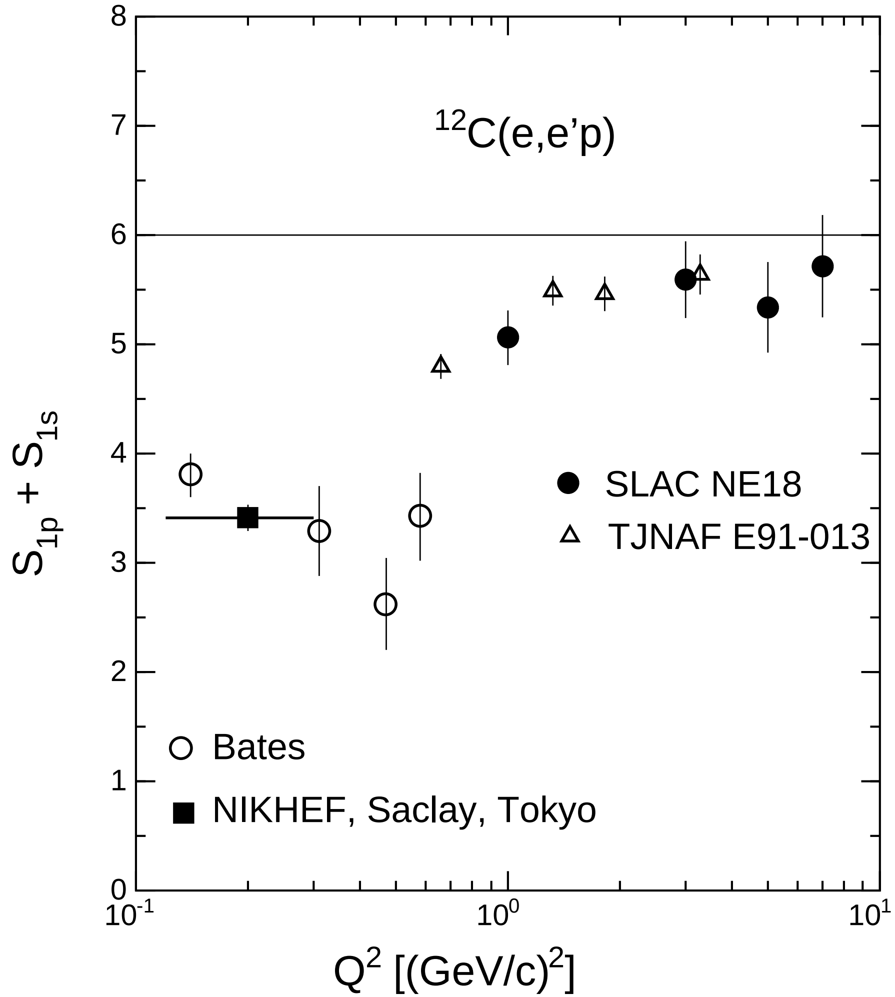
- om variations negligible
- $\approx$ full IPSM in $10 < E_m < 80$ MeV
- depletion by short-range correlations $\Longrightarrow$
  - potentials too absorptive, or
  - $\approx 10\%$ multinucleon continuum, or
  - S too large with NLSH wave functions



$S_\alpha(Q^2)$?

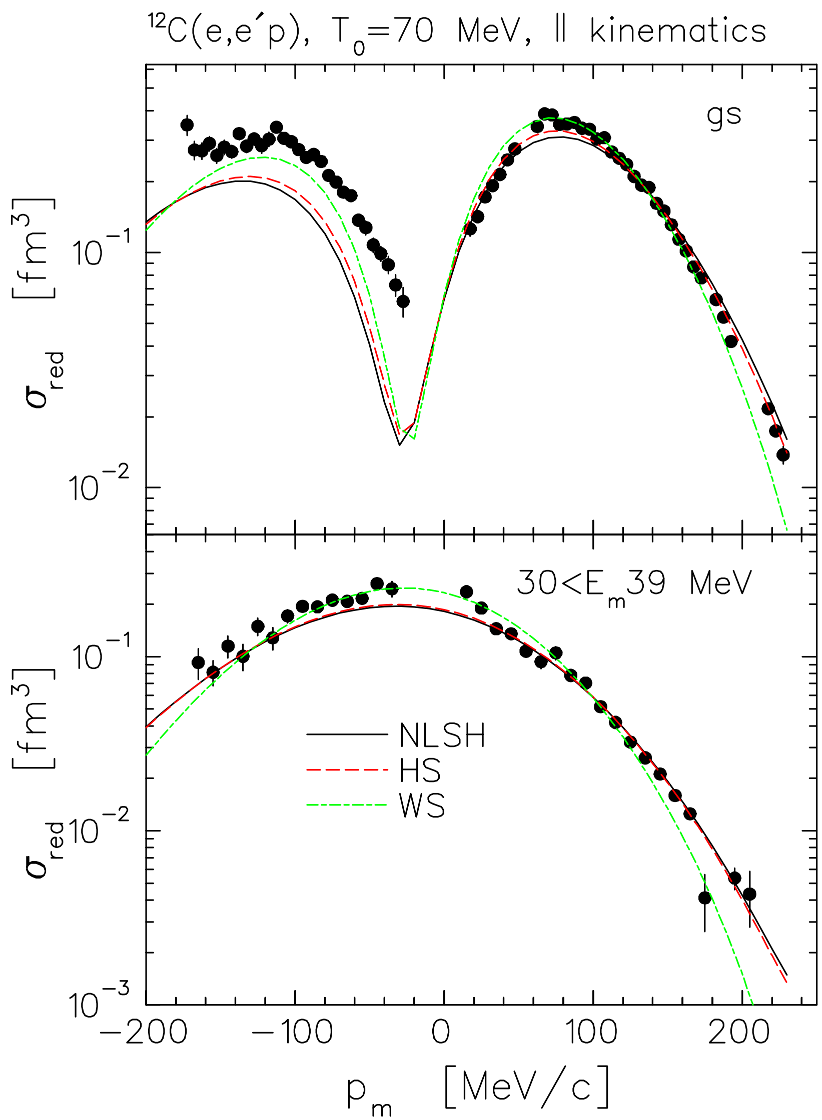
Lapikàs *et al.* claim transition in quasiparticle response near $Q^2 \sim 0.6$, but

- larger constant S_α in ^{12}C for $0.6 < Q^2 < 1.8$
- ^{16}O$(e, e'p)$ for $0.2 < Q^2 < 0.8$
  - S_α consistent with Q^2 independence
  - $\int_{12}^{27} dE_m S_{1p}(E_m) \approx 0.65(9) S_{\text{IPSM}}$
  - $S_{1s}/S_{\text{IPSM}} \sim 1$

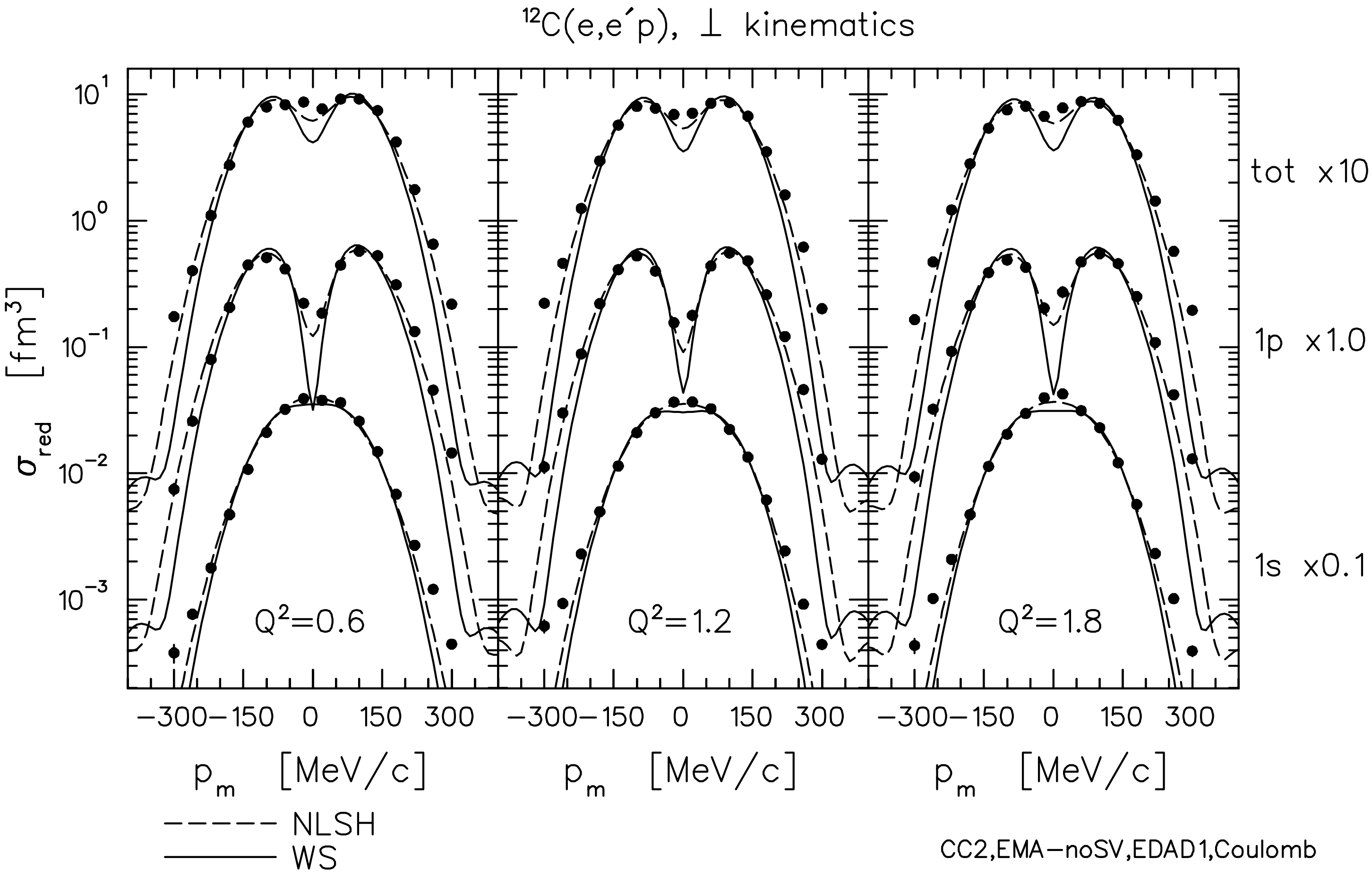


Problems at low Q^2

- parallel kinematics far off shell: $1.5 \gtrsim x_B \gtrsim 0$
- large range of Q^2
- fit to $\rho(p_m)$ can shift DWIA errors into bswf
- cannot see enough of s-shell for reliable normalization
- WS fit to $T_p = 70$ MeV data gives poor decrip-tion of high Q^2 data



Test NIKHEF WSWF at large Q^2



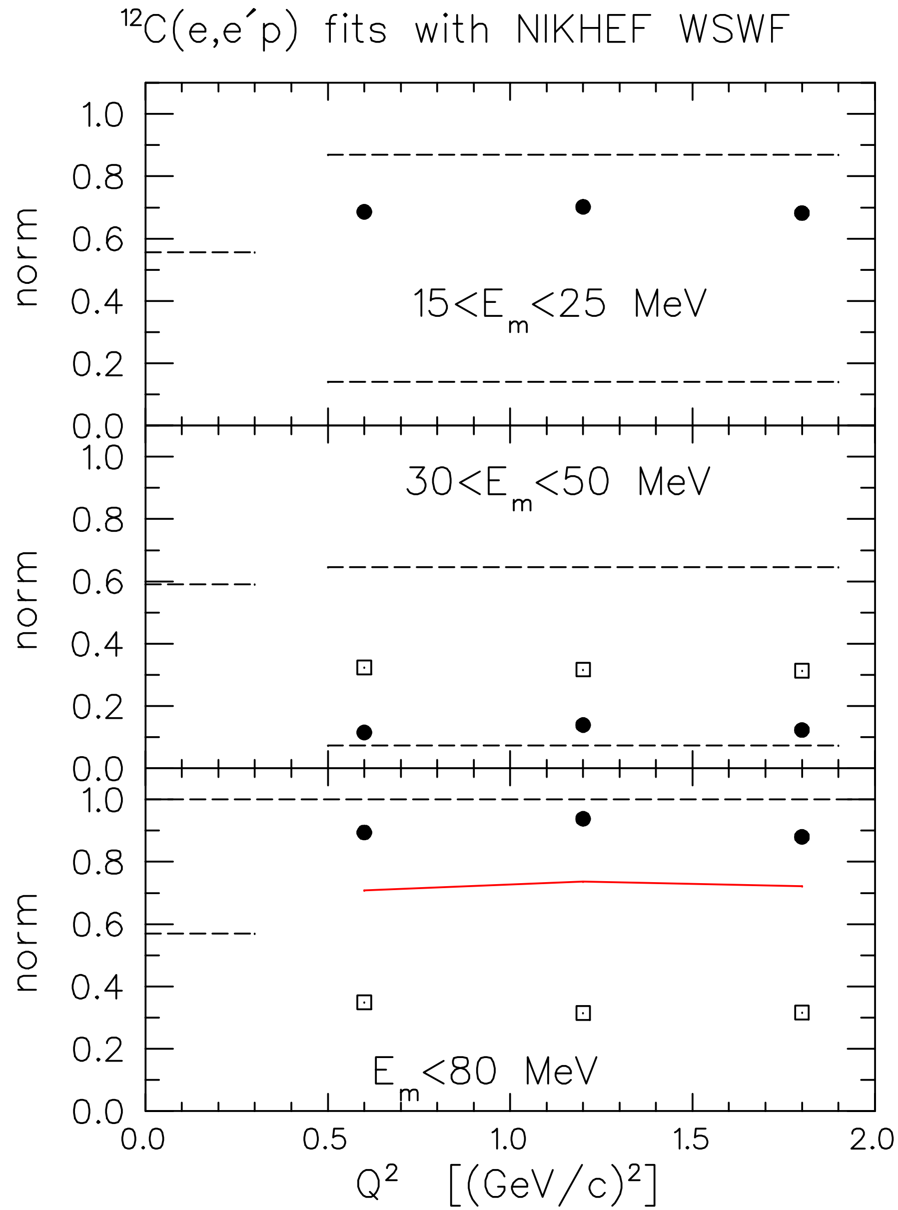
- rather poor fits, no s contibution to $E_m < 25$ MeV
- RDWIA better at high $Q^2 \Longrightarrow$ these WSWF unreliable

High Q^2 fits with NIKHEF WSWF

- p shell reduced by $\approx 25\%$
- s in $30 < E_m < 50$ reduced by $\approx 50\%$

Better approach:

- RDWIA, constant $(\omega, \vec{q})$, large Q^2 to evaluate bswf, adjust if necessary
- use low Q^2 discrepancies to study reaction mechanism and off-shell current



Conclusions

- EMA
  - noSV similar to positive-energy projection
  - dominant defect is neglect of dispersion in lower component
- A_{LT} sensitive to relativistic spinor distortion
  - SV effect most important for bound state
  - sea contribution small but visible for $1p_{1/2}$ at low p_m
  - Gordon ambiguity reduced at high Q^2
  - Gauge and om ambiguities negligible for modest p_m
- spectroscopic factors
  - model dependence remains $\sim \pm 15\%$
  - are low Q^2 problems experimental or failure of reaction model?