ECT Trento 2004
Workshop on Spectroscopic Factors

Spectroscopic Factors from (e,e'p) reactions

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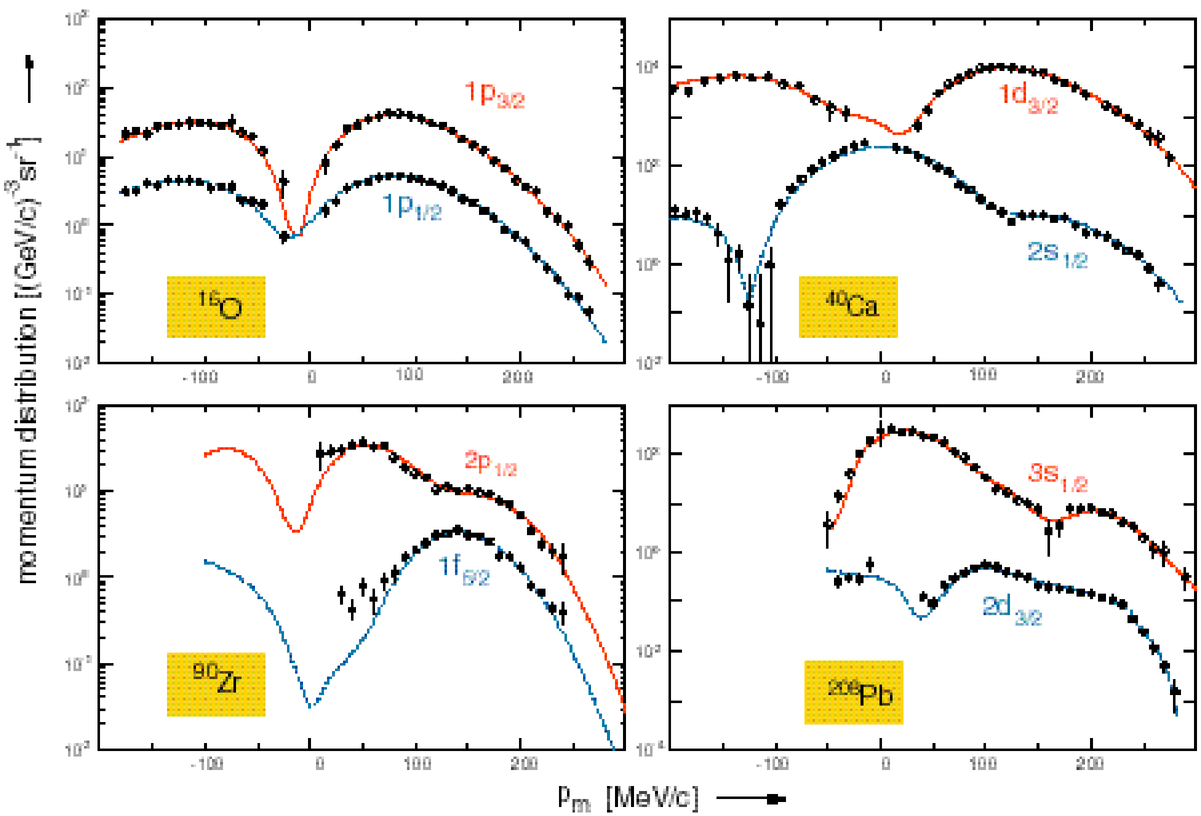
1. **Introduction**
   some early (e,e'p) results, spectroscopic factors
   effective mass, theoretical approaches
2. **Beyond Mean Field Theory**
   Variational Monte Carlo, ^{7}Li(e,e'p)
3. **Towards larger momentum**
   ^{208}Pb(e,e'p), relativistc effects
4. **Towards deeper energies**
   ^{208}Pb(e,e'p), Rescattering, MEC
5. **Towards higher Q^2**
   ^{12}C(e,e'p), FSI, Transparencies
6. **Summary and Conclusion**

Introduction
Some early (e,e'p) results

Spectroscopic strength with the reaction (e,e'p)

- seventies : pioneering experiments Frascati, Tokyo, Saclay
- eighties : high res. NIKHEF (e,e'p) program for nuclei A=2-209
  - spectral function at low (E_m, p_m)
  - Momentum distributions of valence orbits
- nineties –present : NIKHEF/Mainz/Bates also 2N knockout
- present : JLAB towards higher Q^2, larger p_m , E_m

NIKHEF RESULTS

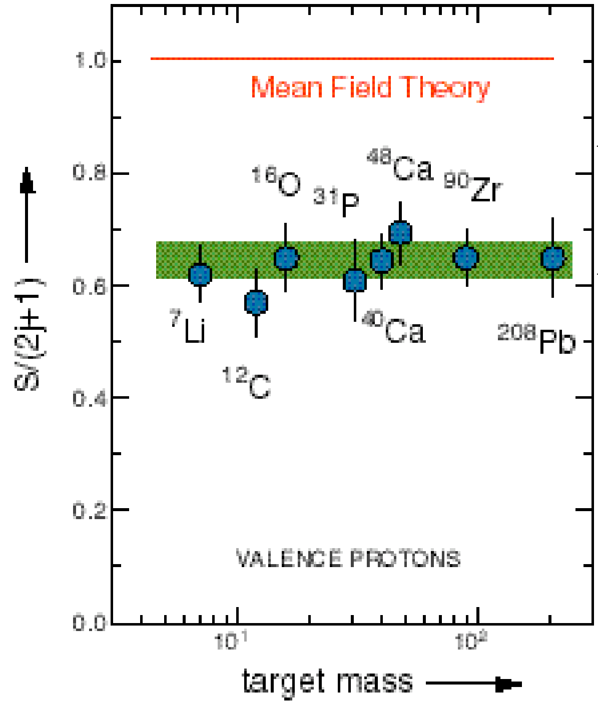


Results for valence orbits in closed-shell nuclei:

Curves scaled by about **0.65** wrt. mean field theory !!

Explanation : Effect of long-range and short-range correlations

Introduction
Spectroscopic Factors



RPA (LRC), G-matrix

NM (SRC) +RPA (LRC)

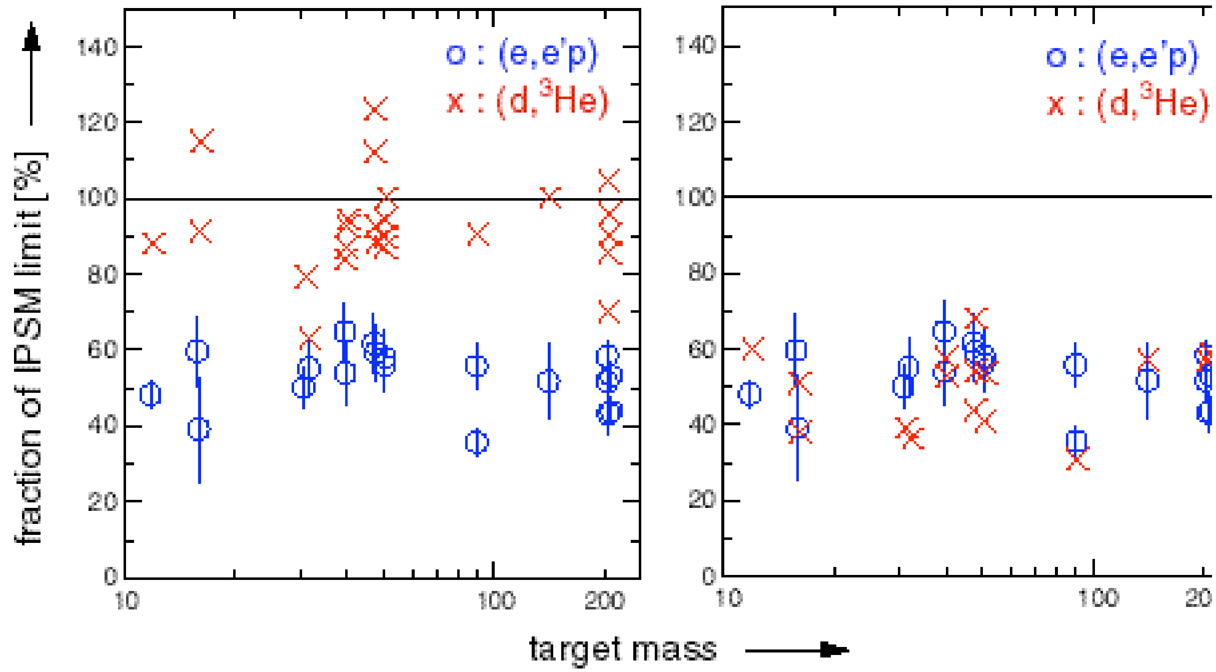
35 % reduction w.r.t MFT

- **10-15 % due to LRC for finite nuclei in RPA**
- **10-15 % effect due to SRC calculated for infinite NM**

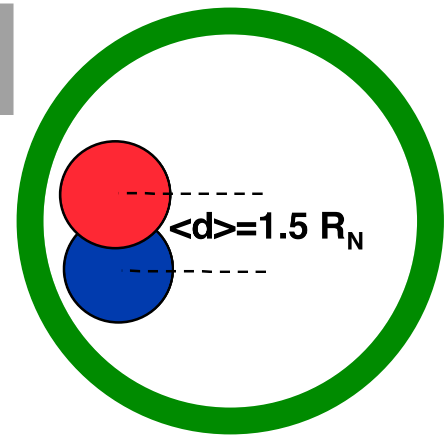
Seeming discrepancy between (d,^{3}He) and (e,e'p) data solved

Original data
(d,^{3}He) Local/Zero-range arbitrary BSWF

NIKHEF Reanalysis
(d,^{3}He) Non-Local/Finite-range BSWF from (e,e'p)



Introduction
Theoretical Approaches



Perturbation theory

- need to calculate ME : $< \square^*_i \square^*_j | V | \square_k \square_l >$
- diverge with realistic interaction V
- take soft-core effective V
- RPA : typically 10 % reduction

G-matrix approach

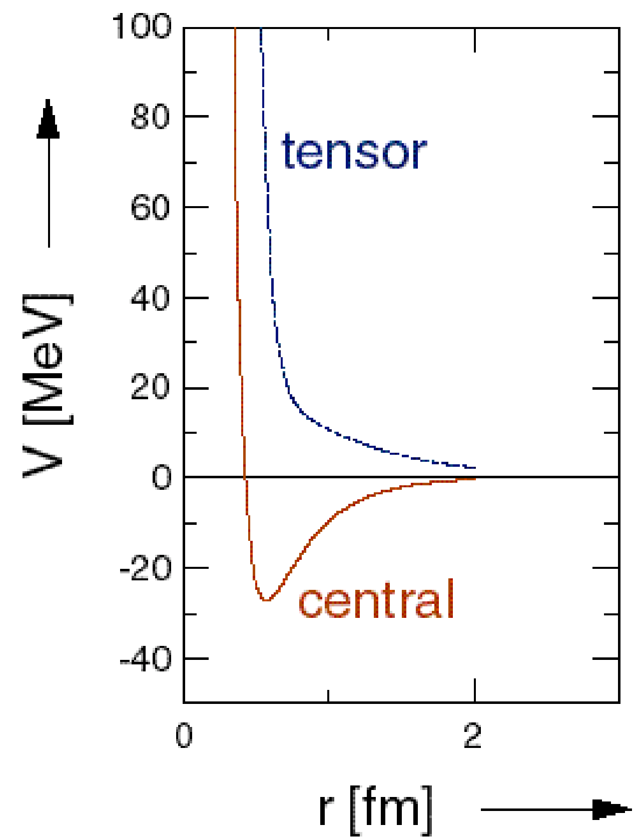
- G-matrix : replace V by $G = V - V Q e^{-1} G$
  results in < 20 % reduction (^{16}O, ^{40}Ca, ^{90}Zr)

Phenomenological approach

- Introduce an effective nucleon mass : $m^*(r,E) / m$
- Calculate overlap matrix elements with m^*
  results in 15-25 % reduction

Realistic approach

- Variational Monte Carlo ➔ Light Nuclei
- Correlated Basis Functions ➔ Nuclear Matter



Introduction
Effective mass

Introduce an effective mass in the overlap function to account for correlations

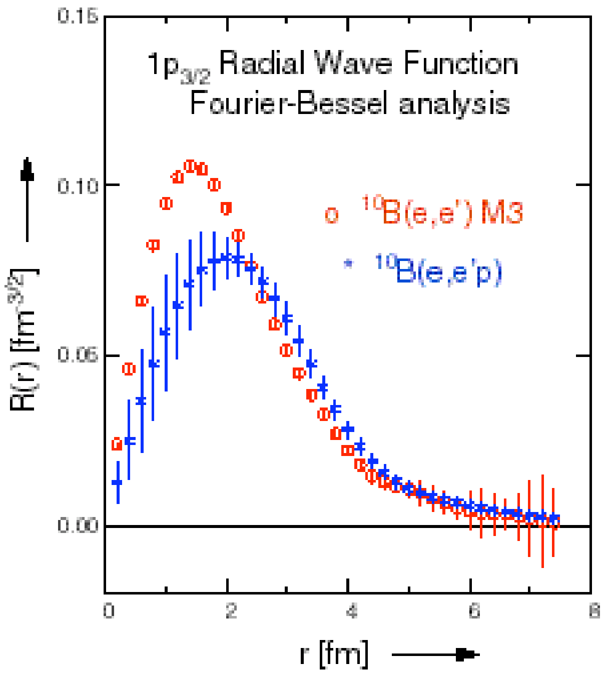
$$m^*(r,E) / m = 1 - {}^{d}/_{dE}\, V(r,E) \quad \rightarrow \quad < \Box \ \mathrm{l} \ \exp(i\, p_m\, r)\ m^*(r,E)/m \ \mathrm{l} \ \Box >$$

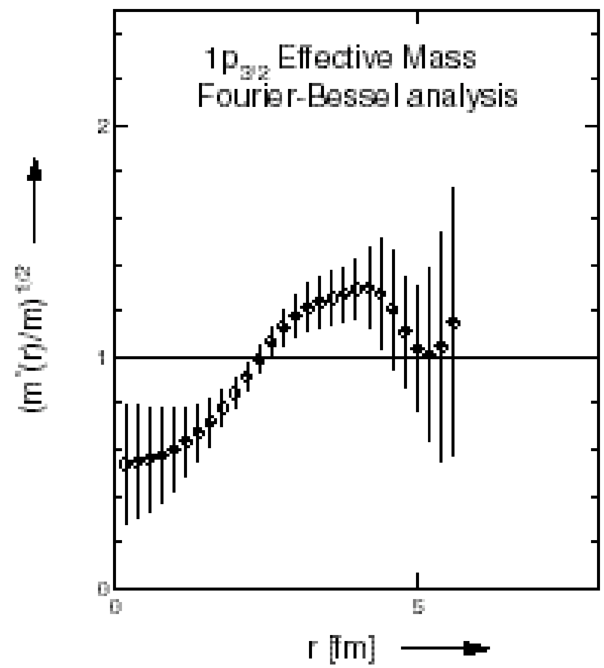
----------- Experimental determination of the effective mass ---------

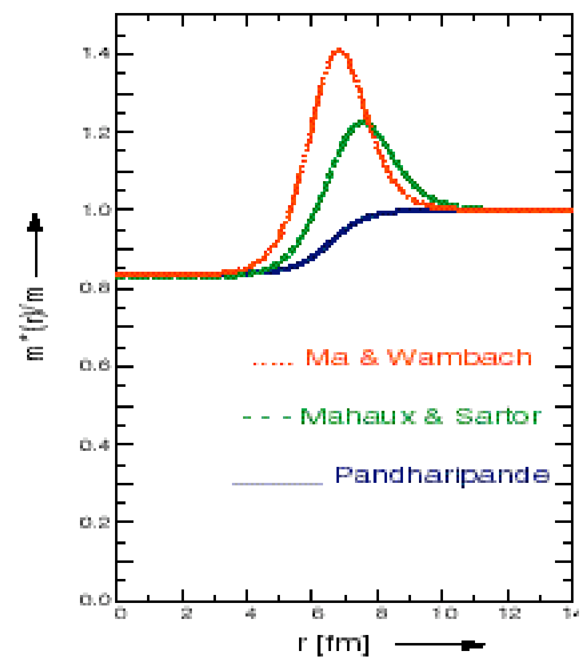
1p3/2 radial wave functions from
^{10}B(e,e') no eff. mass
^{10}B(e,e'p) with eff. mass

Ratio of wave functions yields effective mass for 1p3/2 wave function

Effective mass calculation for ^{208}Pb with SRC (+LRC)





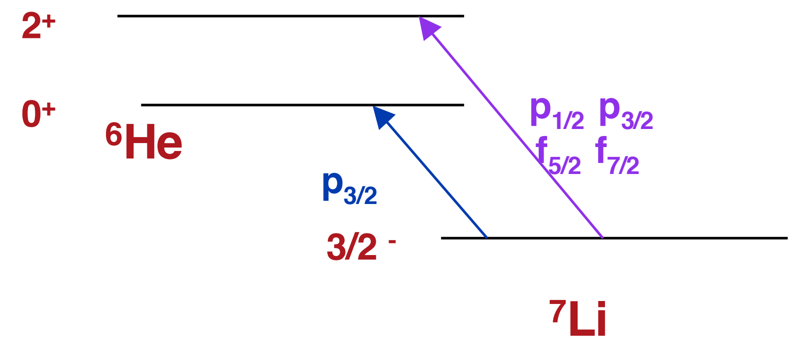


Beyond MFT -> VMC
^{7}Li(e,e'p)

Full calculation

- Variational Monte Carlo (VMC)
- V = AV18 / UIX

  (Argonne 2-nucleon + Urbana 3-nucleon interaction)
- Done for few- body systems
- Now available for A = 6, 7, 9



Technique

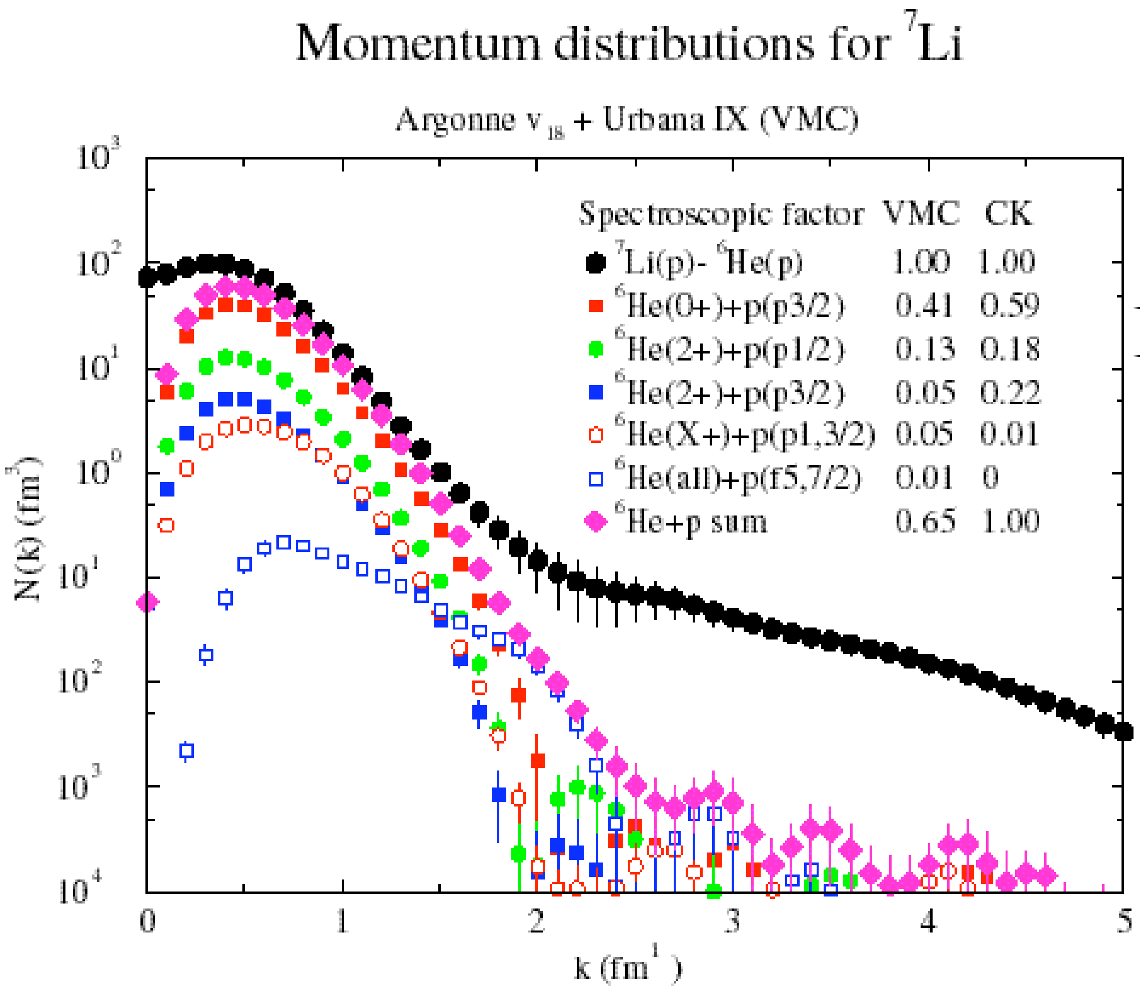
- Minimize and diagonalize $< \square_V \,|\, H \,|\, \square_V >$
- Trial wave function $\square_V = [\, 1 + \square\, U_{ijk} \,]\, [S \square\, (\, 1 + U_{ij}) \,]\, \square_J$
- Two/three body correlation functions U_{ij} , U_{ijk}
- $< \square_V\, (\, ^6He^* \,) \,|\, a(p_m) \,|\, \square_V\, (\, ^7Li \,) >$ measured in (e,e'p)

	MFT (1p)	VMC (1p+1f)
$3/2^- \to 0^+$	0.59	0.41
$3/2^- \to 2^+$	0.40	0.19
Sum	0.99	0.60

Pudliner, Pandharipande, Carlson,
Wiringa, Pieper, Forest

Variational Monte Carlo
^{7}Li(e,e'p)

Pudliner, Pandharipande, Carlson, Wiringa, Pieper, Forest

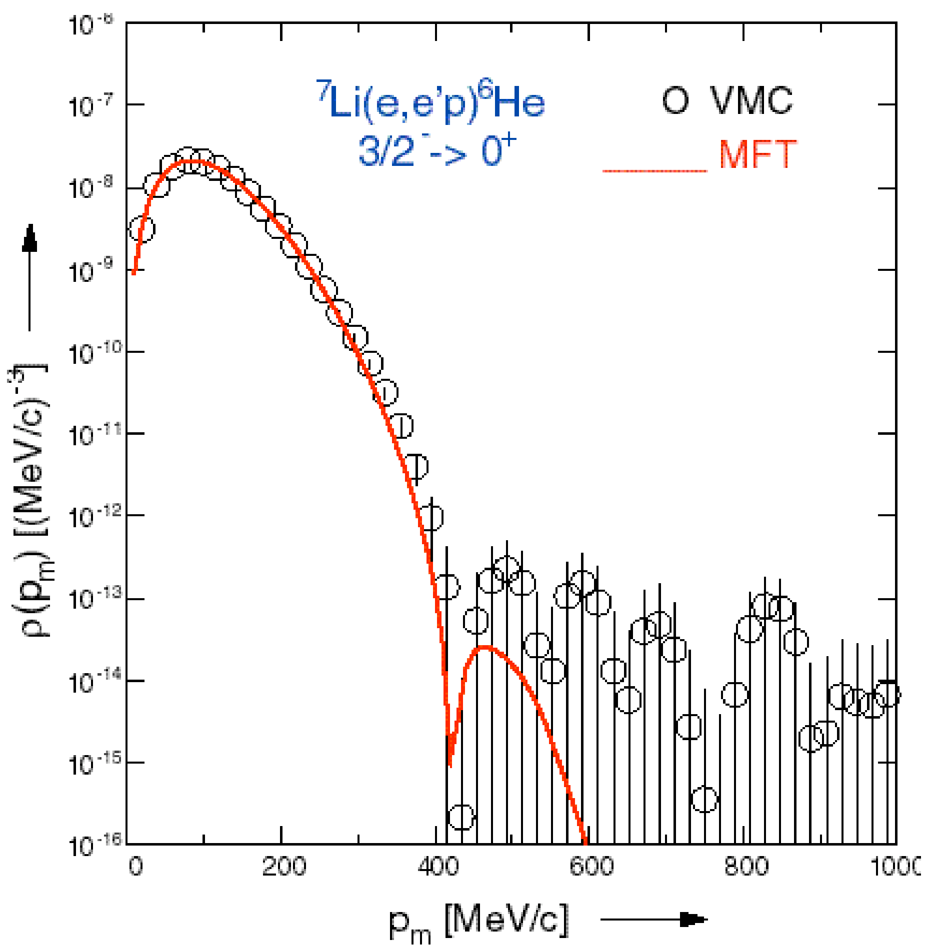


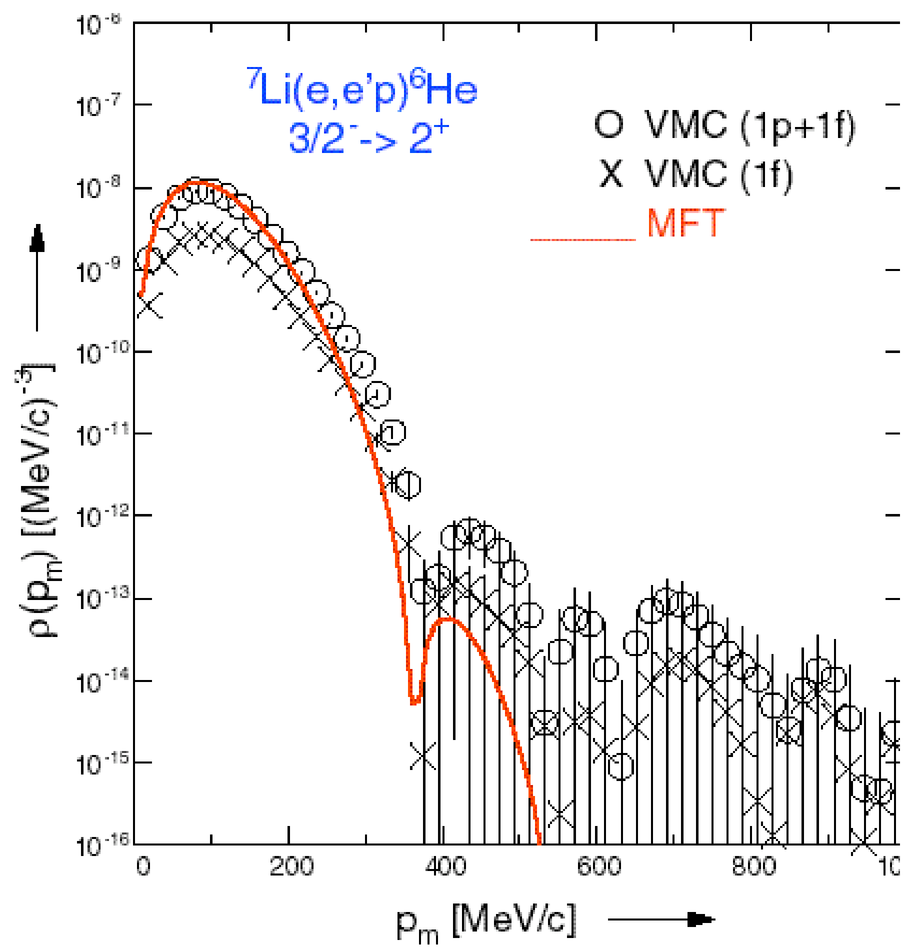
measured in (e,e'p) at NIKHEF at low E_m

total strength at all E_m

VMC versus MFT
^{7}Li(e,e'p)

Compare MFT and VMC overlap wave functions
- normalize both overlaps to 1
- choose MFT rms radii equal to VMC rms radii





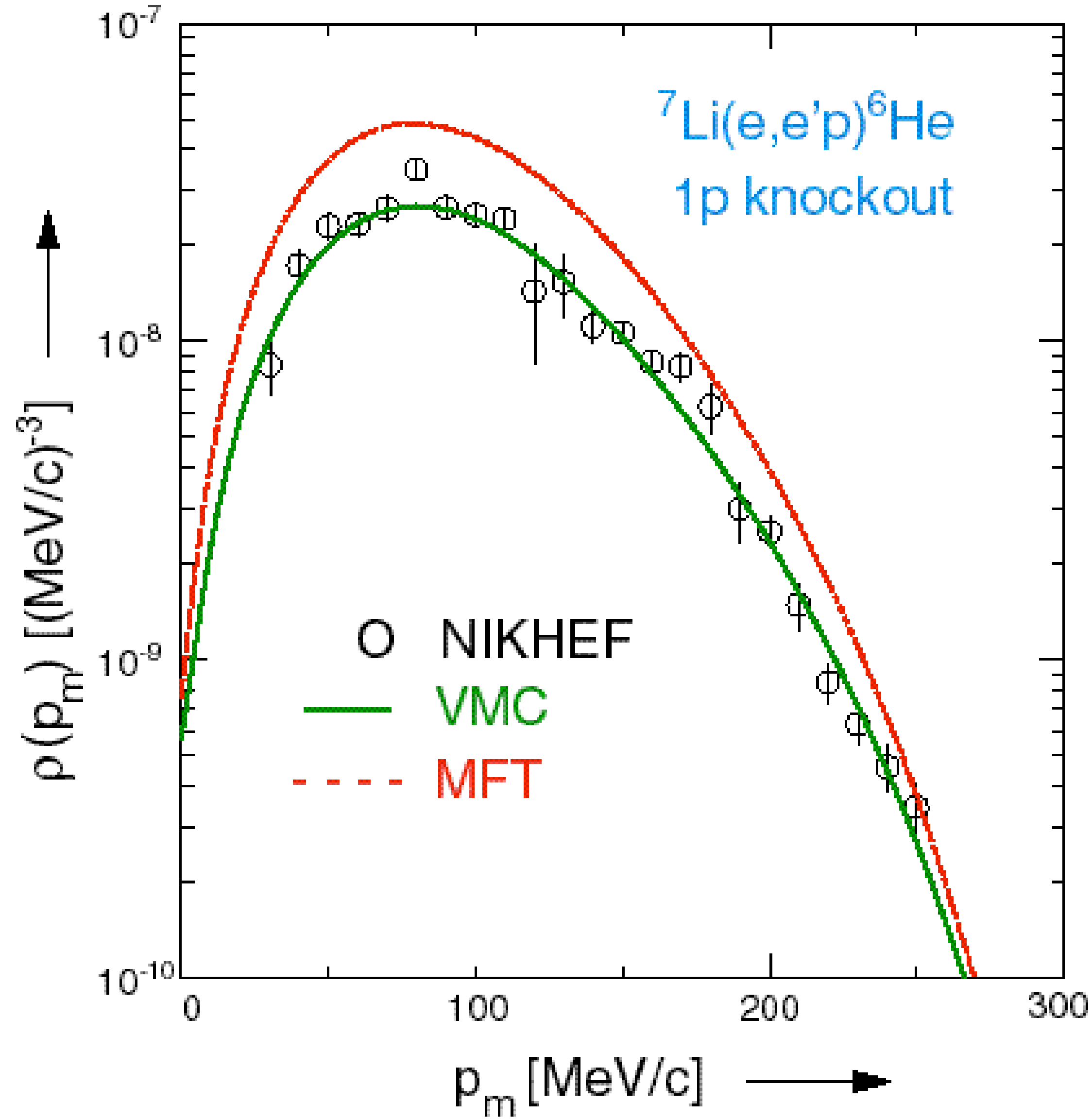
$^{7}Li(e,e'p)$ Spectroscopic Strength

Summarized results

	spectroscopic strength		
	0^+	2^+	$0^+ + 2^+$
Exp	0.42(4)	0.16(2)	0.58(5)
VMC	0.41	0.19	0.60
MFT	0.59	0.40	0.99

- for < 6He I 7Li > overlap VMC explains exactly measured 40% reduction w.r.t. MFT

- for successful description of (e,e'p) momentum distributions (size and shape) full correlations necessary in nuclear-structure calculations



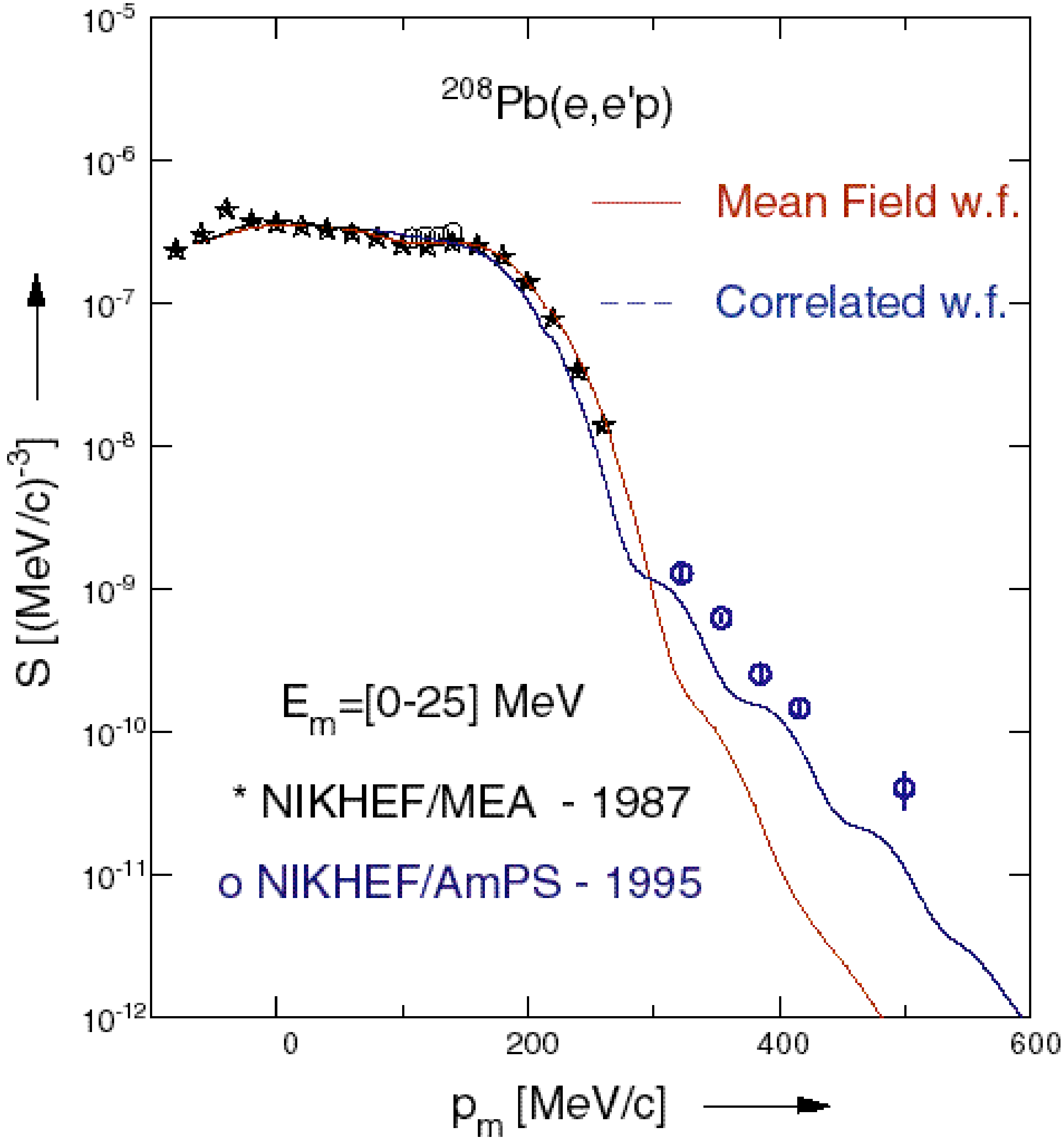
Towards larger momentum
$^{208}Pb(e,e'p)$

Correlations reduce
wave functions at small radius,
hence the
momentum distributions
at large momentum

Is this measurable?

Warning! : at large p_m interpretation difficult :

- Coulomb distortions (Gent, Madrid, Ohio)
- Relativistic effects (Madrid, Gent, Ohio)
- 2-body currents (Gent)



Towards deeper energies
$^{208}Pb(e,e'p)$

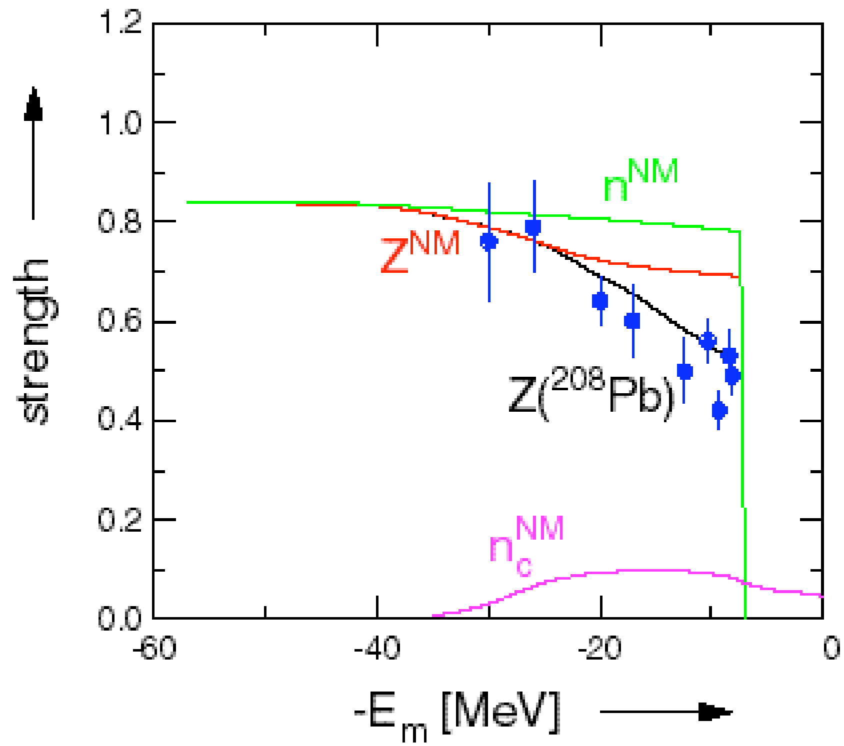
Measured spectroscopic strengths below E_m=30 MeV are influenced both by LRC and SRC.

For deeper lying shells (at large E_m):

- LRC tend to disappear
- Quasi-hole strength Z_h approaches occupation n_h

Most realistic NM calculations predict n_h=0.80-0.85

Measure $^{208}Pb(e,e'p)$ to large E_m



^{208}Pb(e,e'p) Experiment

Experiment @ AmPS :

- Measured ^{208}Pb(e,e'p) in spectral function range $\{E_m,p_m\}$={0-100 MeV, 0-270 MeV/c}
- Difficulties above E_{2N} (16 MeV) :
  - MEC and Δ-excitation may contribute
  - Rescattering (e,e'N) (Np) may contribute
- Data measured at two beam energies
  --> study MEC

Calculate experimental spectral functions

$S^{exp}(E_m,p_m) = \sigma^{exp} / K\sigma_{ep}$

Model spectral function

$n_\alpha(E_\alpha)$ fractional occupations

$\rho_\alpha(p_m)$ distorted momentum distributions: CDWIA

Woods-Saxon MFT wave functions ,

optical Model Potential that describes ^{208}Pb(p,p) at T_p = 161 MeV,

2nd order eikonal Coulomb distortion

non-relativistic σ_{ep}

$P_\alpha(E_m)$ Breit-Wigner shape for energy distributions, two fragments

$\Gamma_\alpha(E_m)$ level width depends on distance to E_F (Brown-Rho)

$$S(E_m,p_m) = \sum_{\alpha<F} n_\alpha(E_\alpha)\,\rho_\alpha^{CDWIA}(p_m)\;P_\alpha(E_m)$$

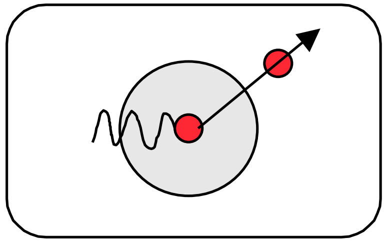
$$P_\alpha(E_m) = \frac{\Gamma_\alpha}{2\pi\left((E_m - E_\alpha)^2 + \Gamma_\alpha^2\right)} \quad \text{with} \quad \Gamma_\alpha(E_m) = \frac{a\,(E_m - E_F)^2}{b^2 + (E_m - E_F)^2}$$

First calculate contributions due to :
1. MEC
2. Rescattering

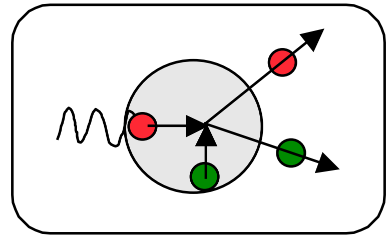
^{208}Pb(e,e'p) Rescattering

Pandharipande -> van Batenburg
Barbieri

plotted $S(E_m,p_m)p_m^2$ □p_m

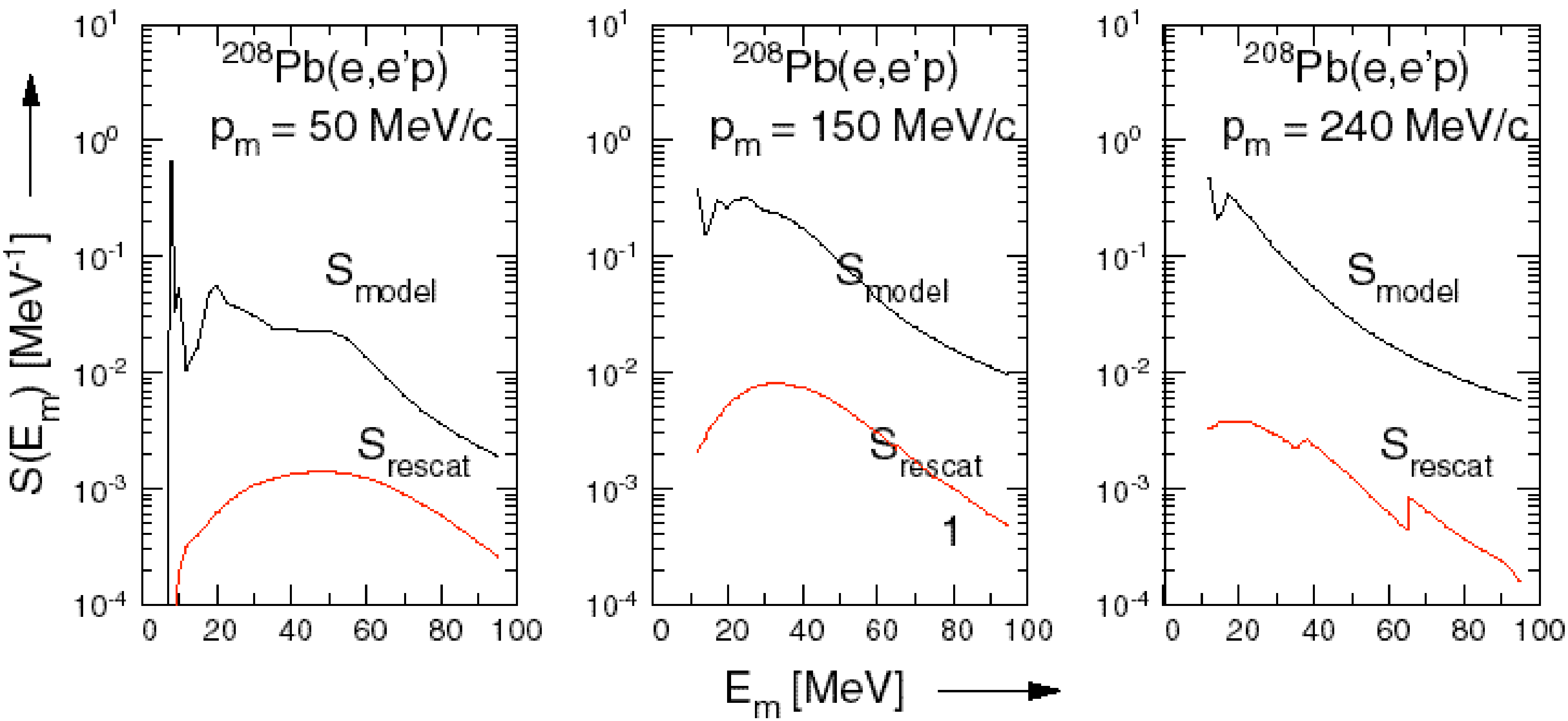


direct process
(e,e'p) -> S(model)



rescattering process
(e,e'N)(N,P) -> S(rescat)

Rescattering below E_m=60 MeV is <3%



^{208}Pb(e,e'p)
MEC contributions

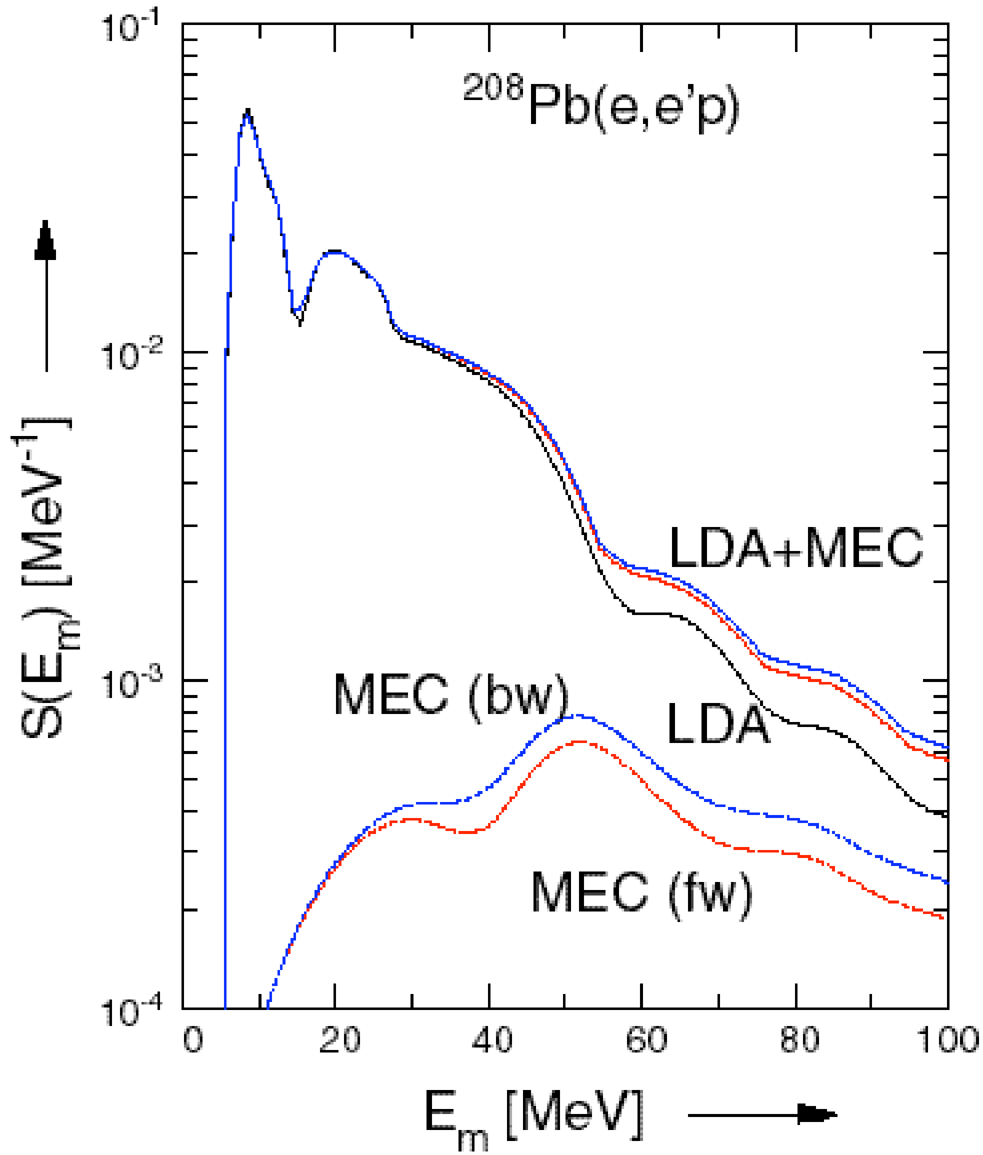
Full calculations of MEC do not exist so use

<u>Quasi Deuteron Model</u>

$$□ = □_L + □_T \{1 + □\ f(q,□)\ \ L\ N^D(^3S_1, E_m)\ /A\}$$

□f(q,□)	**enhancement due to MEC and □ calculated for D(e,e'p)**
L	**Levinger factor**
$N^D(^3S_1, E_m)$	**number of deuteron pairs p{nlj}+n{n'l'j'} in a $^3S_1$ state (E_{nlj} +$E_{n'l'j'}$ >E_m) multiplied by n^2= 0.7^2**

over range 0 < E_m< 60 MeV
effect of MEC <3%



^{208}Pb(e,e'p)
Missing Energy Distributions

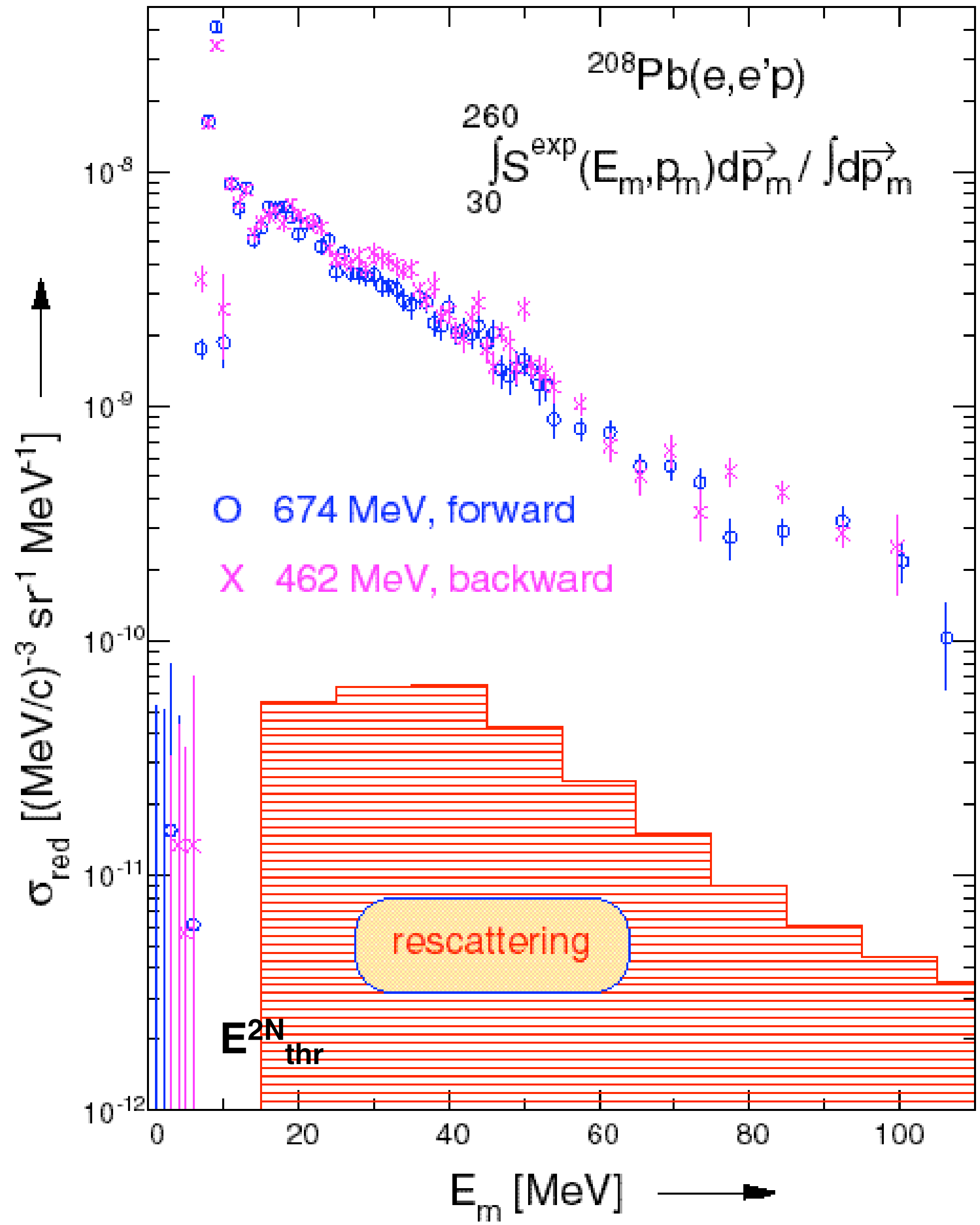
Measured at
high-energy / forward angle
and
low-energy / backward angle

Em = 0 - 100 MeV
Pm = 30 - 260 MeV/c

No large differences between data at
high-energy / forward angle
and
low-energy / backward angle

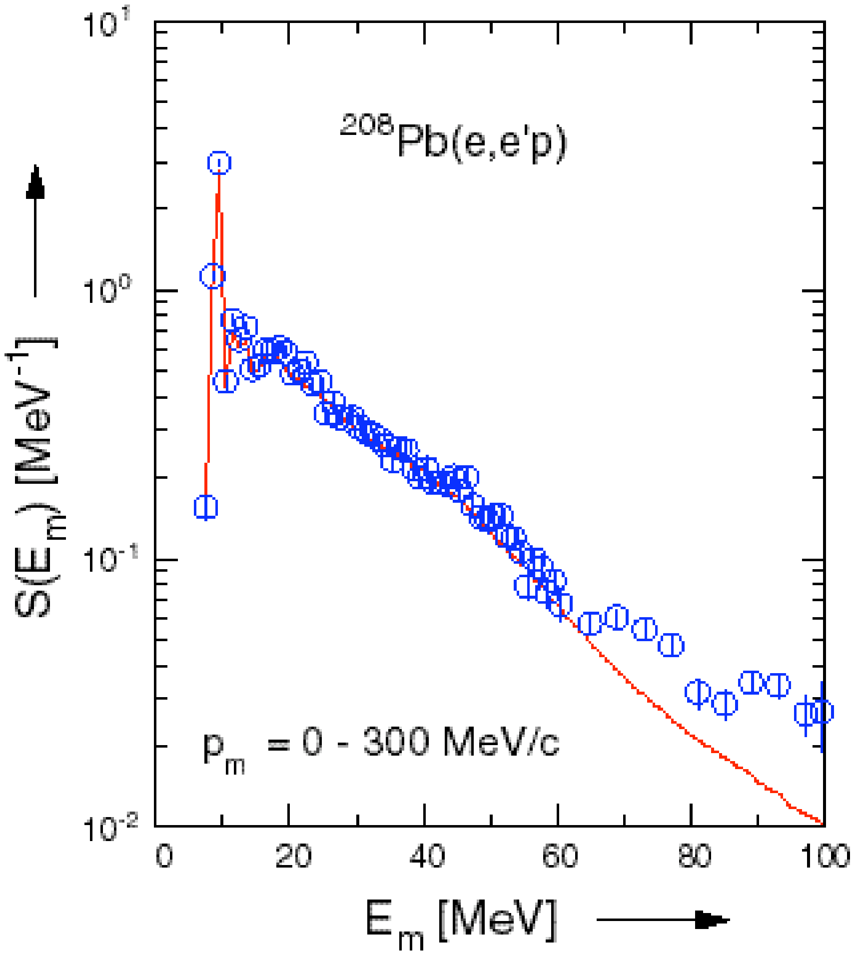
- **MEC indeed small**
- **Calculated rescat. few % of measured data**

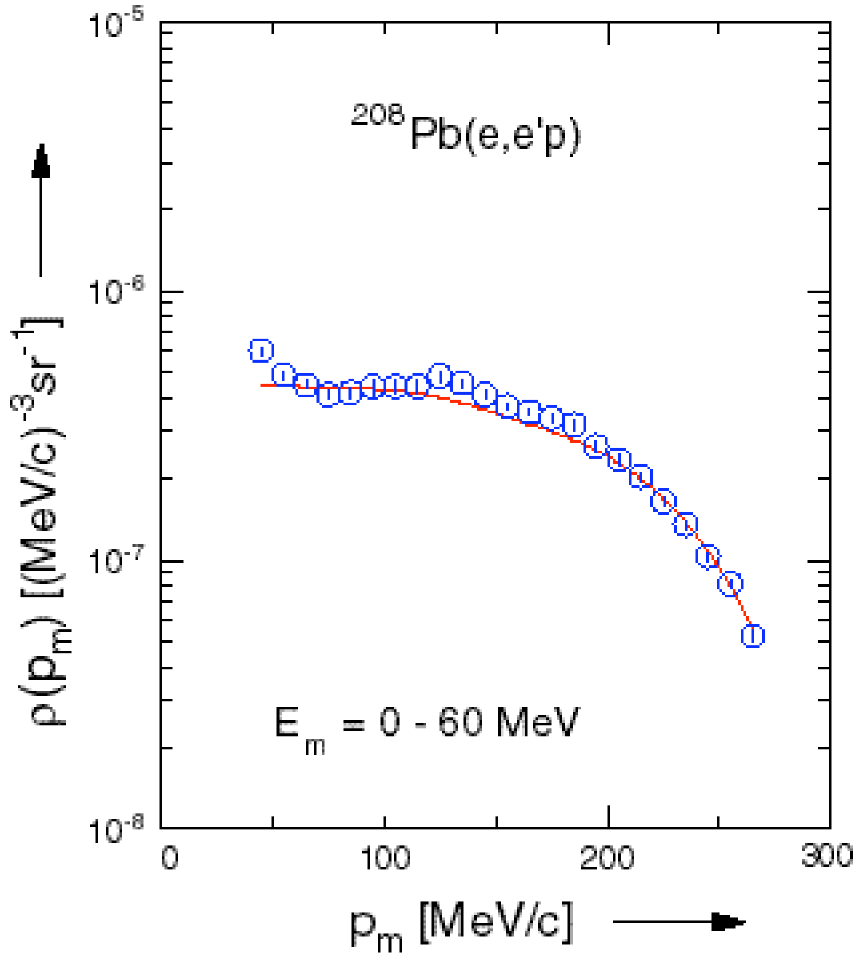
We can safely fit :
model spectral function
to experimental spectral function
=> deduce spectroscopic strength



$^{208}Pb(e,e'p)$ model fits

Comparison of Fitted Model to Spectral Function integrated over
Momentum (left) **and** **Energy (right)**





^{208}Pb(e,e'p) Spectroscopic Strength

From model fits :

- strengths of all orbits
- for deep lying orbits spin-orbit partners taken together
- combined information from
  - this experiment (AmPS -1997)
  - earlier one (MEA -1988)

Total depletion of Fermi sea ($E_m < 60$ MeV):

$\Sigma S_{\ell} / 82 = \underline{0.77} \pm 0.01 \pm 0.05 \pm 0.02$

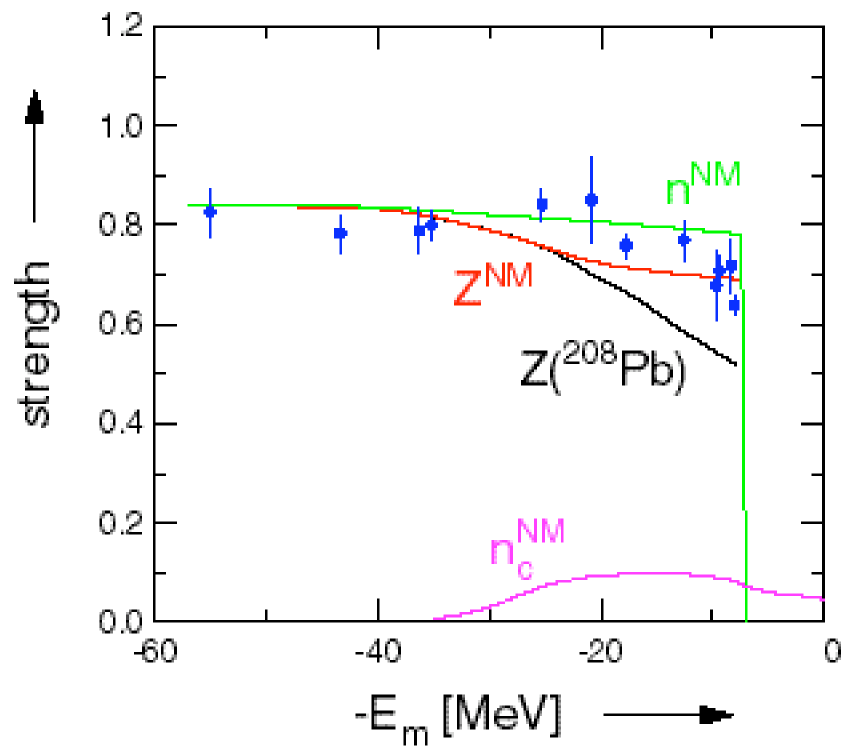
stat syst model

Average deep hole strength ($30 < E_m < 60$ MeV)

$Z_{deep} = \underline{0.80} \pm 0.02 \pm 0.05 \pm 0.03$

stat syst model

NM : $n_h(p=0) = \underline{0.80 - 0.85}$



Towards higher Q^2
$^{12}C(e,e'p)$

$^{12}C(e,e'p)$ comparison of low and high Q^2 data

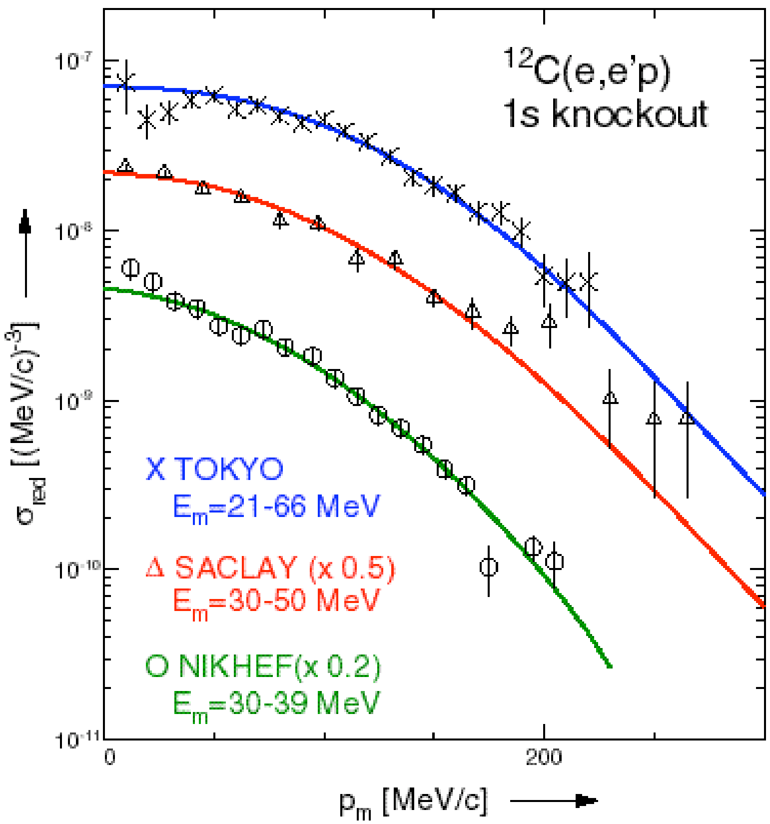
1) determine accurate wave functions for the 1p and 1s strength :
   Consistent reanalysis of world's $^{12}C(e,e'p)$ 1p + 1s data
2) establish 1p and 1s spectroscopic factors
3) compare with SLAC and TJNAF data at high Q^2

Analysis ingredients (CDWIA)

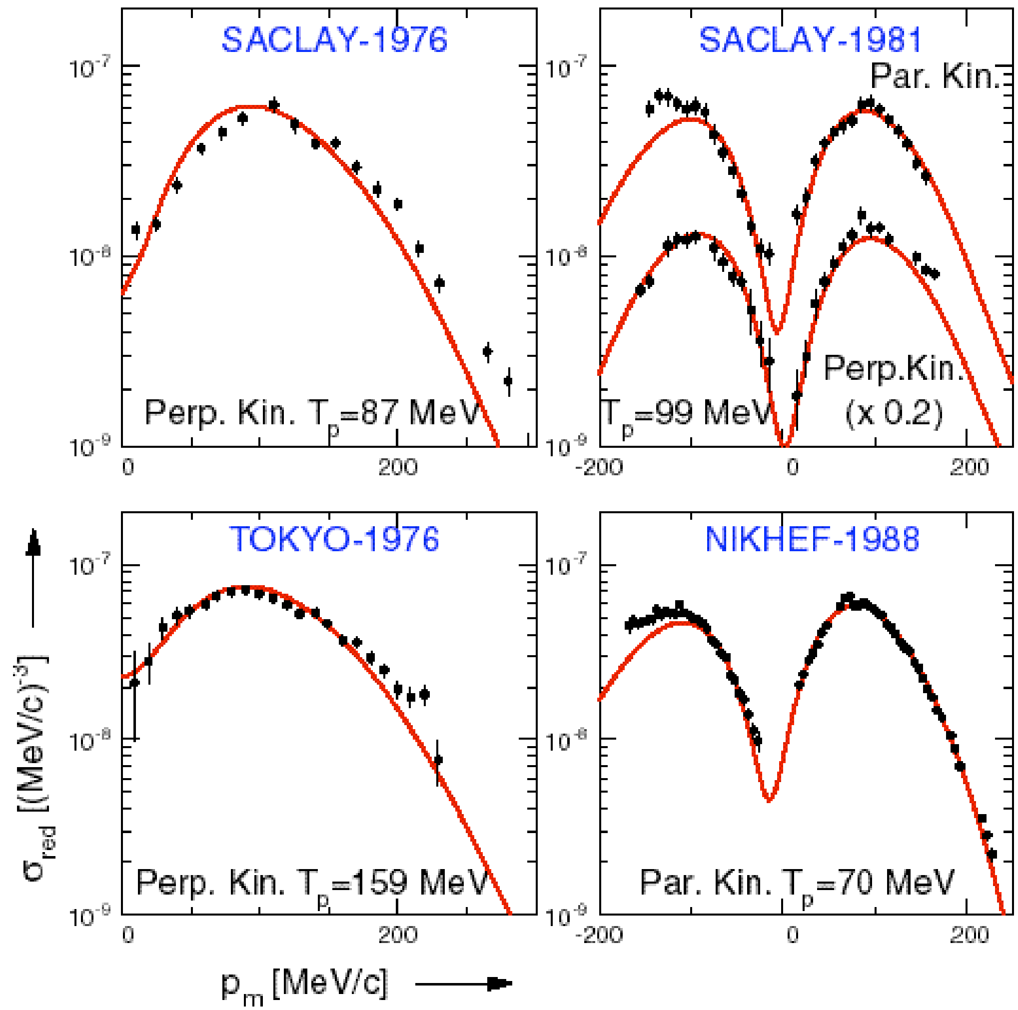
- Impulse Approximation
- Coulomb distortion
- Final State Interaction
  Comfort-Karp potential, modified for channel couplings
- Woods-Saxon bound state wave functions
  adapted to describe momentum distributions
- non-relativistic σ_{ep} McVoy-Van Hove

$^{12}C(e,e'p)$ at low Q^2
Fits to 1p and 1s world data

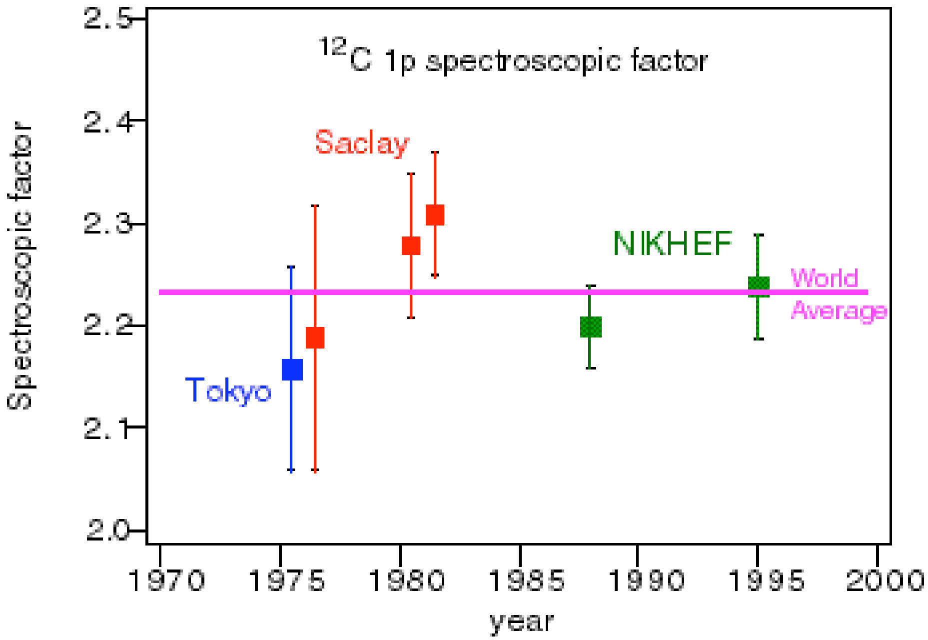
1s momentum distributions

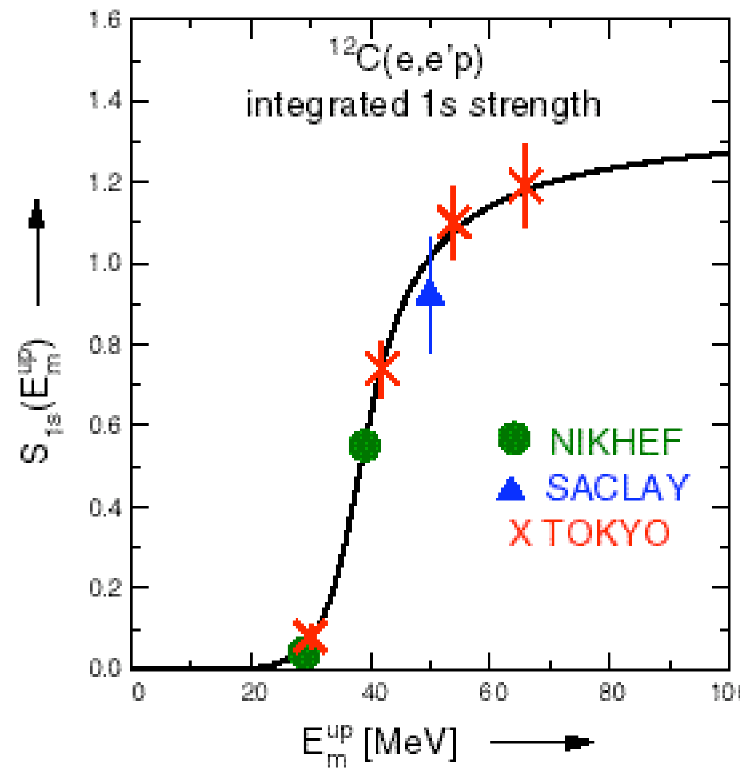


1p momentum distributions



^{12}C(e,e'p)
1p and 1s spectroscopic factors





$$S_{1s}(E_m^{up}) = n_{1s} \int_{E_F}^{E_m^{up}} dE_m \frac{\Gamma(E_m)/2\pi}{(E_m - E_{1s})^2 + \frac{1}{4}\Gamma^2(E_m)}$$

Summed Spectroscopic Strength (up to E_m=80 MeV)

$S_{1p} + S_{1s} = 2.23 + 1.25 = 3.48 \pm 0.10$

We see only 58% of the protons in ^{12}C

$^{12}C(e,e'p)$ SLAC data compared to Glauber calculations

SLAC NE18 data at Q^2= 1.1 (GeV/c)^2

Red Curves : with S_{1p} = 4 , S_{1s} = 2 (full shells)

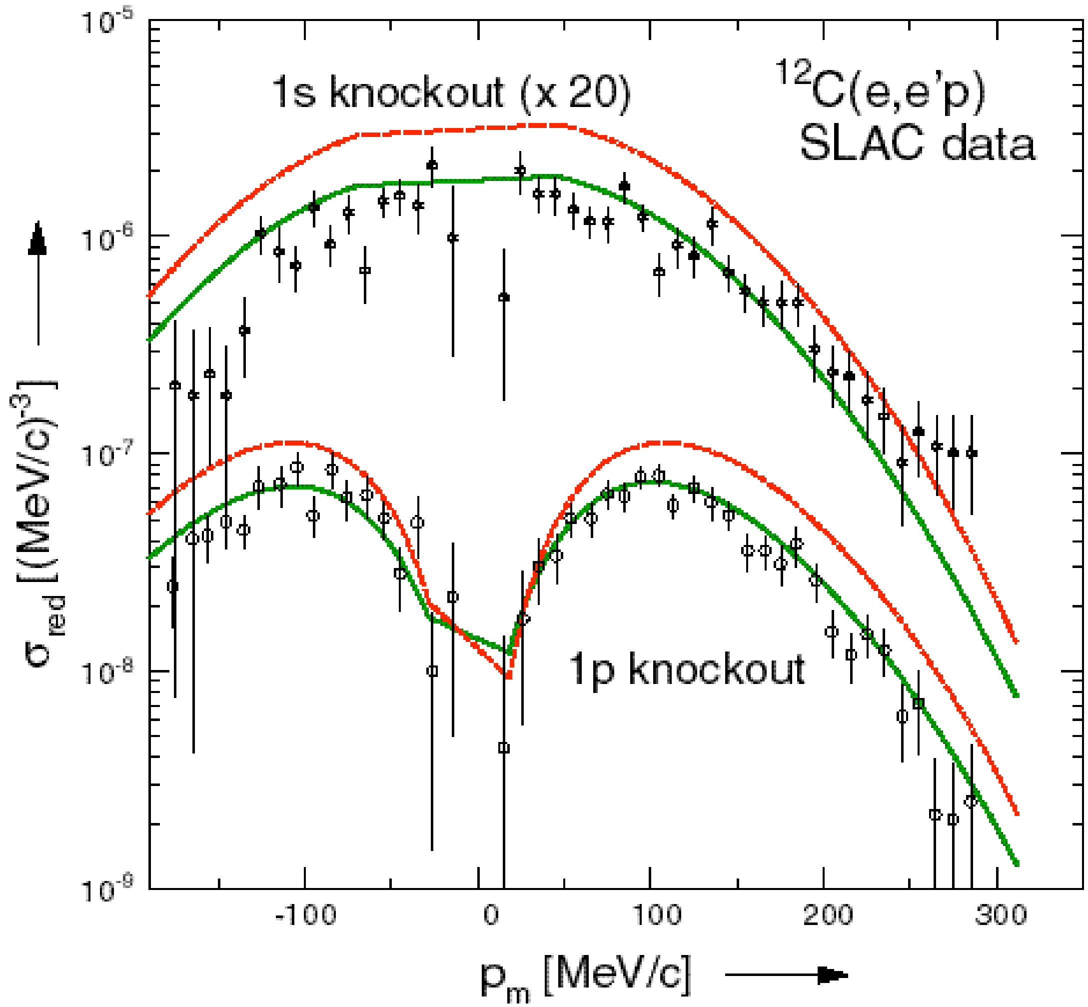
Green Curves : add T(ransparency)

to account for FSI (Frankfurt, Strikman, Zhalov)

__ Glauber T_{1p} =0.6-0.7, T_{1s} = 0.5-0.6 (p_m dependent!)

Fit Glauber curves to data : ➔

S_{1p} = 3.56±0.12 , S_{1s} = 1.50±0.08

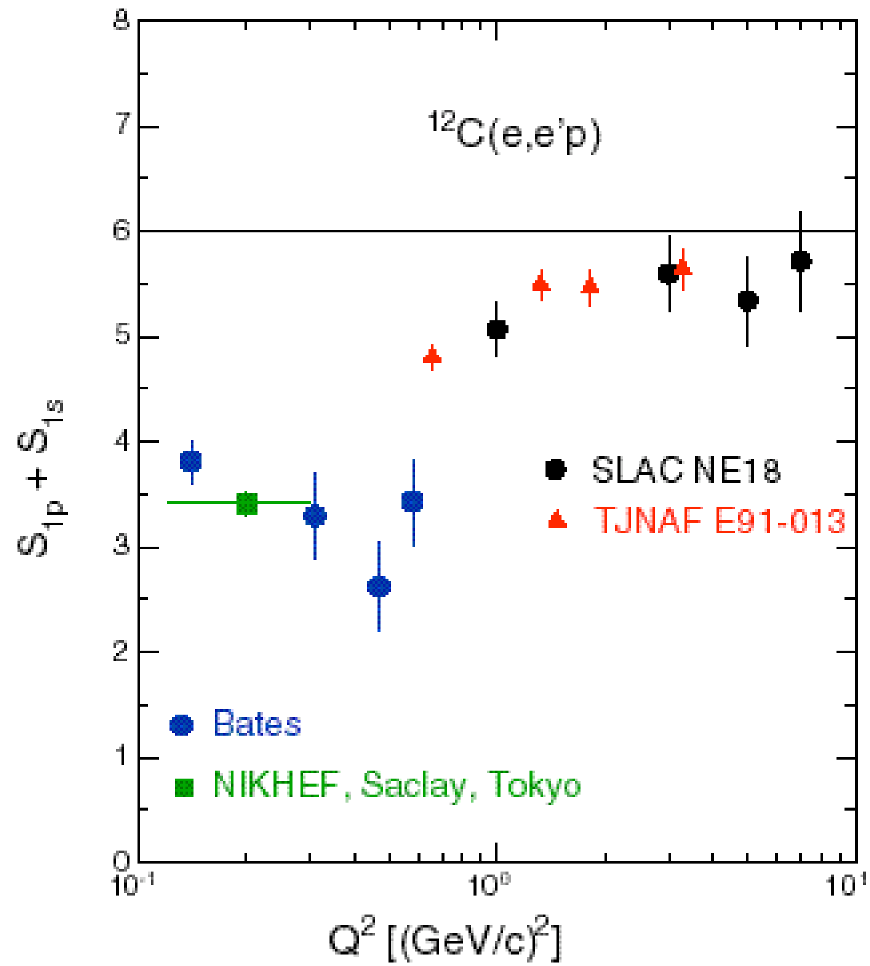


Summed Spectroscopic Strength is Q^2 dependent !

At Q^2= 1.1 (GeV/c)^2 $S_{1p} + S_{1s}$ = 5.06 ± 0.15 (84%)

At Q^2= 0.2 (GeV/c)^2 $S_{1p} + S_{1s}$ = 3.48 ± 0.10 (58%)

$^{12}C(e,e'p)$ Q^2 dependence of Spectroscopic Strength



Are spectroscopic factors Q^2 dependent??

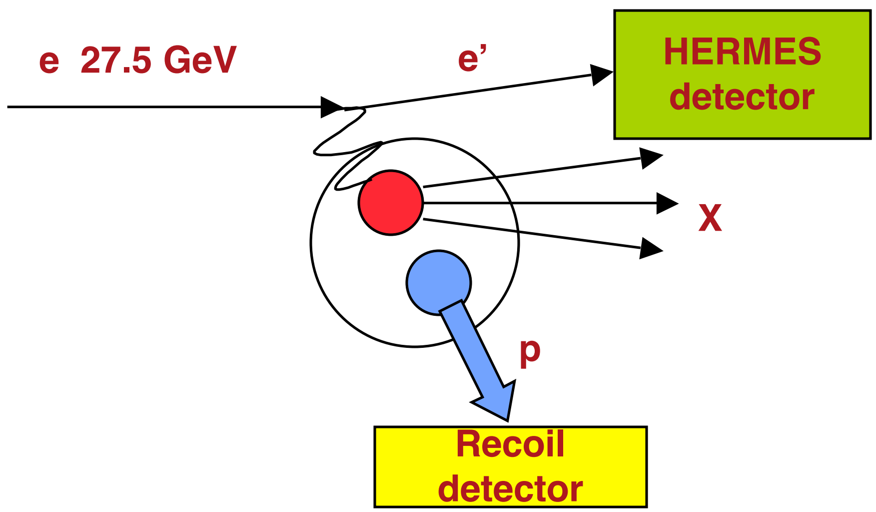
Differences between the analyses :

- **low Q^2 data : FSI -> opt. model, non-relativistic**
- **high Q^2 data : FSI -> Glauber, relativistic**

Possible Explanations :

- **Breakdown of quasi-particle concept at high Q^2**
- **Reaction mechanism effects (2BC ?)**
- **Modified nucleon form factors**
- **Relativistic effects**

$^{2}H(e,e'p)$
HERMES experiment (DESY)
$^{2}H(e,e'n\text{->}X)p$ with detection of recoil proton



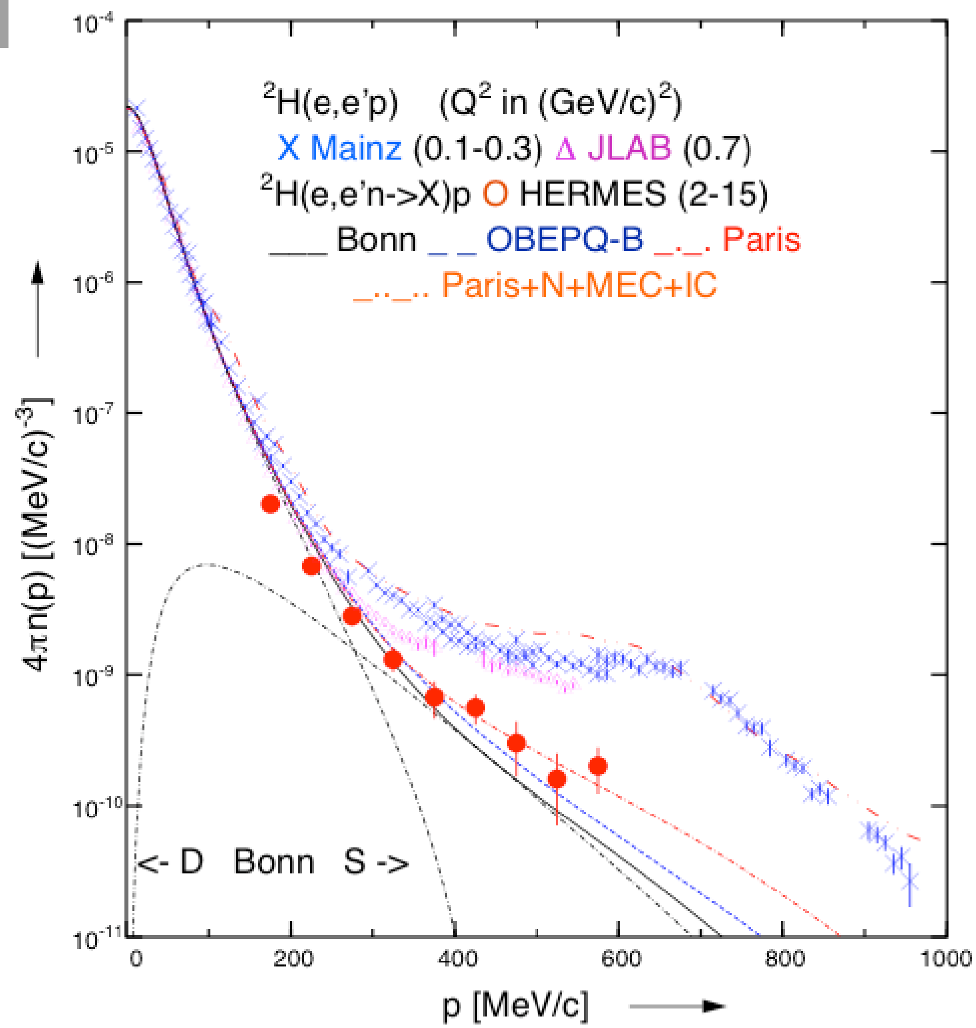
$\sigma_{exp} = K\, F_{2}{}^{n}{}_{tag}\,(x,Q^{2})\; n(p)$

assume $F_{2}{}^{n}{}_{tag}\,(x,Q^{2}) = F_{2}{}^{n}\,(x,Q^{2})$

➔red datapoints (HERMES)

Compare to world $^{2}H(e,e'p)$ data

& models with various NN-interactions



Summary

RESULTS

Exp. Spectroscopic strength at low E_m ~ 60% of IPSM

Exp. Spectroscopic strength for deeplying orbits ~ 80% of IPSM

Wave functions with correlations explain this (VMC, NM)

High-momentum components seen (not very accurate, interpretation?)

For a further interpretation these subjects would be nice to have

(some speakers will show first results!)

EXPERIMENT

Measure at large E_m (> 100 MeV)

➔ (correlated tail)

Measure at large p_m (> k_F)

➔ (modifications w.r.t. MF)

Measure at higher Q^2 for several A

➔ (separate orbits)

THEORY

Extend VMC technique to heavier nuclei

➔ (^{12}C, ^{16}O)

Relativistic description of (e,e'p) [low Q^2 - high Q^2]

➔ (opt. model vs. Glauber)

Decent estimate of 2-body currents and rescattering

➔ (q,□ , kinematics dependence)