Studies of nuclear halo formation with (e,e‘,p), (e,e‘,d) reactions

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Motivation

- (e,e',p), (e,e',d) experiments may be used to study the halo formation in exotic nuclei
- Necessity to develop proposals for new experiments
- Key factor: the interplay between nuclear structure and scattering experiments
- New ways towards modern physics
- Outlook

Topics

- Review of the theory for scattered electron in continuum
- Proton and deuteron knock-out reaction
- Microscopic correlation in the dynamic of even nuclei
- Example: ^{6}Li
- Momentum distribution in exotic nuclei
- Effect of the halo in the scattering cross-section
- Correlated three-particle system

Cross-section for electron scattering in continuum

$$\frac{d^2\sigma}{d\Omega dE_2} = \frac{\sigma_{Mott}}{M}\left(W_2 + 2W_1 \tan^2\frac{\theta}{2}\right)$$

where

$$W_1 = 2\pi \ M F_T^2$$

$$W_2 = 4\pi \ M\left(\frac{\Delta^2}{q^2}\right)\left[\left(\frac{\Delta^2}{q^2}\right)F_C^2 + \frac{1}{2}F_T^2\right]$$

with

$$\sigma_{Mott} = \frac{\alpha^2}{4k_1^2}\cdot\frac{\cos^2\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\cdot\frac{1}{M}$$

$$\vec{q} = \vec{k}_1 - \vec{k}_2$$

$$\omega = k_1^0 - k_2^0 \equiv E_1 - E_2$$

$$\Delta^2 = -\omega^2 + q^2$$

$\vec{q}$ momentum transfer

ω energy transfer

M mass of the target nucleus

F_T transversal form factor

F_C longitudinal form factor

Inverse relations and dimensionless response functions

Inverse relations

$$F_T^2 = \frac{2W_1}{4\pi M}$$

$$F_C^2 = \frac{1}{4\pi M}\left(\frac{\vec{q}^{\,2}}{\Delta^2}\right)\left[\left(\frac{\vec{q}^{\,2}}{\Delta^2}\right)W_2 - W_1\right]$$

The dimensionless functions are defined by:

$$S_T = MF_T^2 \qquad S_T \equiv S_{Transversal}$$

$$S_L = MF_C^2 \qquad S_L \equiv S_{Longitudinal}$$

Scattering cross-section and nuclear model

$$\frac{d^2\sigma}{d\Omega_2 dE_2} = \sigma_M K\left(A + B\tan^2\frac{\theta}{2}\right)$$

with: $K \equiv \left|f(\Delta^2)\right|^2\left(\frac{V}{4\pi^3}\right)$

$$\left(\frac{V}{4\pi^3}\right)n(\vec{p}) \xrightarrow{\text{to be replaced by}} \left|\Psi_{DCM}(\vec{p})\right|^2$$

Taken from the Fermi gas model

Totally correlated n-particle momentum distribution

$$F_T^2 = \frac{K}{4\pi M}B$$

$$F_C^2 = \frac{1}{4\pi}\left(\frac{\vec{q}^{\,2}}{\Delta^2}\right)^2\left[A - \frac{1}{2}B\right]$$

Calculation of A and B

For the computation of A and B we use Ueberall:

„Electron scattering from complex nuclei"

$$\frac{1}{2}A(q,\omega) = I_0 - \frac{\omega}{m}\left(I_0 + \frac{2I_1}{q}\right) + \frac{1}{m^2}\left[\left(\frac{G}{2} - I_2\right)\left(1 - \frac{\omega^2}{q^2}\right) + I_2\right] - \frac{\omega^2}{m^2}\left(2.63 I_0 - \frac{I_1}{q}\right) + 1.17 I_0 \frac{q^2}{m^2}$$

$$\frac{1}{2}B(q,\omega) = \frac{1}{m^2}\left[\left(G + 5.77 q^2 I_0\right) - I_2\right]$$

To calculate the integrals I_0, I_1, I_2 ... we are not using the momentum distribution of the Fermi model but the distribution calculated in the BDCM model.

Momentum integrals

$$I_l = \int d^3p \left|\Psi_{DCM}(p)\right|^2 \left[1 - \left|\Psi_{DCM}\left(\left|\vec{p}+\vec{q}\right|\right)\right|^2\right] (p\cos\phi)^l \, \delta\left(\omega - \varepsilon\left(\left|\vec{p}+\vec{q}\right|\right) + \varepsilon(p)\right)$$

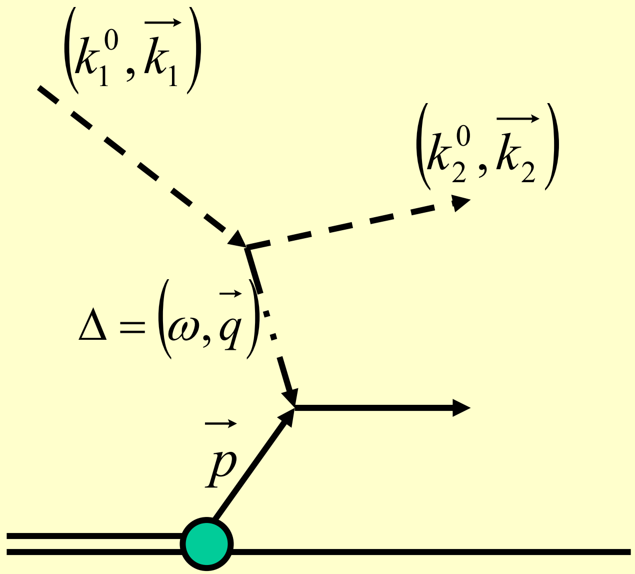
$$G = \int d^3p \left|\Psi_{DCM}(p)\right|^2 \left[1 - \left|\Psi_{DCM}\left(\left|\vec{p}+\vec{q}\right|\right)\right|^2\right] p^2 \delta\left(\omega - \varepsilon\left(\left|\vec{p}+\vec{q}\right|\right) + \varepsilon(p)\right)$$

$$\phi = angle(\vec{p}, \vec{q})$$

$\varepsilon(p)$ the energy of two particle excitation as a function of ist momentum

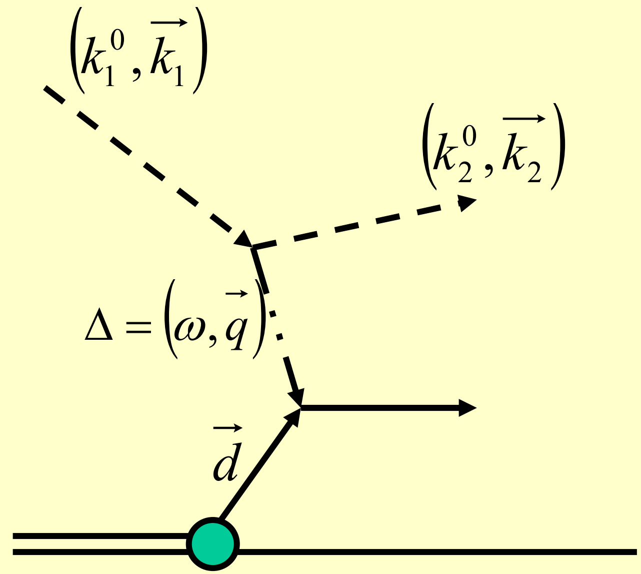
An effective mass m* for the bound nucleons has been introduced.

Electron scattering on ^{6}Li



^{6}Li

(ee‘p)

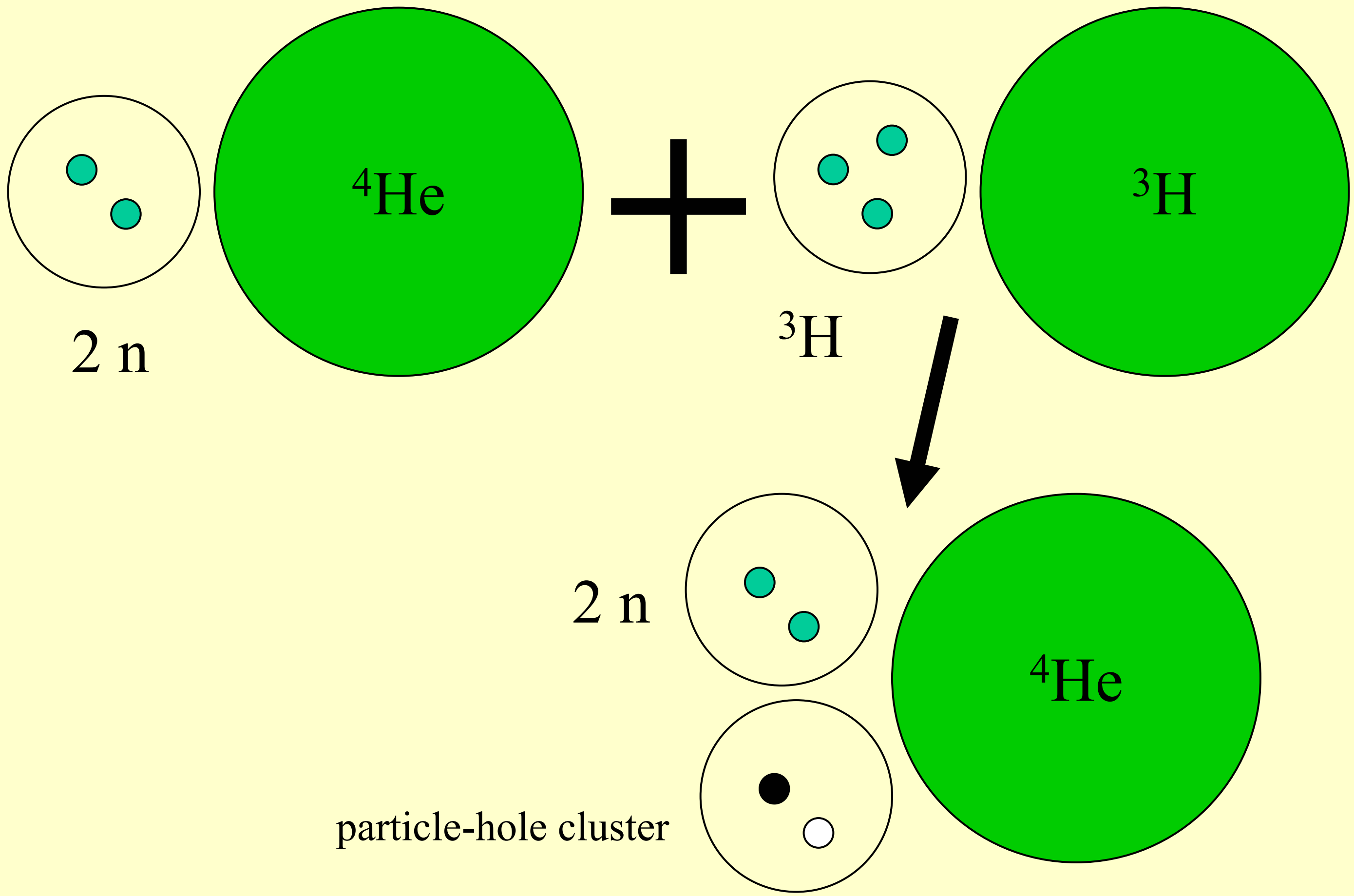


^{6}Li

(ee‘d)

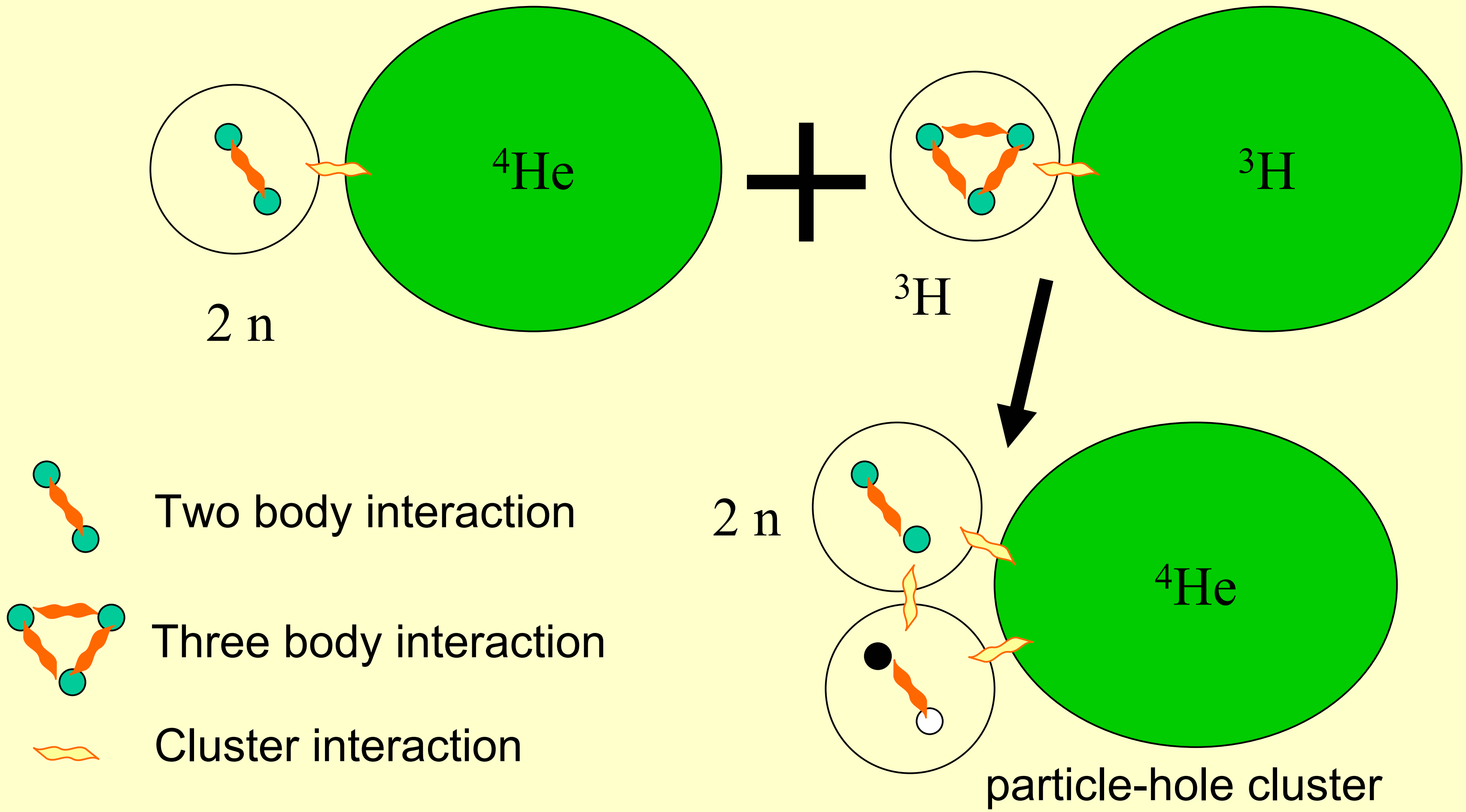
Shape coexistence in the vacuum of BDCM: Example ^{6}He

In the ground state of ^{6}He two different clusters coexist:



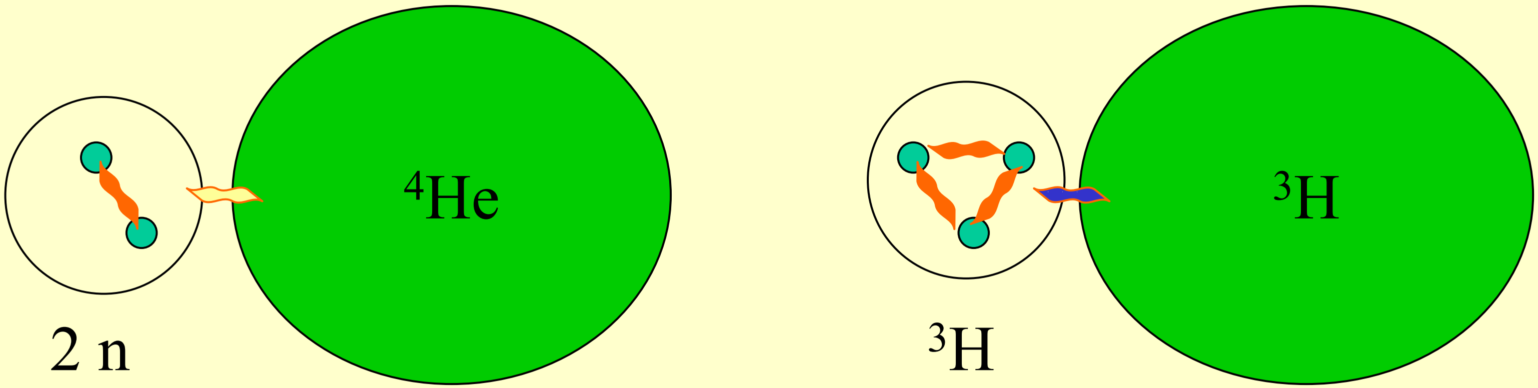
Interaction between clusters: deformation of the vacuum

In the ground state of ^{6}He two different clusters coexist:



From interacting clusters to DCM

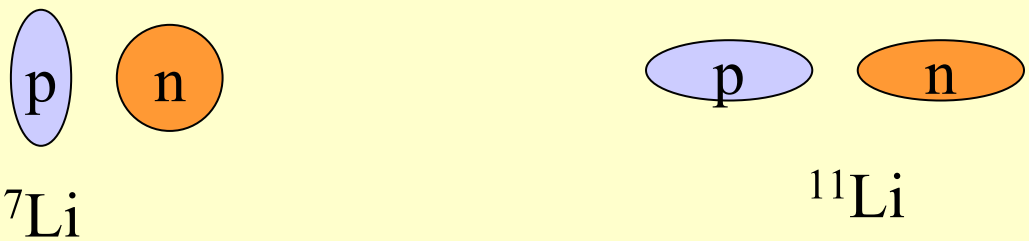
- <u>Macroscopic cluster interaction</u> → ^{2}H cluster move in the optical potential formed by the alpha particle optical model.
- <u>Microscopic cluster interaction</u> → N-particles which were partitioned in small and big clusters interact.



Can the dynamic correlation model reproduce the interacting cluster picture?

Prolate or oblate proton distributions?

- In AMD (Antisymetrized Molecular Dynamics) protons and neutrons are treated as separated clusters. By increasing the neutron number proton distribution can vary between two shapes.



Consequence: strong variation of proton charge radius as the neutron number increases.
Reason: the neutron average potential (mean field) is strongly deformed.

Introducing correlation in nuclei via the Unitarity Model Operator (UMO)

$$e^{iS}\prod_{i=1}^{N} a_i^{+}\left|0\right\rangle = (1 + S_1 + S_2 + \ldots)\prod_{i=1}^{N} a_i^{+}\left|0\right\rangle$$

$$S_1 \propto a_1^{+} a_2$$

$$S_2 \propto a_1^{+} a_2 a_3^{+} a_4$$

.....

Disadvantage:

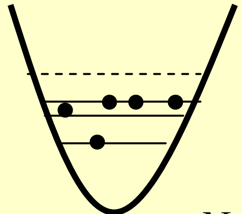
1. Perturbative
2. No Pauli Principle
3. Works with effective operator

$$H_{eff} = e^{-iS^{+}} H e^{iS}$$

$$O_{eff} = e^{-iS^{+}} O e^{iS}$$

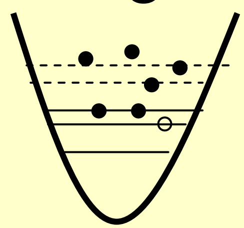
Nuclear model based on Dynamic Correlation Model (DCM)

$$e^{iS}\left|\Psi_N\right\rangle \propto e^{iS}\prod_{i=1}^{N} a_i^+\left|0\right\rangle$$



$$=\alpha\left|\Psi_N\right\rangle+\beta\left|\Psi_{N+1,1}\right\rangle+\gamma\left|\Psi_{N+2,2}\right\rangle+\ldots=\alpha\prod_{i=1}^{N}a_i^+\left|0\right\rangle+\beta\prod_{i=1}^{N+1}a_i^+a_1\left|0\right\rangle+\gamma\prod_{i=1}^{N+2}\prod_{i'=1}^{2}a_i^+a_{i'}\left|0\right\rangle\ldots$$

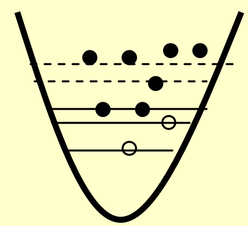
$\alpha,\beta,\gamma,\ldots$ are calculated from the following comutator chain:



$$[H,\prod_{i=1}^{N}a_i^+]\left|0\right\rangle\propto\prod_{i=1}^{N}a_i^+\left|0\right\rangle+\prod_{i=1}^{N+1}a_i^+a_1\left|0\right\rangle$$

$$[H,\prod_{i=1}^{N+1}a_i^+a_1]\left|0\right\rangle\propto\prod_{i=1}^{N+1}a_i^+a_1\left|0\right\rangle+\prod_{i=1}^{N+2}\prod_{i'=1}^{2}a_i^+a_{i'}\left|0\right\rangle$$

……



Boson Dynamic Correlation Model (BDCM)

$$[H,\prod_{i=1}^{N=2} a_i^+]|0\rangle \propto \prod_{i=1}^{N=2} a_i^+|0\rangle + \prod_{i=1}^{N=3} a_i^+ a_1|0\rangle$$

$$[H,\prod_{i=1}^{N=3} a_i^+ a_1]|0\rangle \propto \prod_{i=1}^{N=3} a_i^+ a_1|0\rangle + \prod_{i=1}^{N=4}\prod_{i'=1}^{N=2} a_i^+ a_{i'}|0\rangle$$

……

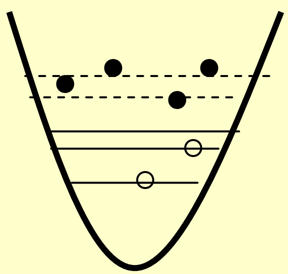
Advantage:

1. The commutator chain reduced to an eigenvalue problem by introducing dynamic linearisation aproximation
2. Pauli Principle
3. Microscopic calculation without effective operators

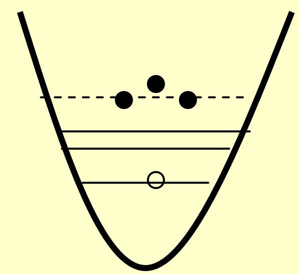
Example for linearisation approximation

- In this work we are mainly concerned with calculating admixture coefficients for the ground state wavefunction.
- That means that vacuum boiling configuration of higher complexity are poorly admixed.

Under these considerations we introduce the following aproximations:



$$\prod_{i=1}^{N=4}\prod_{i'=1}^{N'=2} a_i^{+} a_{i'}|0\rangle \propto \sum_{l}^{N'}\prod_{j=1}^{N=3} a_{lj}^{+} a_1 |0\rangle$$



With the linearisation the 4 particle – 2 holes configuration are approximated with effective 3 particle – 1 hole configurations

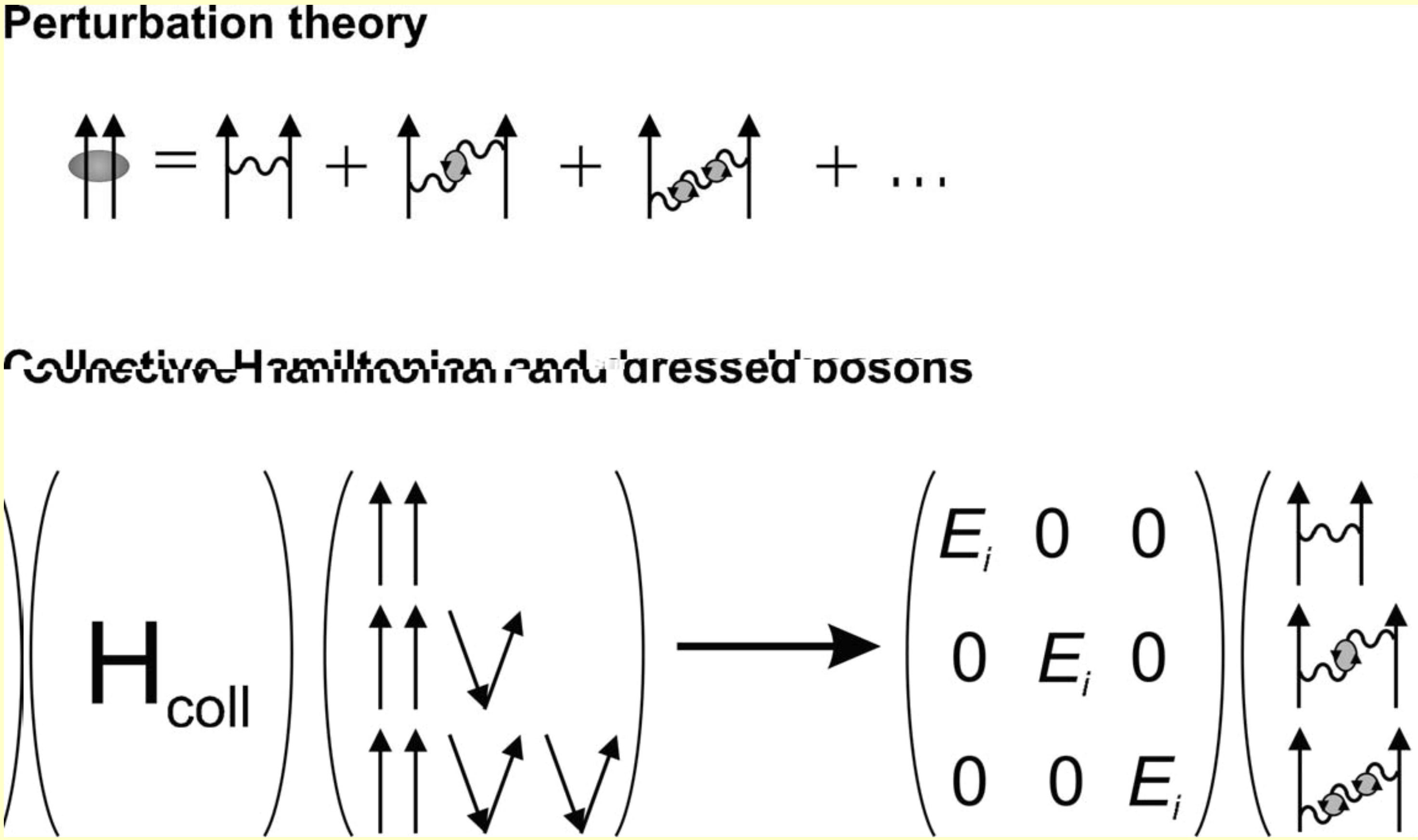
Effect of linearisation on commutator chain

- Use the linearisation approximation defined in the previous transparency
- Collect the resulting terms

$$\begin{vmatrix} E - E_{2p} + \langle 2p \| V \| 2p \rangle & \langle 2p \| V \| 3p1h \rangle \\ \langle 3p1h \| V \| 2p \rangle & E - E_{3p1h} + \langle 3p1h \| V \| 3p1h \rangle \end{vmatrix} \begin{vmatrix} \chi_{2p} \\ \chi_{3p1h} \end{vmatrix} = 0$$

Dynamic eigenvalue equations for mixed mode amplitudes
2 particles => 3 particles – 1 hole

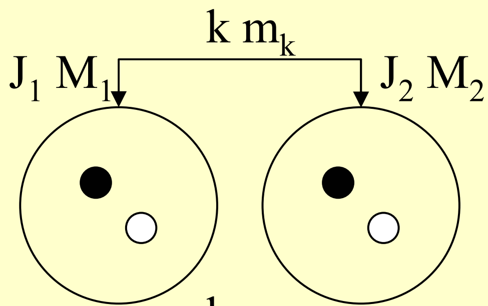
Dynamics eigenvalue equation for one dressed boson which is solvable self-consistently



Simmetry properties of the configuration mixing wavefunction (CMWF)

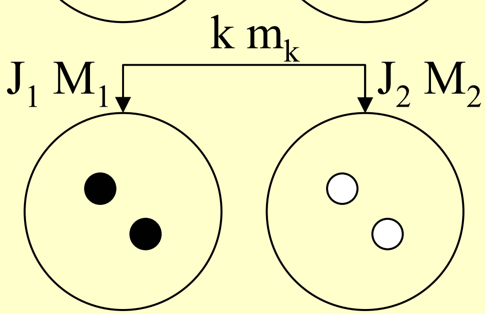
Define operators:

$u^k_{mk}=$



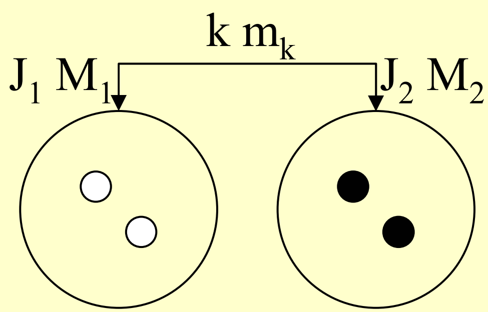
Destroy a hole-particle pair
and create a particle-hole pair

$p^k_{mk}=$



Destroy a hole-hole pair
and create a particle-particle pair

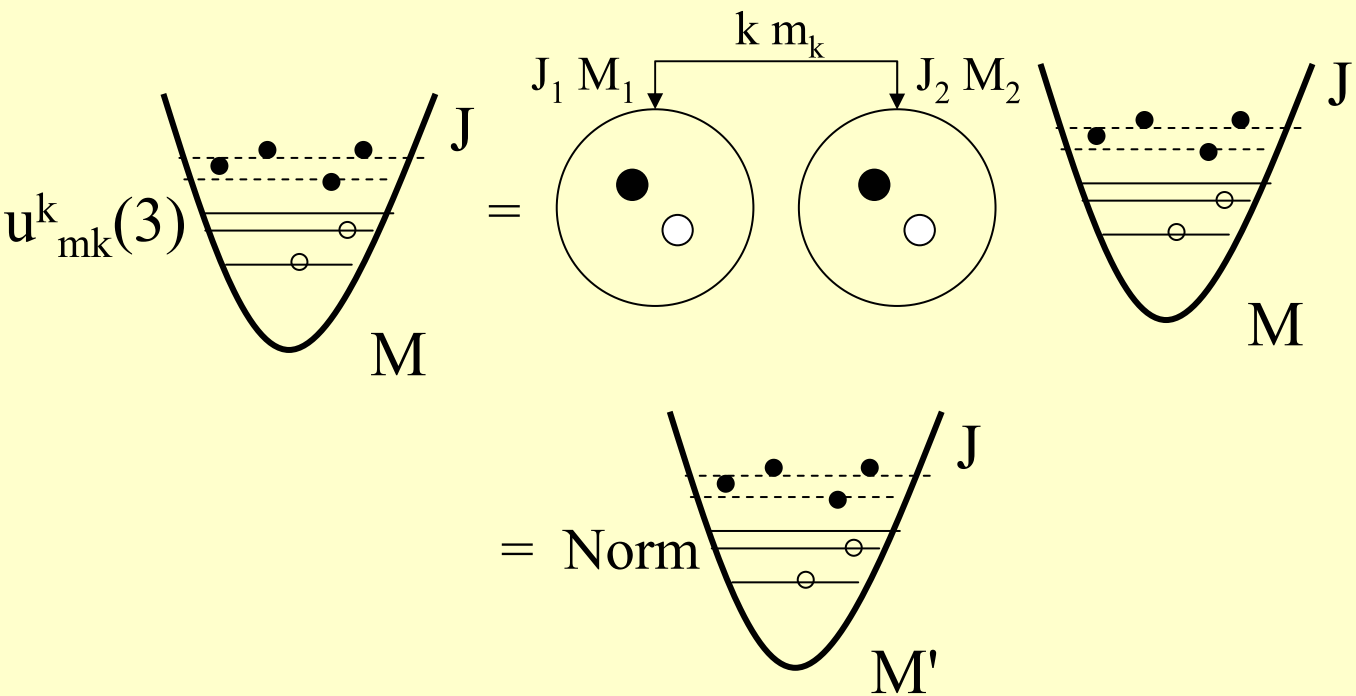
$h^k_{mk}=$



Destroy a particle-particle pair
and create a hole-hole pair

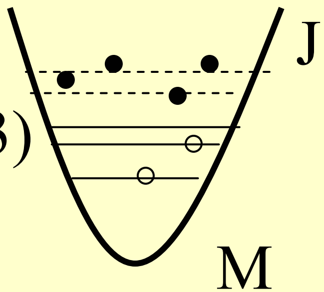
Group property of the defined operators

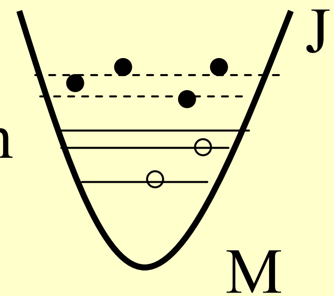
Action of the operator on totally antisymetric wavefunction

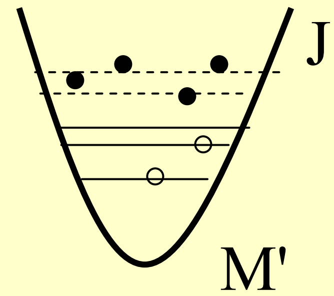


The same action is valid for p^k_{mk} and h^k_{mk}

Unitary operator

$\underline{u}^{k}{}_{mk}(3)$ 

$=$ $u^{k}{}_{mk}(3)/\mathrm{Norm}$ 

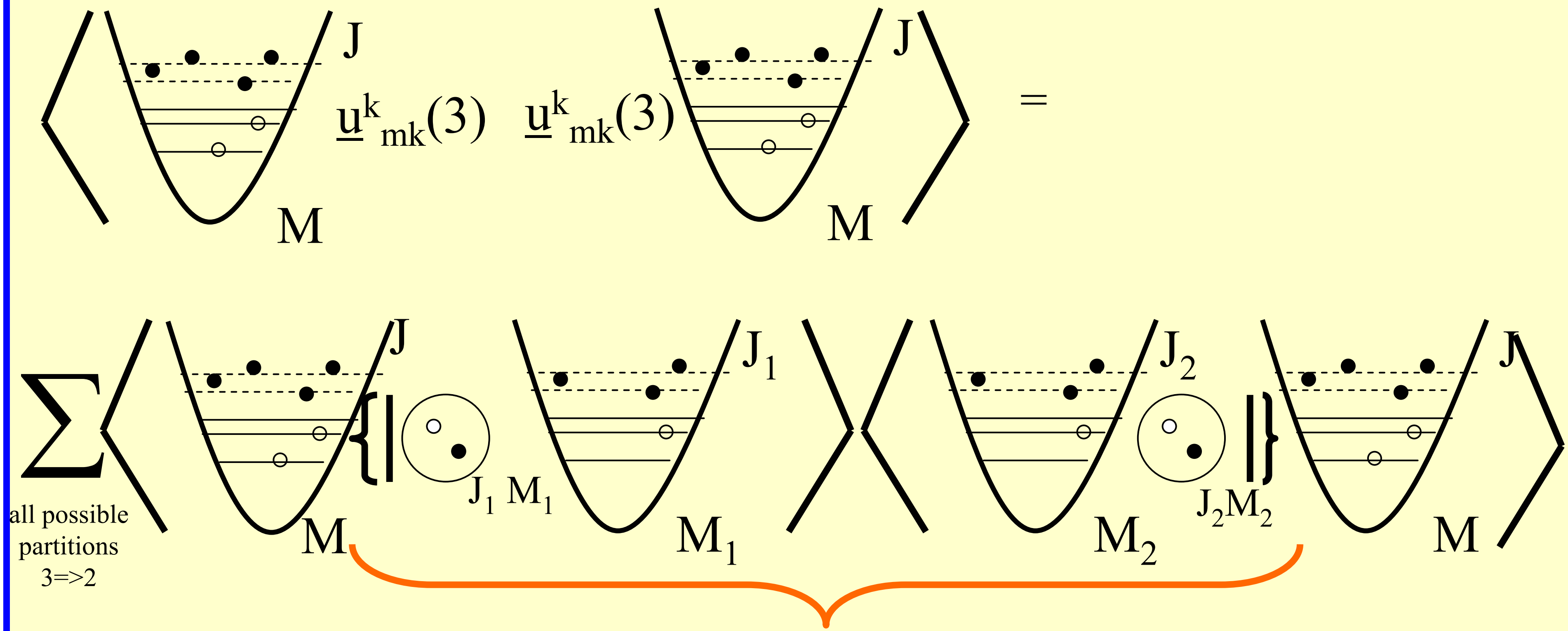
$=$ 

Commutator relation :

$$\underline{u}^{k1}{}_{mk1}(3)\ \underline{u}^{k2}{}_{mk2}(3) = \text{geometric factor} * \underline{u}^{k3}{}_{mk3}(3)$$

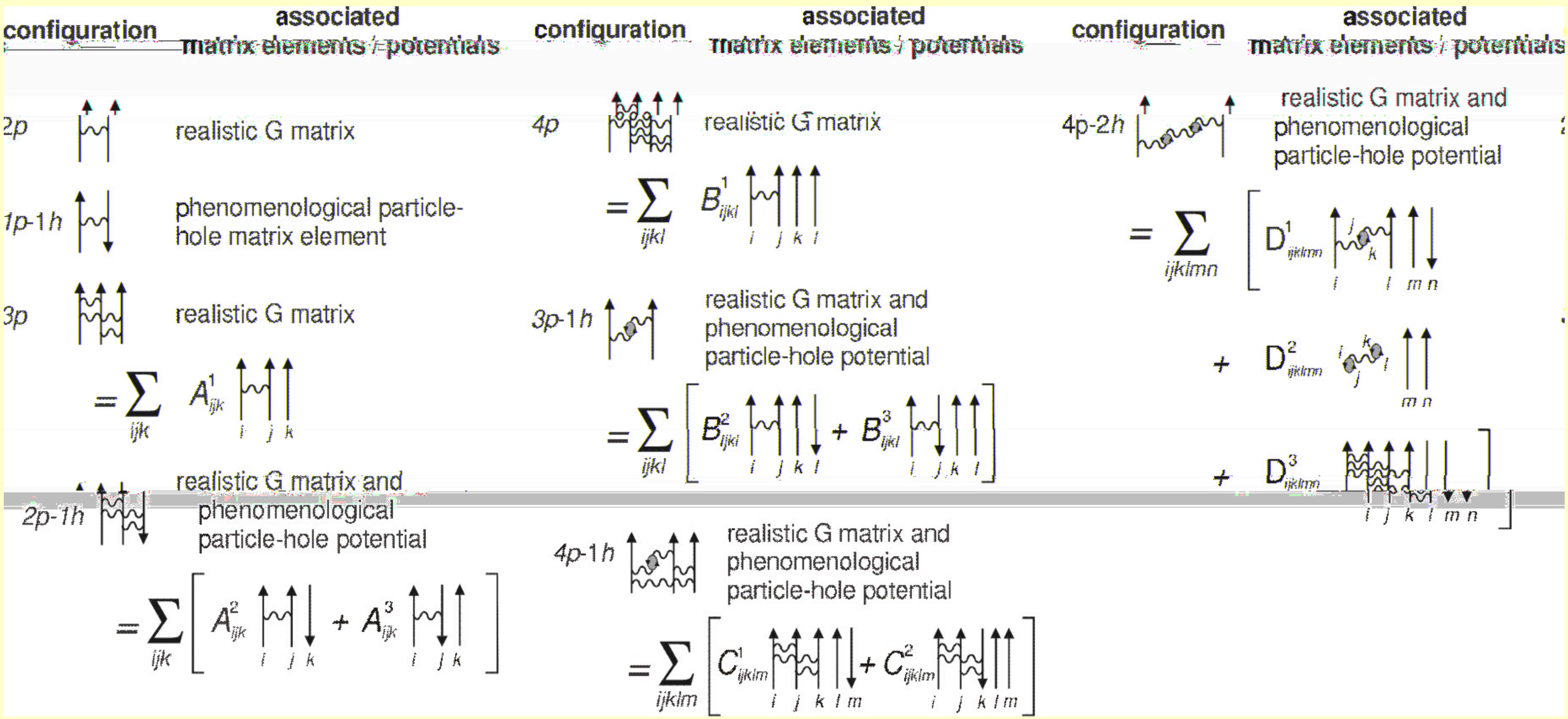
Therefore the unitary operators are generators of the $SU_{2J+1}(3)$

Cluster transformation coefficients



Cluster Transformation Coefficients

Factorisation of the model CMWFs (for electrons and nucleons) in terms of cluster coefficients



Distribution in BDCM

$$\rho_{CM}(r) = \frac{1}{A}\langle\psi^{(2n-or-pn)}_{dressed}| \sum_{i=1,6} \delta(\vec{r}-\vec{r_i})|\psi^{(2n-or-pn)}_{dressed}\rangle$$

$$= \frac{1}{A}\langle\psi(\vec{r_1},\vec{r_2})| \sum_{i=1,6} \delta(\vec{r}-\vec{r_i})|\psi(\vec{r_3},\vec{r_4})\rangle$$

$$+C^1_{cluster}\langle\psi^a(\vec{r_i},\vec{r_j})\psi^p(\vec{r_k},\vec{r_l})| \sum_{i=1,6} \delta(\vec{r}-\vec{r_i})|\psi(\vec{r_3},\vec{r_4})\rangle \qquad (1)$$

$$+C^2_{cluster}\langle\psi(\vec{r_1},\vec{r_2}))| \sum_{i=1,6} \delta(\vec{r}-\vec{r_i})|\psi^a(\vec{r_k},\vec{r_l})\psi^p(\vec{r_k},\vec{r_l})\rangle$$

$$+C^1_{cluster} * C^2_{cluster}\langle\psi^a(\vec{r_i},\vec{r_j})\psi^p(\vec{r_k},\vec{r_l})| \sum_{i=1,6} \delta(\vec{r}-\vec{r_i})|\psi^a(\vec{r_k},\vec{r_l})\psi^p(\vec{r_k},\vec{r_l})\rangle$$

a=active, p=passive

for the pairs we use the Moshinski transformation:

$$\psi_{n_1l_1}(\vec{r_1}),\psi_{n_2l_2}(\vec{r_2}) = \sum_{nlNL\lambda}\langle n_1l_1n_2l_2\lambda|\}nlNL\lambda\rangle\phi_{nl}(\vec{r})\phi_{NL}(\vec{R}) \qquad (2)$$

Distribution in BDCM

Before performing the Moshinski trasformation we go from jj to ls coupling scheme.

JJ→LS transformation:

$$|\Phi(n_1 l_1 j_1)\Phi(n_2 l_2 j_2); JT\rangle =$$

$$\hat{j}_1 \hat{j}_2 \sum_{\lambda S J_i n l N L} \langle n_1 l_1 n_2 l_2 \lambda | \} n l N L \lambda \rangle \tag{1}$$

$$9j(j_1 l_1 \frac{1}{2}; j_2 l_2 \frac{1}{2}; J\lambda S) 6j(Ll\lambda; SJJ_i) |\psi_{nl}(\vec{r}), \psi_{NL}(\vec{R})\rangle |\chi_1^{\frac{1}{2}} \chi_2^{\frac{1}{2}} S\rangle |\tau_1^{\frac{1}{2}} \tau_2^{\frac{1}{2}} T\rangle$$

This should be used also for the $\langle bra|$.

Distribution in BDCM

For one pair of particle the distibution is then:

$$\langle\Phi(n_1'l_1'j_1')\Phi(n_2'l_2'j_2');JT|\delta(\vec{r}-\vec{r_i})|\Phi(n_1l_1j_1)\Phi(n_2l_2j_2);JT\rangle =$$

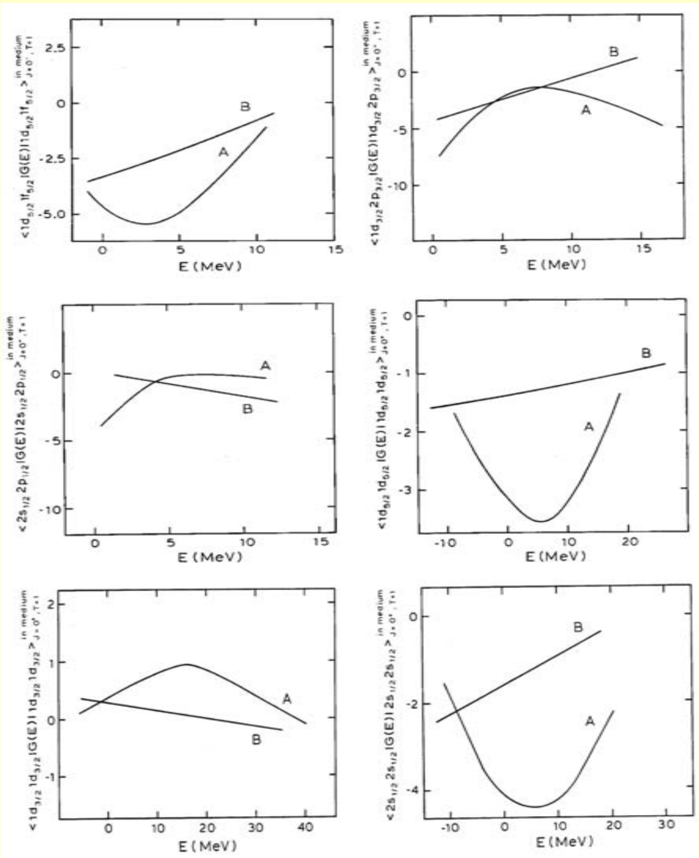
$$\hat{j}_1\hat{j}_2\hat{j}_1'\hat{j}_2' \sum_{i\lambda SJ_inlNL\lambda'n'l'L'N'} \langle n_1l_1n_2l_2\lambda|\}nlNL\lambda\rangle\langle n_1'l_1'n_2'l_2'\lambda'|\}n'l'N'L'\lambda'\rangle$$

$$9j(j_1l_1\frac{1}{2};j_2l_2\frac{1}{2};J\lambda S)6j(Ll\lambda;SJJ_i)9j(j_1'l_1'\frac{1}{2};j_2'l_2'\frac{1}{2};J\lambda'S)6j(L'l'\lambda';SJJ_i) \tag{1}$$

$$\langle\psi_{n'l'}(\vec{r}),\psi_{N'L'}(\vec{R}_i)|\langle\chi_1^{\frac{1}{2}}\chi_2^{\frac{1}{2}}S|\langle\tau_1^{\frac{1}{2}}\tau_2^{\frac{1}{2}}T\rangle|\delta(\vec{R}-\vec{R}_i)|\psi_{nl}(\vec{r}),\psi_{NL}(\vec{R}_i)\rangle|\chi_1^{\frac{1}{2}}\chi_2^{\frac{1}{2}}S\rangle|\tau_1^{\frac{1}{2}}\tau_2^{\frac{1}{2}}T\rangle$$

where; $\hat{j}_1=\sqrt{2j_1+1}$, 9j are the 9j symbols and 6j the 6j symbols.

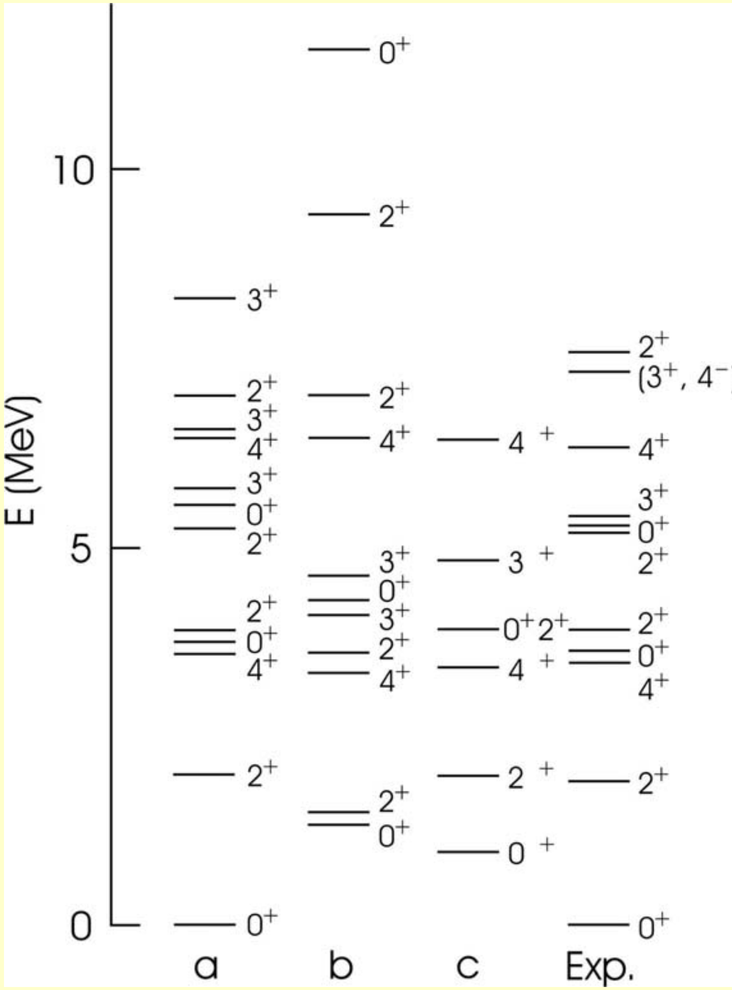
In medium G-matrix elements and corresponding two body potentials for ^{18}O

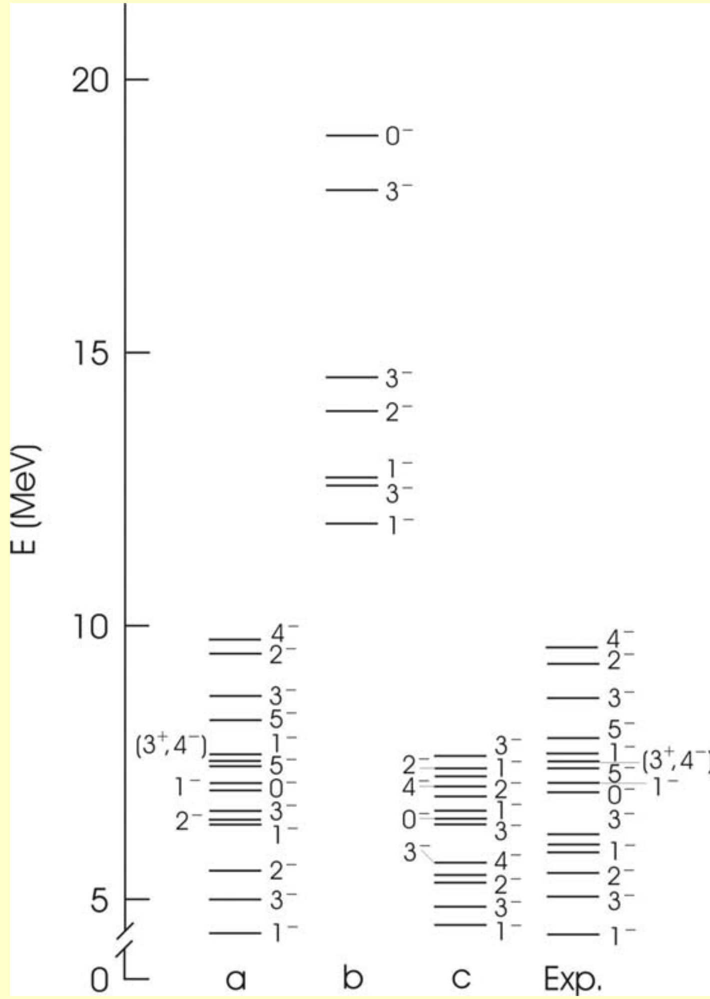


Matrix elements	<44|V|44>	<55|V|55>	<66|V|66>
Bare G-matrix	-0.68	-4.49	2.59
$G^{\text{in medium}}$ (E=0. MeV)	-2.01	-1.47	-1.30
$G^{\text{in medium}}$ (E=3.65 MeV)	-2.63	-1.59	-0.91
$G^{\text{in medium}}$ (E=5.33 MeV)	-2.91	-1.92	-0.89
Folded G-matrix and Paris potential	-2.22	-1.61	-0.95
Folded G-matrix and Bonn A potential	-2.77	-2.05	-1.28
Empirical	-2.82	-2.12	-2.18

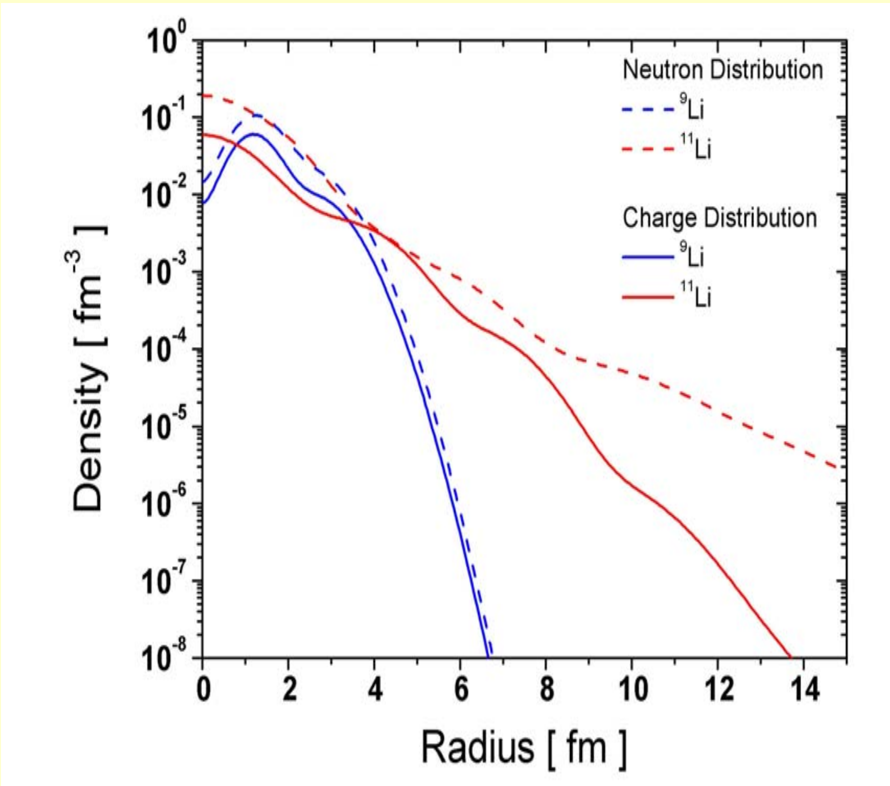
Matrix elements for the first three positive 0^+ levels.

Spectrum of the positive and negative parity state of ^{18}O



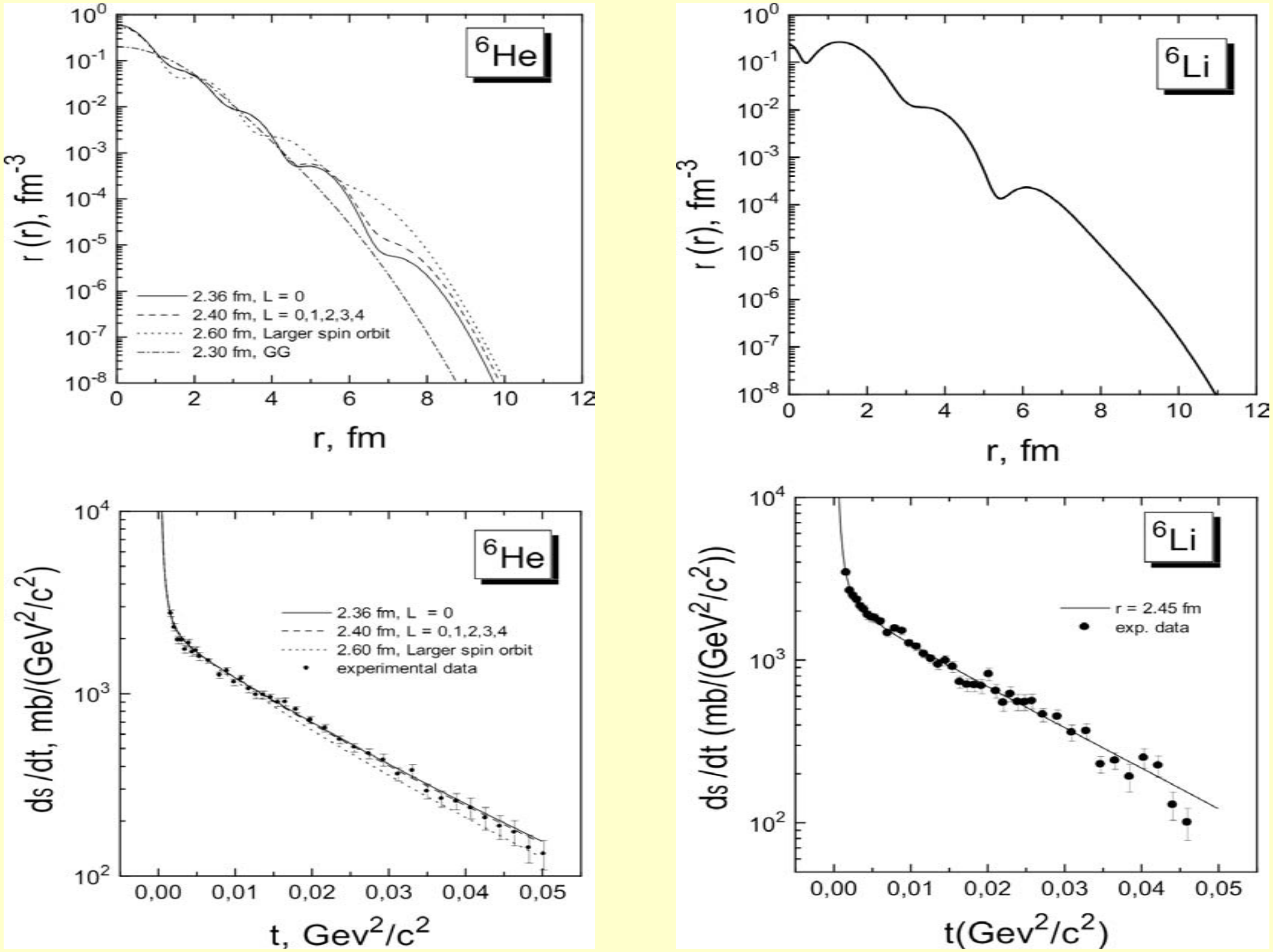


Nuclear results for Li isotopes



	R^{calc}_{matter}	R^{exp}_{matter}	R^{calc}_{charge}	R^{exp}_{charge}	μ^{calc}	μ^{exp}
^{6}Li	2.41	2.45(7)	2.55	2.55(4)	0.82	0.82204(6)
		2.35(2)				
^{7}Li	2.36	2.35(3)	2.41	2.39(20)	3.25	3.2564
^{9}Li	2.42	2.43(7)	2.42	?	3.44	3.43(6)
		2.32(2)				
	2.30(2)					
^{11}Li	3.64	3.62(19)	2.67	?		
		3.12(16)				
	3.53(10)					

Distribution and proton scattering cross-section



Spectroscopic factor and groundstate wavefunction of ^{6}Li

Spectroscopic factor for ^6Li

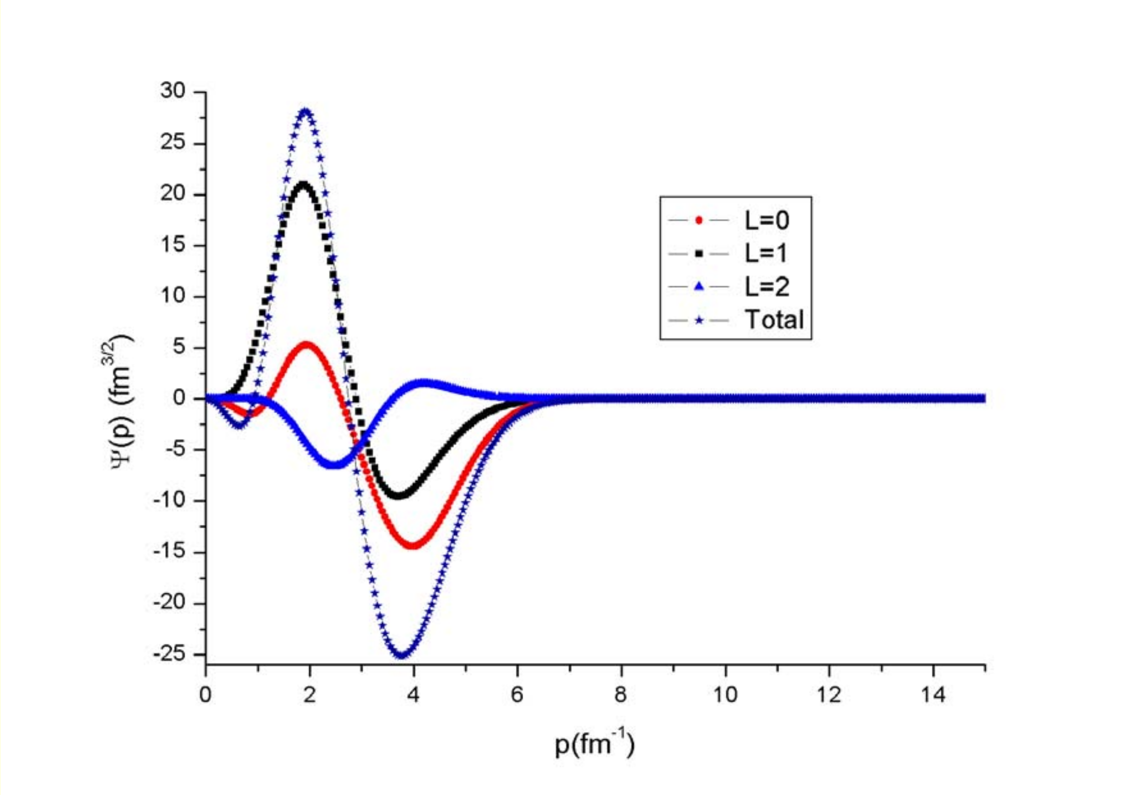
pn in $p_{\frac{3}{2}}$ shell	pn in (psd) shell	pn in (psd) shell and 3p1h
1.0	0.6657	0.6564

Two particle components of the ground state wave function of ^6Li

	$1p_{\frac{3}{2}}1p_{\frac{3}{2}}$	$1p_{\frac{3}{2}}1p_{\frac{1}{2}}$	$1p_{\frac{3}{2}}2p_{\frac{3}{2}}$	$1p_{\frac{3}{2}}1f_{\frac{5}{2}}$	$1p_{\frac{3}{2}}2p_{\frac{1}{2}}$	$1p_{\frac{1}{2}}1p_{\frac{1}{2}}$	$1p_{\frac{1}{2}}2p_{\frac{3}{2}}$	$1p_{\frac{1}{2}}2p_{\frac{1}{2}}$
pn	.8159	.5576	.0133	.0091	.0154	-.1133	-.0179	-.0106
pn+3p1h	.8102	.5598	.0142	.0091	.0158	-.1165	-.0175	-.0110
	$1d_{\frac{5}{2}}1d_{\frac{5}{2}}$	$1d_{\frac{5}{2}}1d_{\frac{3}{2}}$	$2s_{\frac{1}{2}}2s_{\frac{1}{2}}$	$2s_{\frac{1}{2}}1d_{\frac{3}{2}}$	$1d_{\frac{3}{2}}1d_{\frac{3}{2}}$	$1f_{\frac{7}{2}}1f_{\frac{7}{2}}$	$1g_{\frac{9}{2}}1g_{\frac{9}{2}}$	-

[redacted]

Momentum components of the in medium proton neutron wavefunction in ^{6}Li

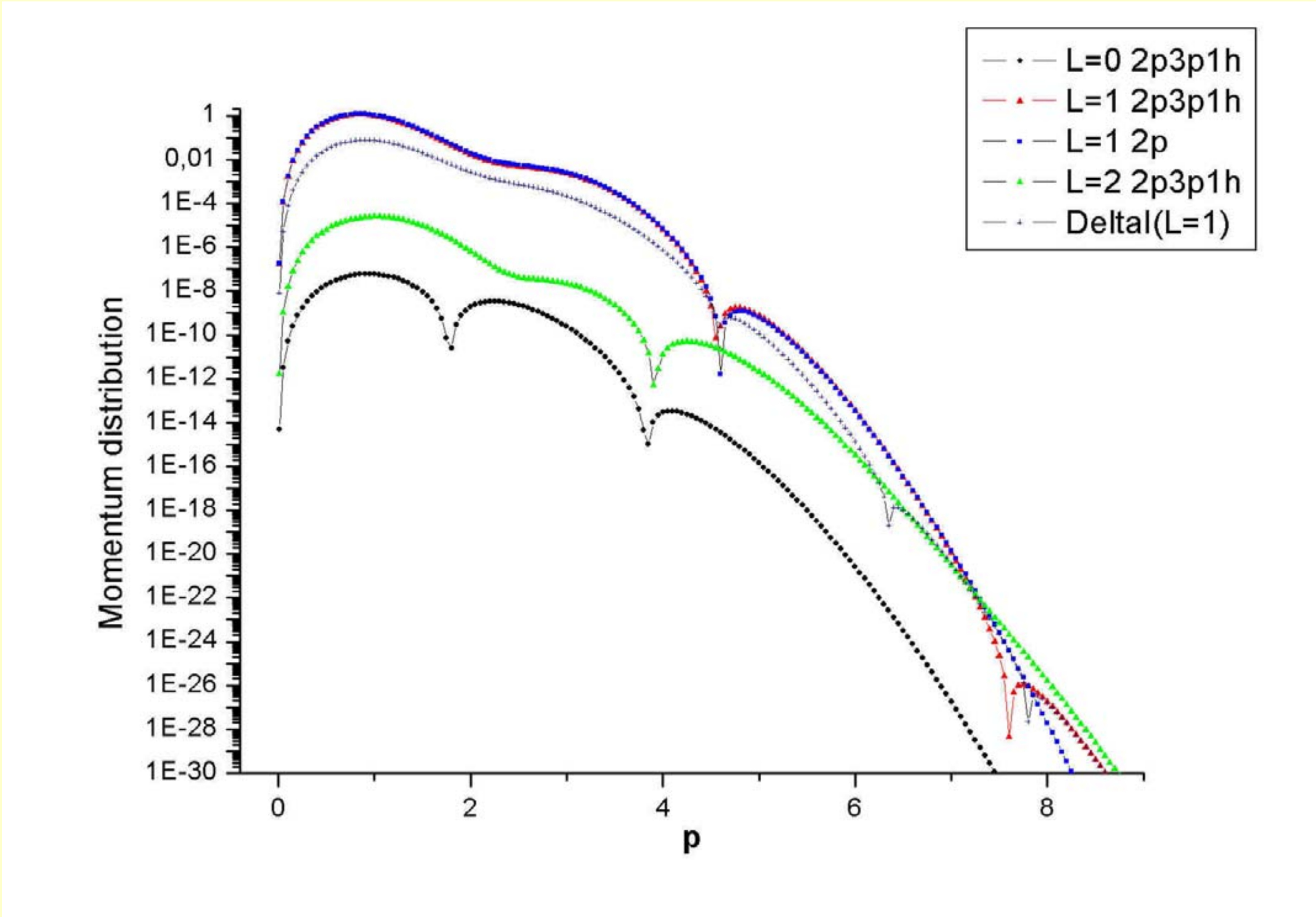


Summary of Charge Radii

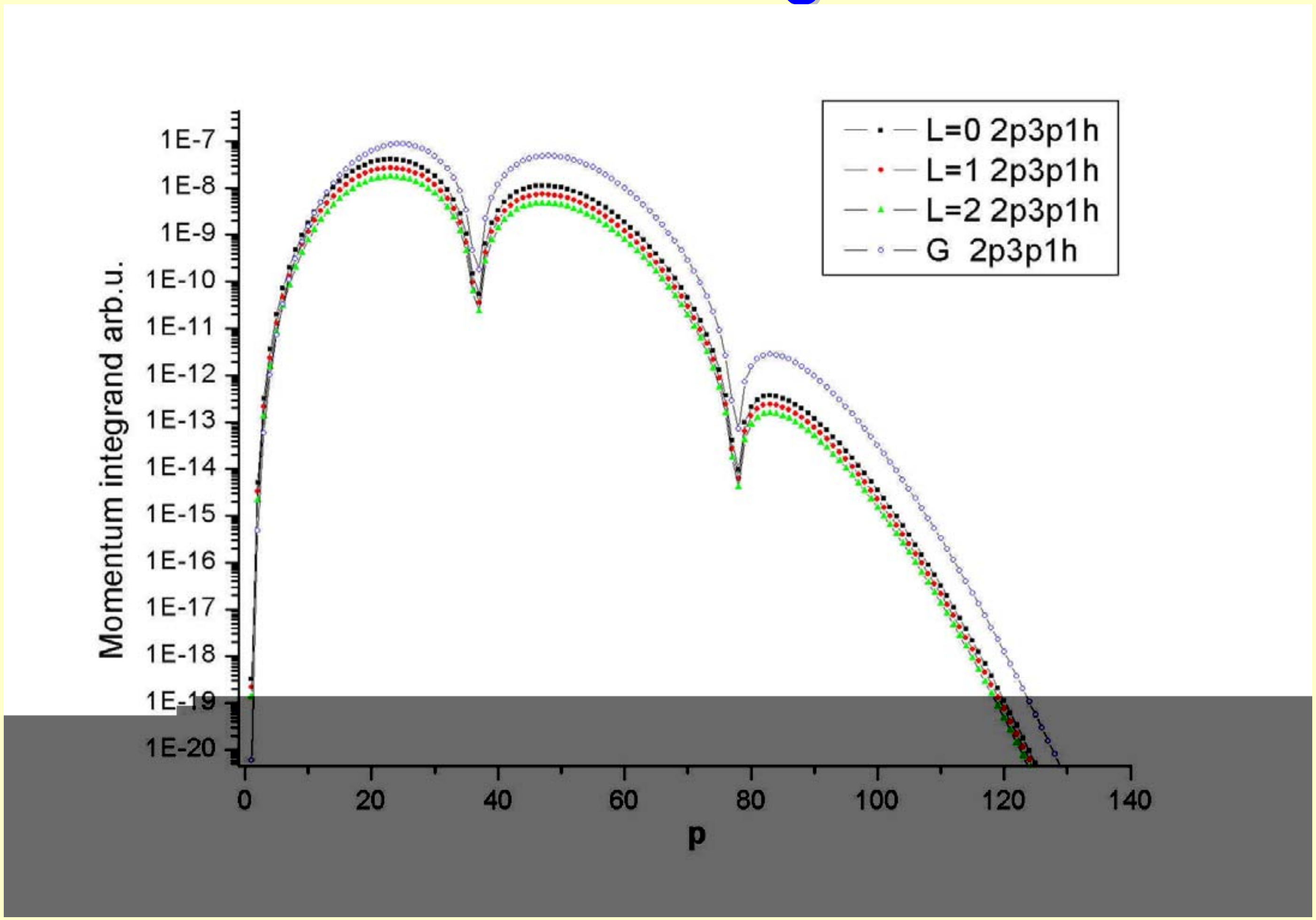
Ref.	this	[8]	[1]	[1]	[2]	[3]	[4]	[5]	[6]	[7]
	Exp.	Exp.	Exp.+Th.	Exp.+Th.	Theory	Theory	Theory	Theory	Theory	Exp.+Th.
Li -	rms R_c	rms R_c	rms R_p	rms R_c	rms R_p	rms R_p		rms R_p	rms R_c	rms R_c
6	2.55	2.55 (4)	2.32 (3)	2.46 (2)	2.045	2.39			2.55	
7	2.46	2.37 (3)	2.27 (2)	2.40 (2)	1.941	2.25		2.27	2.41	
8	2.37		2.26 (2)		1.946	2.09		2.18		
9	2.30		2.18 (2)		1.986		2.04	2.10	2.42	
11			2.88 (2)						2.67	2.235

References:	Method:
[1] I. Tanihata, Phys. Lett B 206,592 (1988)	Interaction Cross Sections with Glauber model, HO distributions
[2] P. Navratil, PRC 57,3119 (1998)	Large-basis shell-model calculations
[3] S. Pieper, Annu.Rev.Nucl.Part.Sci. 51, 53 (2001)	Greens Function Monte Carlo AV18/IL2
[4] S. Pieper, PRC 66, 044310 (2002)	Greens Function Monte Carlo AV18/IL2
[5] Suzuki, Progr.Theo.Phys.Suppl. 146, 413 (2002)	Stochastic Variational Multicluster Method on a correlated gaussian basis
[6] M. Tomaselli, priv. comm. (2004)	Dynamic Correlation model
[7] Penionzhkevich, Nucl.Phys. A 616, 247 (1997)	coupled channel calculations, double-folding optical potential, M3Y effective interaction
[8] C.W. de Jager, At.Dat.Nucl.Dat.Tab. 14, 479 (1974)	Electron Scattering

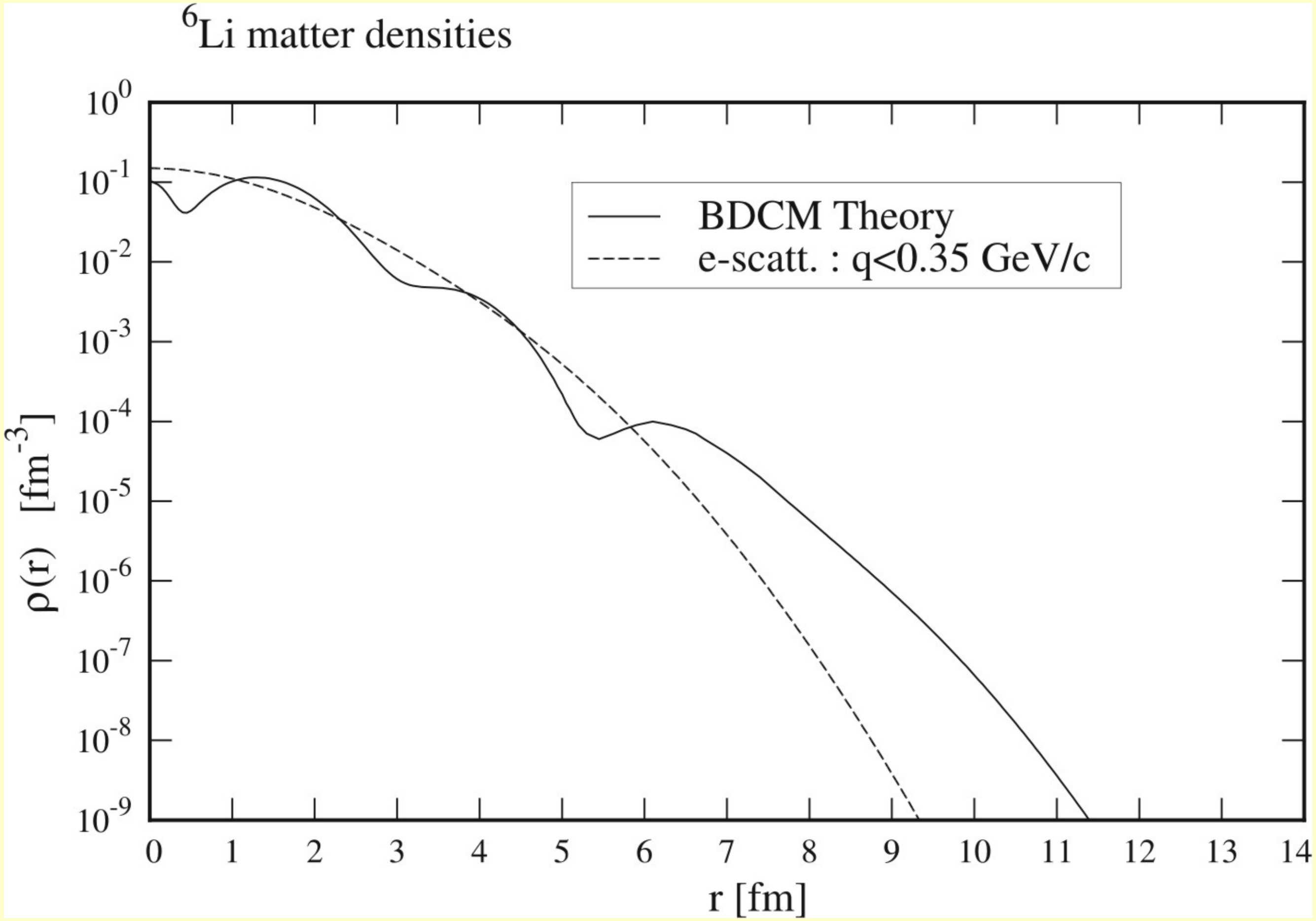
Nuclear results for Li isotopes: momentum distribution



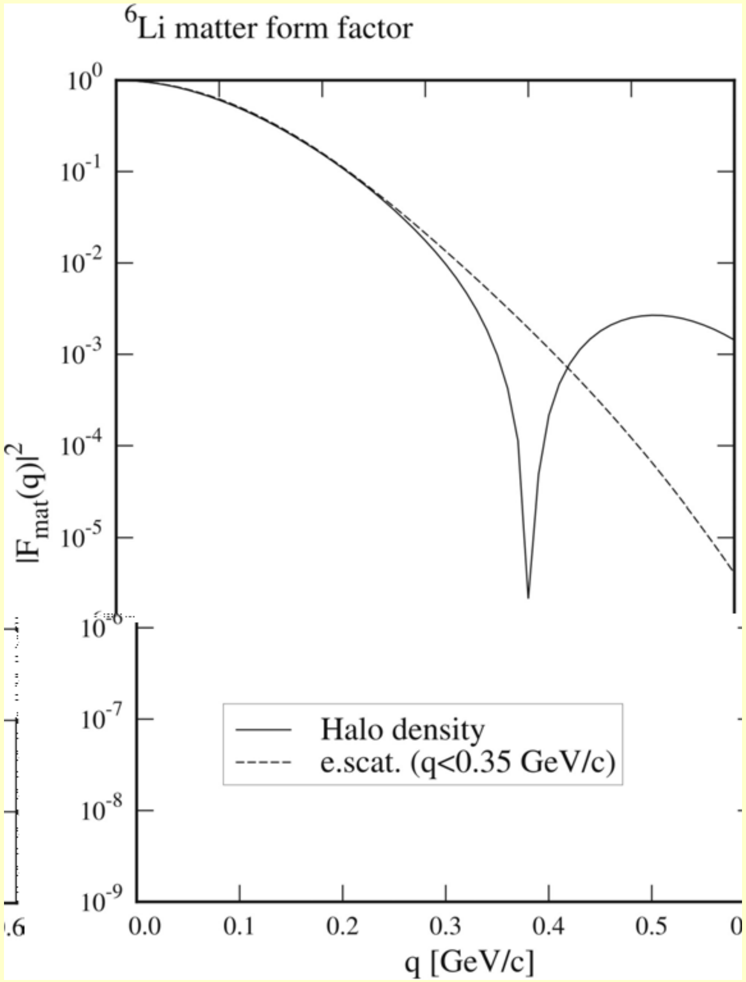
Nuclear results for Li isotopes: Momentum integrands

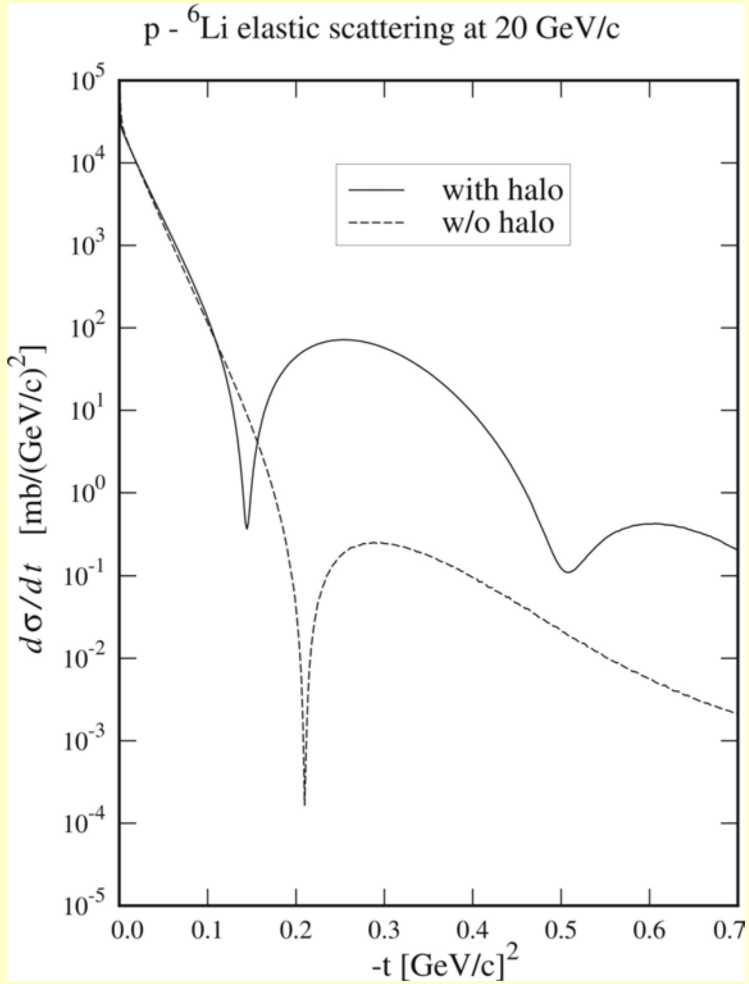


Nuclear results for Li isotopes

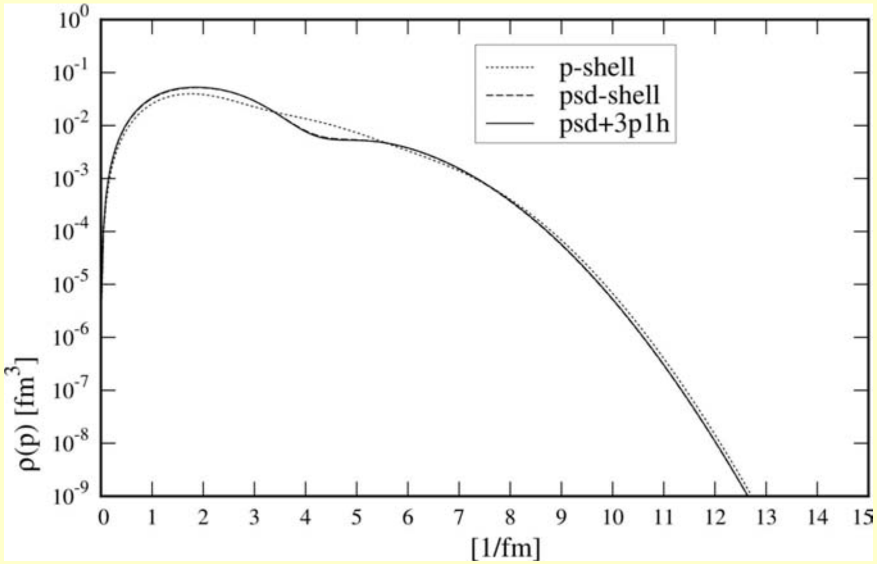


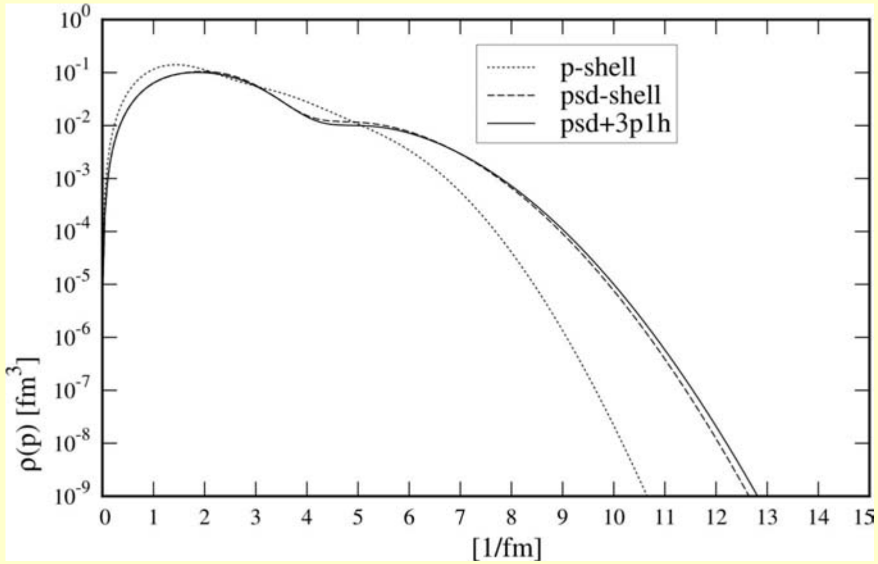
Nuclear results for Li isotopes





Momentum distribution for p and d in ^{6}Li

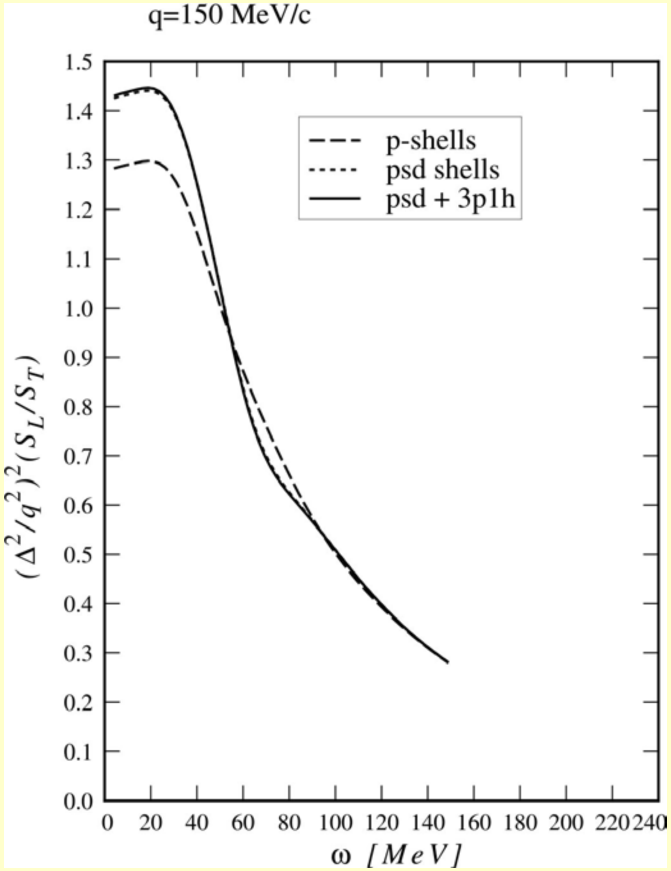


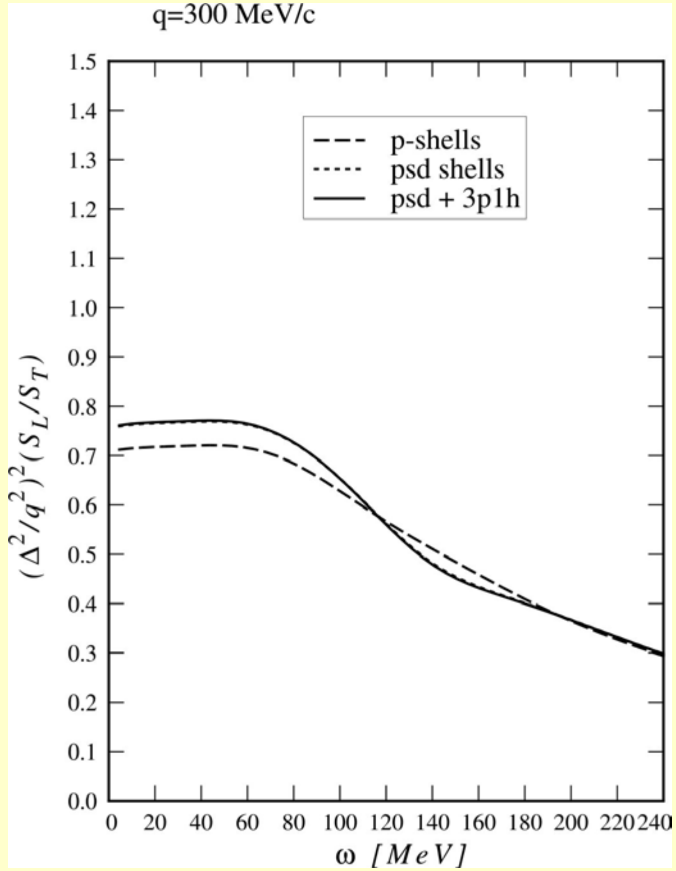


Proton

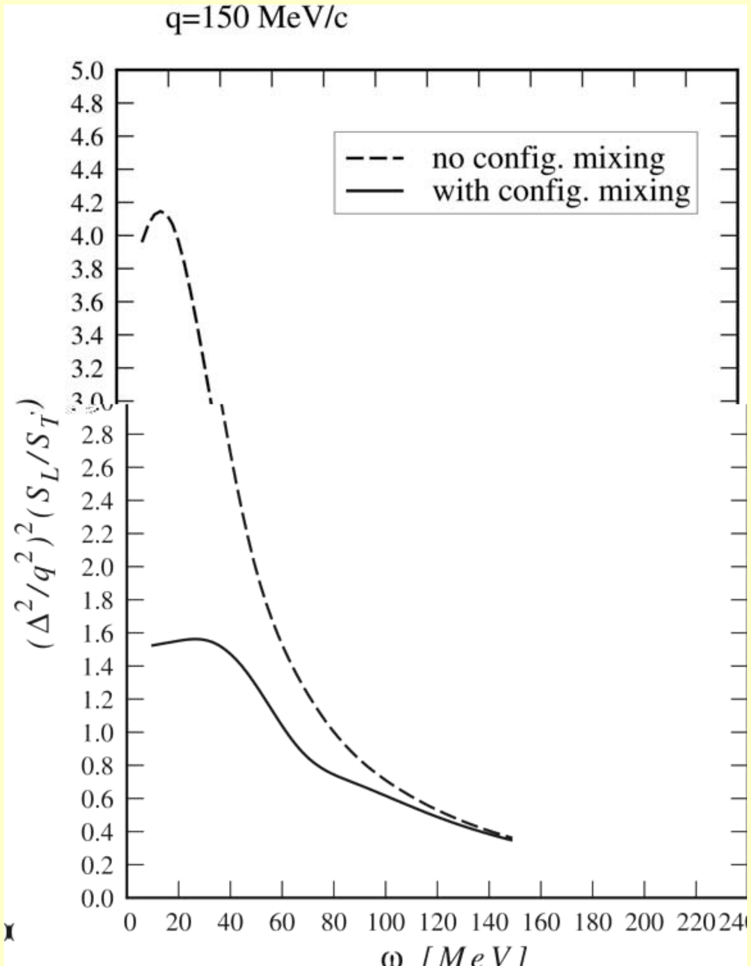
Deuteron

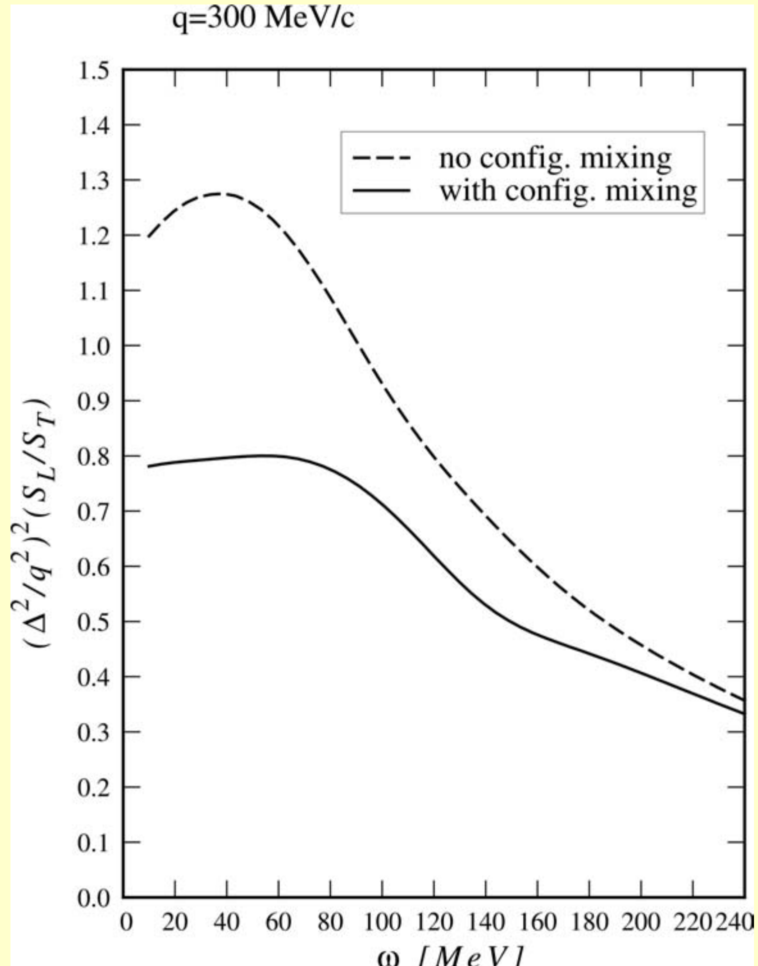
Calculated singularity-free longitudinal to transverse response ratio R for ee'p





Calculated singularity-free longitudinal to transverse response ratio R for ee'd





Conclusions

- (ee'p) and (ee'd) give a good test for the halo structure of exotic nuclei
- Microscopic structure of the nucleus play an important role
- Result can be used as prediction for new experiments at the future GSI
- Presented method should be sistematically applied to nuclei with pronounced halo like ^{11}Li
- Effective method to predict also the proton halo