# Methods and lessons from direct reaction theory <br> Spectroscopic Factors Workshop Trento, Italy, $2^{\text {nd }}-12^{\text {th }}$ March 2004 

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## Direct (knockout, break-up, transfer) reactions - generics

1) Reactions in which there is a minimal rearrangement, or excitation involving a very small number of active (effective) degrees of freedom of the projectile and/or target: single-particle (sp) or collective inelastic excitation, sp or cluster transfer - 'reactions are fast'
2) Reaction energies are such that average, effective (complex) interactions can be used between the reacting constituents - regions of high level density
3) Because of complex effective interactions and short mean free paths, reactions are localised / dominated by interactions in the nuclear surfaces and by hence by peripheral and grazing collisions - 'so fast'

## Surface localisation of knockout reactions

## Intermediate energy: ${ }^{12} \mathrm{Be}+{ }^{9} \mathrm{Be} \rightarrow{ }^{11} \mathrm{Be}(\mathrm{gs})+\mathrm{X}, 80 \mathrm{~A} \mathrm{MeV}$



## Few-body reaction models for sp spectroscopy

There are no practical many-body reaction theories - we construct model 'effective' few-body models ( $\mathrm{n}=2,3,4 \ldots$...)

$$
\mathrm{H}=\underbrace{\mathrm{T}_{\mathrm{r}}+\mathrm{V}_{v c}}_{\text {projectile } \mathrm{H}_{\mathrm{p}}}+\mathrm{T}_{\mathbf{R}}+\underbrace{\mathrm{V}_{c T}+\mathrm{V}_{v T}}_{\mathrm{U}(\mathbf{r}, \mathbf{R})}
$$



Solve, as best we can, the Schrödinger equation:

$$
\mathrm{H} \Psi=\mathrm{E} \Psi
$$

## (1) Dynamics - we need effective interactions

$$
\mathrm{H}=\mathrm{T}_{\mathbf{r}}+\mathrm{V}_{v c}+\mathrm{T}_{\mathrm{R}}+\mathrm{V}_{c T}+\mathrm{V}_{v T}
$$

binds projectile
effective (complex) interactions of $c$ and $v$ individually with target (nuclear + Coulomb potentials)
(a) From experiment: potentials fitted to available data for $\mathrm{c}+\mathrm{T}$ or $\mathrm{v}+\mathrm{T}$ scattering at the appropriate energy per nucleon
(b) From theory: multiple scattering or folding models, for example

nucleon-nucleon t-matrix or effective NN interaction
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## Skyrme Hartree-Fock radii and densities (1)



W.A. Richter and B.A. Brown, Phys. Rev. C67 (2003) 034317

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## Skyrme Hartree-Fock radii and densities (2)



B.A. Brown, S. Typel, and W.A. Richter, Phys. Rev. C65 (2002) 014612

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(2) Structure - we need sp overlap integrals

Nucleon removal from $\Phi_{A+1}$ will leave mass A residue in the ground or an excited state - even in extreme sp model More generally: amplitude for finding nucleon with sp
 quantum numbers $\ell, j$, about core state $\Phi_{\mathrm{c}}$ in $\Phi_{\mathrm{A}+1}$ is

$$
\Phi_{\mathrm{c}}
$$

$$
\begin{gathered}
\mathrm{F}_{\ell j}^{\mathrm{c}}(\mathbf{r})=\left\langle\mathbf{r}, \Phi_{\mathrm{c}} \mid \Phi_{\mathrm{A}+1}\right\rangle, \mathrm{S}_{\mathrm{N}}=\mathrm{E}_{\mathrm{A}+1}-\mathrm{E}_{\mathrm{c}} \\
\int \mathrm{~d} \mathbf{r}\left|\mathrm{~F}_{\ell j}^{\mathrm{c}}(\mathbf{r})\right|^{2}=\mathrm{C}^{2} \mathrm{~S}(\ell j)\left\{\begin{array}{l}
\text { Spectroscopic } \\
\text { factor - occupancy } \\
\text { of the state }
\end{array}\right.
\end{gathered}
$$

Usual to write

$$
\mathrm{F}_{\ell j}^{\mathrm{c}}(\mathbf{r})=\sqrt{\mathrm{C}^{2} \mathrm{~S}(\ell j)} \phi_{0}(\mathbf{r}) ; \quad \int \mathrm{d} \mathbf{r}\left|\phi_{0}(\mathbf{r})\right|^{2}=1
$$

## (2) Structure - sensitivity to overlap integrals

$$
\begin{aligned}
& \Phi_{\mathrm{A}+1}^{\mathbf{r}} \mathrm{F}_{\ell, j}^{\mathrm{c}}(\mathbf{r})=\left\langle\mathbf{r}, \Phi_{\mathrm{c}} \mid \Phi_{\mathrm{A}+1}\right\rangle, \mathrm{S}_{\mathrm{N}}=\mathrm{E}_{\mathrm{A}+1}-\mathrm{E}_{\mathrm{c}} \\
& \Phi_{\mathrm{c}} \quad \mathrm{~F}_{\ell j}^{\mathrm{c}}(\mathbf{r})=\sqrt{\mathrm{C}^{2} \mathrm{~S}(\ell j)} \phi_{0}(\mathbf{r}) ; \quad \int \mathrm{d} \mathbf{r}\left|\phi_{0}(\mathbf{r})\right|^{2}=1
\end{aligned}
$$

Usually $\phi_{0}(\mathbf{r})$ calculated in a simple potential model, e.g. Woods-Saxon with 'reasonable' geometry; encompasses a mass of experimental systematics - use HF theory?

Major sensitivity of cross sections in transfer, break-up and knockout reactions is (linear in) $\left\langle r^{2}\right\rangle^{1 / 2}$ of overlap

## 2 N correlations - on single-nucleon overlaps

2 N correlations in non-Borromean two-nucleon halo nuclei:
[ A, A-1, A-2 nuclei all particle-stable ]

| $A$ | $A-1$ | $A-2$ | $S_{A}(2 N)$ |  |  | $S_{A}(1 N)$ | $S_{A-1}(1 N)$ |
| :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: |
| $(\mathrm{MeV})$ | $(\mathrm{MeV})$ | $(\mathrm{MeV})$ |  |  |  |  |  |
| ${ }^{12} \mathrm{Be}$ | ${ }^{11} \mathrm{Be}\left(\frac{1}{2}^{+}\right)$ | ${ }^{10} \mathrm{Be}$ | 3.670 | 3.170 |  |  |  |
| ${ }^{12} \mathrm{Be}$ | ${ }^{11} \mathrm{Be}\left(\frac{1}{2}^{-}\right)$ | ${ }^{10} \mathrm{Be}$ | 3.670 | 3.490 |  |  |  |
| ${ }^{15} \mathrm{~B}$ | ${ }^{14} \mathrm{~B}\left(1^{-}\right)$ | ${ }^{13} \mathrm{~B}$ | 3.740 | 3.510 |  |  |  |
| ${ }^{9} \mathrm{C}$ | ${ }^{8} \mathrm{~B}($ g.s. $)$ | ${ }^{7} \mathrm{Be}$ | 1.433 | 1.296 |  |  |  |
| ${ }^{16} \mathrm{C}$ | ${ }^{15} \mathrm{C}\left(\frac{1}{2}^{+}\right)$ | ${ }^{14} \mathrm{C}$ | 5.469 | 4.251 |  |  |  |
| ${ }^{16} \mathrm{C}$ | ${ }^{15} \mathrm{C}\left(\frac{5}{2}^{+}\right)$ | ${ }^{14} \mathrm{C}$ | 5.469 | 4.991 |  |  |  |

+ many others
L.D. Blokhintsev, Bull. Acad. Russ. Sci. Phys. 65 (2001), 77.
N.K. Timofeyuk, L.D. Blokhintsev and J.A. Tostevin, Phys Rev C 68 (2003) 021601(R)

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## Past: DR analyses with light-ions: questions?

1) How important is it to take account of the loosely bound nature of the deuteron/triton/3 He and three-body breakup channels in direct reactions and how can one treat these 'practically'?
2) How accurate are first-order (BA, DWBA) approaches, and the spectroscopic information (spectroscopic factors B(E2)'s, deformations and angular momentum assignments) deduced, as a test of structure models?
3) How do we treat the required single-particle overlaps of many-body wave functions? (often assumed known)
4) How does one best deal with sensitivity of direct reaction calculations to the assumed effective interactions?

Present: ingredients/questions with exotic beams:

1) It is vital to take into account non-perturbatively the loosely bound nature of exotic nuclei and their break-up channels in calculations of reaction observables
2) How accurate is the spectroscopic information (spectroscopic factors) deduced from approximate fewbody models as a test of structure models?
3) How to / can we (?) obtain 'practically' the required single-particle overlaps from realistic many-body wave functions of the best structure theory?
4) How should we best choose the assumed effective interactions between reacting constituents? - we should make best use of theoretical models - sizes, densities.

## Eikonal theory reveals bare requirements

Reaction mechanism complications stripped away:
dynamics

$$
\mathrm{S}_{\alpha \beta}(\mathrm{b})=\underbrace{\left\langle\phi_{\beta}\right| \overbrace{\mathrm{S}_{\mathrm{c}}\left(\mathrm{~b}_{\mathrm{c}}\right) \mathrm{S}_{\mathrm{v}}\left(\mathrm{~b}_{\mathrm{v}}\right)}^{\text {aynamics }}\left|\phi_{\alpha}\right\rangle}_{\text {structure }}
$$

Can use overlaps from the best available few- or manybody sp wave functions if can be provided/extracted in a suitable form

More generally,

$$
S_{\alpha \beta}(b)=\left\langle\varphi_{\beta}\right| S_{1}\left(b_{1}\right) S_{2}\left(b_{2}\right) \ldots \ldots S_{n}\left(b_{n}\right)\left|\varphi_{\alpha}\right\rangle
$$

for any choice of $1,2,3, \ldots . . n$ clusters if $\varphi$ is available

## The continuum-coupled channels methodology



## Core fragment differential cross sections - CDCC



## Adiabatic/sudden approximation - few-bodies

The time-dependent equation is
$H \Psi(\mathbf{r}, \mathbf{R}, \mathrm{t})=i \hbar \frac{\partial \Psi}{\partial \mathrm{t}}$
and can be written

$$
\Psi(\mathbf{r}, \mathbf{R}, \mathrm{t})=\Lambda \Phi(\mathbf{r}(\mathrm{t}), \mathbf{R}), \mathbf{r}(\mathrm{t})=\Lambda^{+} \mathbf{r} \Lambda
$$

$\Lambda=\exp \left\{-i\left(\mathrm{H}_{\mathrm{p}}+\varepsilon_{0}\right) \mathrm{t} / \hbar\right\}$ and where

$$
\left[\mathrm{T}_{\mathrm{R}}+\mathrm{U}(\mathbf{r}(\mathrm{t}), \mathbf{R})-\varepsilon_{0}\right] \Phi(\mathbf{r}(\mathrm{t}), \mathbf{R})=i \hbar \frac{\partial \Phi}{\partial \mathrm{t}}
$$

Adiabatic equation


## Adiabatic step assumes

$\mathbf{r}(\mathrm{t}) \approx \mathbf{r}(0)=\mathbf{r}=\mathrm{fixed}$ or $\Lambda=1$ for the collision time $\mathrm{t}_{\text {coll }}$ requires
$\left[\mathrm{T}_{\mathrm{R}}+\mathrm{U}(\mathbf{r}, \mathbf{R})\right] \Phi(\mathbf{r}, \mathbf{R})=\left(\mathrm{E}+\varepsilon_{0}\right) \Phi(\mathbf{r}, \mathbf{R})$
$\left(\mathrm{H}_{\mathrm{p}}+\varepsilon_{0}\right) \mathrm{t}_{\text {coll }} / \hbar \ll 1$

Models for transfer reactions: e.g. (d,p)

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{dp}}=\left\langle\chi_{\mathrm{p}}^{(-)}\left(\mathbf{r}_{\mathrm{p}}\right) \phi_{\mathrm{n}}\left(\mathbf{r}_{\mathrm{n}}\right)\right| \mathrm{V}_{\mathrm{np}}\left|\Psi_{\mathrm{K}}^{(+)}(\mathbf{r}, \mathbf{R})\right\rangle \text { note }|\mathbf{r}| \leq \text { range of } \mathrm{V}_{\mathrm{np}} \\
& \longrightarrow \begin{array}{c}
{\left[\mathrm{T}_{\mathrm{R}}+\mathrm{U}(\mathbf{r}, \mathbf{R})+\mathrm{H}_{\mathrm{d}}-\mathrm{E}\right] \Psi_{\mathrm{K}}^{(+)}=0} \\
\mathrm{H}_{\mathrm{d}} \rightarrow-\varepsilon_{0}, \Psi_{\mathrm{K}}^{(+)} \rightarrow \Psi_{\mathrm{K}}^{\mathrm{AD}}
\end{array} \\
& \left.\hline \mathbf{U}(\mathbf{r}, \mathbf{R})-\mathrm{E}_{0}\right] \Psi_{\mathrm{K}}^{\mathrm{AD}}=0
\end{aligned}
$$

DWBA $\left(|\mathbf{r}| \leq\right.$ range of $\left.\phi_{0}\right) \quad \underline{\text { ADIABATIC }}\left(|\mathbf{r}| \leq\right.$ range of $\left.\mathrm{V}_{\mathrm{np}}\right)$

$$
\begin{array}{cc|c}
\Psi_{\mathrm{K}}^{(+)} \rightarrow \phi_{0}(\mathbf{r})\left\langle\phi_{0}(\mathbf{r}) \mid \Psi_{\mathrm{K}}^{(+)}\right\rangle_{\mathbf{r}} & \begin{array}{c}
\Psi_{\mathrm{K}}^{\mathrm{AD}} \approx \phi_{0}(\mathbf{r}) \widetilde{\chi}_{\mathrm{K}}^{\mathrm{AD}}(\mathbf{R}) \\
\\
=\phi_{0}(\mathbf{r}) \chi_{\mathrm{K}}^{(+)}(\mathbf{R})
\end{array} & {\left[\mathrm{T}_{\mathrm{R}}+\widetilde{\mathrm{V}}(\mathbf{R})-\mathrm{E}_{0}\right] \widetilde{\chi}_{\mathrm{K}}^{\mathrm{AD}}=0} \\
& { }^{4} & \widetilde{\mathrm{~V}}(\mathbf{R})=\frac{\left\langle\phi_{0}\right| \mathrm{V}_{\mathrm{np}} \mathrm{U}(\mathbf{r}, \mathbf{R})\left|\phi_{0}\right\rangle}{\left\langle\phi_{0}\right| \mathrm{V}_{\mathrm{np}}\left|\phi_{0}\right\rangle} \approx \mathrm{U}(\mathbf{r}=0, \mathbf{R})
\end{array}
$$

## Large number of semi-classical methods



## Few-body eikonal model - adiabatic, trajectories

Modulation function after collision, $\quad \omega(\mathbf{r}, \mathbf{R})=\mathrm{S}_{\mathrm{c}}\left(\mathrm{b}_{\mathrm{c}}\right) \mathrm{S}_{\mathrm{v}}\left(\mathrm{b}_{\mathrm{v}}\right)$

$$
\Psi_{\mathrm{K}}^{\mathrm{Eik}}(\mathbf{r}, \mathbf{R}) \rightarrow \mathrm{e}^{\mathrm{iK} \cdot \mathbf{R}} \mathrm{~S}_{\mathrm{c}}\left(\mathrm{~b}_{\mathrm{c}}\right) \mathrm{S}_{\mathrm{v}}\left(\mathrm{~b}_{\mathrm{v}}\right) \phi_{\alpha}(\mathbf{r})
$$

with $\mathrm{S}_{\mathrm{c}}$ and $\mathrm{S}_{\mathrm{v}}$ the eikonal approximations to the S -matrices for the independent scattering of c and v from the target


So, inelastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b-i.e. The amplitude that it emerges in state $\varphi_{\beta}(\mathbf{r})$ is
adiabatic

$$
\mathrm{S}_{\alpha \beta}(\mathrm{b})=\left\langle\phi_{\beta}\right| \mathrm{S}_{\mathrm{c}}\left(\mathrm{~b}_{\mathrm{c}}\right) \mathrm{S}_{\mathrm{v}}\left(\mathrm{~b}_{\mathrm{v}}\right)\left|\phi_{\alpha}\right\rangle
$$

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## Non-adiabatic - but trajectory based

Time-dependent (finite difference) solution of the valence particle motion - assuming the heavy core, or c.m., follows a trajectory: [See: Bertsch and Esbensen, Baur and Typel, Suzuki, Melezhik and Baye]


Not exact - but non-adiabatic Dynamics of $\mathrm{V}_{\mathrm{cT}}$ is not included and no energy transfer/sharing between core and internal motion. For heavy targets - Coulomb path

Solved on an ( $\mathbf{r}, \mathrm{t}$ ) grid and care is needed.

$$
\begin{aligned}
& i \hbar \frac{\partial \psi}{\partial \mathrm{t}}=\left(\mathrm{H}_{\mathrm{p}}+\mathrm{V}_{\mathrm{vT}}\right) \psi(\mathbf{r}, \mathrm{t}) \\
& \text { as } \mathrm{t} \rightarrow-\infty \psi(\mathbf{r}, \mathrm{t}) \rightarrow \phi_{0}(\mathbf{r}) \\
& \mathrm{t} \rightarrow+\infty \psi(\mathbf{r}, \mathrm{t}) \rightarrow \psi_{\mathrm{f}}\left(\mathbf{r}, \mathrm{~T}_{0}\right)
\end{aligned}
$$

## Transfer to the continuum approximation

Related transfer to the continuum model is due to Angela Bonaccorso , David Brink and others (this meeting). Using additional approximations (asymptotic forms of wave function) the time-dependent finite difference solution is avoided in favour of largely analytic approach.


Not exact - but non-adiabatic Dynamics of $\mathrm{V}_{\text {cT }}$ is not included and no energy transfer/sharing between core and internal motion.

$$
\begin{aligned}
i \hbar \frac{\partial \psi}{\partial \mathrm{t}} & =\left(\mathrm{H}_{\mathrm{p}}+\mathrm{V}_{\mathrm{vT}}\right) \psi(\mathbf{r}, \mathrm{t}) \\
\text { as } \mathrm{t} & \rightarrow-\infty \psi(\mathbf{r}, \mathrm{t}) \rightarrow \phi_{0}(\mathbf{r})
\end{aligned}
$$

What is the state of the reaction theory?
Do the different theories agree for the same structure and effective interaction inputs?
Theorists will (sometimes/always?) argue the details but where fair tests and comparisons have been carried out and domains of approximations overlap - answer is YES

## Structure inputs - overlaps <br> Dynamics - effective interactions

## Transfer reactions: choice of distorting potentials

It used to be thought that the best procedure is to measure the elastic scattering by the target nucleus of the incident projectiles and that by the final nucleus of the outgoing particles, all at the proper energies, and then to fit the elastic data as well as possible with optical model potentials. These potentials were then to be used as input to DWBA calculations.
Experience has shown that a more sensible procedure is to use distorting parameters which are appropriate for a wider range of target nuclei and energies. Emphasis on accurate fitting of data on one or two nuclei tends to optimize the fit by selecting a peculiar (and perhaps unphysical) set of parameters.
M.H. Macfarlane and J.P. Schiffer, Nucl. Spectroscopy and Reactions, Vol B, pp 169

What should we use? - appeal to theory

## Spectroscopic factors from individual analyses


X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), in progress

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## Adiabatic model for transfer reactions: e.g. (d,p)



DWBA $\left(|\mathbf{r}| \leq\right.$ range of $\left.\phi_{0}\right)$
$\Psi_{\mathrm{K}}^{(+)} \rightarrow \phi_{0}(\mathbf{r})\left\langle\phi_{0}(\mathbf{r}) \mid \Psi_{\mathrm{K}}^{(+)}\right\rangle_{\mathbf{r}}$
$=\phi_{0}(\mathbf{r}) \chi_{\mathbf{K}}^{(+)}(\mathbf{R})$
d elastic scattering

ADIABATIC $\left(|\mathbf{r}| \leq\right.$ range of $\left.\mathrm{V}_{\mathrm{np}}\right)$

$$
\Psi_{\mathbf{K}}^{\mathrm{AD}} \approx \phi_{0}(\mathbf{r}) \tilde{\chi}_{\mathbf{K}}^{\mathrm{AD}}(\mathbf{R})
$$

$$
\left[\mathrm{T}_{\mathbf{R}}+\tilde{\mathrm{V}}(\mathbf{R})-\mathrm{E}_{0}\right] \tilde{\chi}_{\mathbf{K}}^{\mathrm{AD}}=0
$$

$$
\tilde{\mathrm{V}}(\mathbf{R}) \approx \mathrm{V}_{\mathrm{n}}(\mathrm{R})+\mathrm{V}_{\mathrm{p}}(\mathrm{R})
$$

## Key outcomes for transfer reactions - spectroscopy



## Increased reflection at nuclear surface - less diffuse 'deuteron' channel potential <br> Greater surface localisation - L-space localisation <br> Less nuclear volume contribution and less sensitivity to optical model parameters <br> More consistent sets of deduced spectroscopic factors

J.D. Harvey and R.C. Johnson, Phys. Rev.C 3 (1971) 636

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## Microscopic nucleon optical potentials - JLM



J.S. Petler et al. Phys. Rev. C 32 (1985), 673

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## Consistent analyses of transfer reaction data


X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), submitted

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## Spectroscopic factors - consistent inputs


X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), submitted

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## One- and two-nucleon knockout reactions

## Peripheral collisions ( $\mathrm{E} \geq 50 \mathrm{~A} \mathrm{MeV}$; MSU, RIKEN, GSI)


$\mathrm{T}={ }^{9} \mathrm{Be},{ }^{12} \mathrm{C}$

## Direct from the projectile perspective

Events contributing will be both break-up and stripping both of which leave a mass A residue in the final state

## Absorptive cross sections - target excitation

Since our effective interactions are complex all S(b) include the effects of
 absorption due to inelastic channels

$$
\begin{aligned}
& \sigma_{\mathrm{abs}}=\sigma_{\mathrm{R}}-\sigma_{\mathrm{diff}}=\int \mathrm{d} \mathbf{b}\left\langle\phi_{0}\right| \underbrace{1-\left|\mathrm{S}_{\mathrm{c}} \mathrm{~S}_{\mathrm{v}}\right|^{2}}\left|\phi_{0}\right\rangle \\
& \text { of } v \text { from } \\
& \begin{cases}\left|S_{\mathrm{v}}\right|^{2}\left(1-\left|\mathrm{S}_{\mathrm{c}}\right|^{2}\right)+ & \text { v survives, c absorbed } \\
\left|\mathrm{S}_{\mathrm{c}}\right|^{2}\left(1-\left|\mathrm{S}_{\mathrm{v}}\right|^{2}\right)+ & \text { v absorbed, c survives } \\
\left(1-\left|S_{\mathrm{v}}\right|^{2}\right)\left(1-\left|\mathrm{S}_{\mathrm{v}}\right|^{2}\right) & \text { v absorbed, c absorbed }\end{cases} \\
& \sigma_{\text {strip }}=\int \mathrm{d} \mathbf{b}\left\langle\phi_{0}\right|\left|\mathrm{S}_{\mathrm{c}}\right|^{2}\left(1-\left|\mathrm{S}_{\mathrm{v}}\right|^{2}\right)\left|\phi_{0}\right\rangle \\
& \text { projectile } \\
& \text { exciting } \\
& \text { the target. } \\
& \text { c scatters } \\
& \text { at most } \\
& \text { elastically } \\
& \text { with the } \\
& \text { target }
\end{aligned}
$$

Related equations exist for the differential cross sections, etc.
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## Choice of two-body distorting interactions

- Work at MSU has used the same energy range (60$100 \mathrm{MeV} /$ nucleon) and the same light nuclear target ( ${ }^{9} \mathrm{Be}$ ) combination - systematics across many data sets
- Need nucleon - ${ }^{9}$ Be S-matrices over limited energy. Can use JLM or other absorptive $\mathrm{t}_{\mathrm{NN}}$ effective interaction consistent with $\mathrm{n}+{ }^{9}$ Be reaction cross section (split between diffractive and stripping mechanisms depends on this choice - but not their sum)
- Core-target systems are black (highly absorptive). Calculated using ' $\mathrm{t}_{\mathrm{NN}} \rho \rho$ ' double folding model to incorporate realistic sizes and surface geometries gives results consistent with two-body $\sigma_{\mathrm{R}}$ (core)


## Weakly bound states - with good statistics


P.G. Hansen and J.A.Tostevin, ARNPS 53 (2003), 219

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## More strongly bound states - deep hole states


P.G. Hansen and J.A.Tostevin, ARNPS 53 (2003), 219

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## Neutron removal from the $\mathrm{N}=16$ isotones



## Direct two nucleon knockout - 2N correlations?

$$
\sigma_{\text {strip }}=\sigma_{-2 \mathrm{~N}}=\int \mathrm{d} \mathbf{b}\left\langle\phi_{0}\right|\left|\mathrm{S}_{\mathrm{c}}\right|^{2}\left(1-\left|\mathrm{S}_{1}\right|^{2}\right)\left(1-\left|\mathrm{S}_{2}\right|^{2}\right)\left|\phi_{0}\right\rangle
$$



Estimate assuming removal of a pair 2 of uncorrelated nucleons -

$$
\begin{aligned}
& \phi_{0}\left(\mathrm{~A}, \mathbf{r}_{1}, \mathbf{r}_{2}\right)=\Phi_{\mathrm{c}}(\mathrm{~A}) \phi_{\ell_{1}}\left(\mathbf{r}_{1}\right) \phi_{\ell_{2}}\left(\mathbf{r}_{2}\right) \\
& \sigma_{\text {strip }} \Rightarrow \sigma_{\text {strip }}\left(\ell_{1} \ell_{2}\right)
\end{aligned}
$$

contribution from direct 2 N removal $\sigma_{-2 \mathrm{~N}}$
\(\left.\begin{array}{l}\frac{p particles}{q particles} \ell_{\alpha} <br>

\hline\end{array}\right\} \quad\)| $\sigma_{-2 N}=\frac{p(p-1)}{2} \sigma_{\text {strip }}\left(\ell_{\alpha} \ell_{\alpha}\right)+\frac{\mathrm{q}(\mathrm{q}-1)}{2} \sigma_{\text {strip }}\left(\ell_{\beta} \ell_{\beta}\right)$ |
| :--- |
| $+\mathrm{pq} \sigma_{\text {strip }}\left(\ell_{\alpha} \ell_{\beta}\right)$ |

D. Bazin et al., PRL 91 (2003) 012501

## Two proton knockout from neutron rich nuclei



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## Two proton removal from n-rich - (i) uncorrelated

${ }^{28} \mathrm{Mg} \rightarrow{ }^{26} \mathrm{Ne}$ (inclusive)
D. Bazin et al.,

PRL 91 (2003) 012501


Assuming $\left(1 \mathrm{~d}_{5 / 2}\right)^{4}$ then
$\sigma_{-2 \mathrm{~N}}=\frac{4(4-1)}{2} \sigma_{\text {strip }}(22) \approx 1.8 \mathrm{mb}$
Expt:1.50(1)mb

$$
\sigma_{\text {strip }}(22)=0.29 \mathrm{mb}
$$

There is now no factorisation!!

$$
\begin{aligned}
& \sigma_{\text {strip }}(02)=0.32 \mathrm{mb} \\
& \sigma_{\text {strip }}(00)=0.35 \mathrm{mb}
\end{aligned}
$$

## Two proton removal from n-rich - (ii) correlated

$$
\sigma_{\text {strip }}=\frac{1}{2 J+1} \sum_{M c} \int \mathrm{~d} \mathbf{b}\left\langle\Psi_{J M}^{(c)}\right|\left|\mathrm{S}_{\mathrm{c}}\right|^{2}\left(1-\left|\mathrm{S}_{1}\right|^{2}\right)\left(1-\left|\mathrm{S}_{2}\right|^{2}\right)\left|\Psi_{J M}^{(c)}\right\rangle
$$

$$
Q^{1} \Psi_{J M}^{(c)}=\sum_{\alpha I} C_{\alpha}^{J c}\left[\left[\phi_{j_{1} \ell_{1}}(1) \otimes \phi_{j_{2} \ell_{2}}(2)\right]_{I} \otimes \phi_{c}\right]_{J M}
$$

$j_{2} \ell_{2}$

$$
\alpha \equiv\left(j_{1} \ell_{1}, j_{2} \ell_{2}\right)
$$

## ${ }^{28} \mathrm{Mg} \rightarrow{ }^{26} \mathrm{Ne}\left(\mathrm{O}^{+}\right)$

$$
\begin{aligned}
\mathrm{C}\left(2 \mathrm{~s}_{1 / 2}\right)^{2} & =-0.305 \\
\mathrm{C}\left(1 \mathrm{~d}_{3 / 2}\right)^{2} & =-0.301 \\
\mathrm{C}\left(1 \mathrm{~d}_{5 / 2}\right)^{2} & =-1.05
\end{aligned}
$$

## There is now no factorisation!!

J.A. Tostevin et al., RNB6 proceedings, in press

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## Cross sections - correlated and uncorrelated

$$
{ }^{28} \mathrm{Mg} \rightarrow{ }^{26} \mathrm{Ne}\left(\mathrm{O}^{+}, 2^{+}, 4^{+}\right) \mathrm{S}=\sigma(\text { in } \mathrm{mb}) / 0.29
$$

|  | $\mathbf{S}_{\text {th }}$ <br> unc. | $\mathbf{S}_{\text {exp }}$ | $\mathbf{S}_{\text {th }}$ <br> corr. | $\sigma_{\text {exp }}$ <br> $(\mathrm{mb})$ | $\sigma_{\text {th }}$ <br> $(\mathrm{mb})$ |
| :--- | :---: | :--- | :--- | :--- | :---: |
| $\mathbf{0}^{+}$ | 1.33 | $\mathbf{2 . 4 ( 5 )}$ | 1.83 | $0.70(15)$ | 0.53 |
| $\mathbf{2}^{+}$ | 1.67 | $0.3(5)$ | 0.55 | $0.09(15)$ | 0.16 |
| $\mathbf{4}^{+}$ | 3.00 | $2.0(3)$ | 1.79 | $0.58(9)$ | 0.52 |
| $\mathbf{2}^{+}$ | - | $\mathbf{0 . 5 ( 3 )}$ | 0.76 | $0.15(9)$ | 0.22 |

Inclusive cross section (in mb) 1.50(10) 1.43
J.A. Tostevin, G. Podolyák, et al., in preparation

No suppression?

## Test case - earlier data from Berkeley (~10\%)

## 2 N removal from ${ }^{12} \mathrm{C}$ <br> B.A. Brown, 2N amplitudes

Kidd et al., Phys Rev C 37 (1988) 2613

## Energy/nucleon $250 \mathrm{MeV} \quad 1.05 \mathrm{GeV} \quad 2.10 \mathrm{GeV}$

| ${ }^{12} \mathrm{C} \rightarrow{ }^{10} \mathrm{Be}(2 \mathrm{p})$ | 5.82 mb | 5.33 mb | 5.15 mb |
| :--- | :--- | :--- | :--- |
| $\mathrm{S}(2 \mathrm{p})=27.18 \mathrm{MeV}$ | $\mathbf{5 . 8 8}$ | $\mathbf{5 . 3 0 ( 3 0 )}$ | $\mathbf{5 . 8 1 ( 2 9 )}$ |
|  |  |  |  |
| ${ }^{12} \mathrm{C} \rightarrow{ }^{10} \mathrm{C}(2 \mathrm{n})$ | 4.26 mb | 3.91 mb | 3.84 mb |
| $\mathrm{S}(2 \mathrm{n})=31.84 \mathrm{MeV}$ | $5.33(81)$ | $\mathbf{4 . 4 4 ( 2 4 )}$ | $\mathbf{4 . 1 1 ( 2 2 )}$ |

J.A. Tostevin et al., RNB6 proceedings, in press and in preparation

Spectroscopic Factors Workshop, Trento, $2^{\text {nd }}-12^{\text {th }}$ March 2004

