# Methods and lessons from direct reaction theory Spectroscopic Factors Workshop Trento, Italy, 2<sup>nd</sup>-12<sup>th</sup> March 2004

Jeff Tostevin Department of Physics School of Electronics and Physical Sciences University of Surrey, UK Direct (knockout, break-up, transfer) reactions – generics

- 1) Reactions in which there is a minimal rearrangement, or excitation involving a <u>very small number</u> of active (*effective*) degrees of freedom of the projectile and/or target: single-particle (sp) or collective inelastic excitation, sp or cluster transfer – 'reactions are fast'
- 2) Reaction energies are such that <u>average, effective</u> (complex) interactions can be used between the reacting constituents – regions of high level density
- Because of complex effective interactions and short mean free paths, reactions are localised / dominated by interactions in the nuclear surfaces and by hence by peripheral and grazing collisions – 'so fast'



#### Surface localisation of knockout reactions



Few-body reaction models for sp spectroscopy

There are no practical <u>many-body</u> reaction theories - we construct model 'effective' few-body models (n=2,3,4 ...)





(1) Dynamics – we need effective interactions



### Skyrme Hartree-Fock radii and densities (1)



W.A. Richter and B.A. Brown, Phys. Rev. C67 (2003) 034317



#### Skyrme Hartree-Fock radii and densities (2)



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### (2) Structure – we need sp overlap integrals

Nucleon removal from  $\Phi_{A+1}$  will leave mass A residue in the ground or an excited state - even in extreme sp model

More generally:  $\Phi_{A+1} \mathbf{r}, j$   $\Phi_{c} \mathbf{i} \Phi_{A+1} \mathbf{i} \mathbf{s}$   $F_{\ell j}^{c} (\mathbf{r}) = \langle \mathbf{r}, \Phi_{c} | \Phi_{A+1} \rangle, \ S_{N} = E_{A+1} - E_{c}$   $\mathbf{r} = \langle \mathbf{r}, \Phi_{c} | \Phi_{A+1} \rangle, \ S_{N} = E_{A+1} - E_{c}$ 

 $\int d\mathbf{r} \, |F_{\ell j}^{c}(\mathbf{r})|^{2} = C^{2}S(\ell j) \begin{cases} \text{Spectroscopic} \\ \text{factor - occupancy} \\ \text{of the state} \end{cases}$ 

Usual to write

$$\mathbf{F}_{\ell j}^{c}(\mathbf{r}) = \sqrt{\mathbf{C}^{2} \mathbf{S}(\ell j)} \phi_{0}(\mathbf{r}); \quad \int d\mathbf{r} |\phi_{0}(\mathbf{r})|^{2} = 1$$



### (2) Structure – sensitivity to overlap integrals

$$\Phi_{A+1} \stackrel{\mathbf{r}}{\ell, j} \quad F_{\ell j}^{c}(\mathbf{r}) = \langle \mathbf{r}, \Phi_{c} | \Phi_{A+1} \rangle, \ S_{N} = E_{A+1} - E_{c}$$

$$\Phi_{c} \quad F_{\ell j}^{c}(\mathbf{r}) = \sqrt{C^{2}S(\ell j)} \phi_{0}(\mathbf{r}); \quad \int d\mathbf{r} | \phi_{0}(\mathbf{r}) |^{2} = 1$$

Usually  $\phi_0(\mathbf{r})$  calculated in a simple potential model, e.g. Woods-Saxon with 'reasonable' geometry; encompasses a mass of experimental systematics – use HF theory?

Major sensitivity of cross sections in transfer, break-up and knockout reactions is (linear in)  $\langle r^2 \rangle^{1/2}$  of overlap



2N correlations in <u>non-Borromean</u> two-nucleon halo nuclei:

[A, A-1, A-2 nuclei all particle-stable]

| A                  | A-1                               | A-2               | $S_A(2N)$ | $S_A(1N)$ | $\overline{S_{A-1}(1N)}$ |
|--------------------|-----------------------------------|-------------------|-----------|-----------|--------------------------|
|                    |                                   |                   | (MeV)     | (MeV)     | (MeV)                    |
| <sup>12</sup> Be   | $^{11}\text{Be}(\frac{1}{2}^+)$   | <sup>10</sup> Be  | 3.670     | 3.170     | 0.500                    |
| $^{12}\mathrm{Be}$ | $^{11}\text{Be}(\frac{1}{2}^{-})$ | <sup>10</sup> Be  | 3.670     | 3.490     | 0.180                    |
| $^{15}\mathrm{B}$  | $^{14}B(1^{-})$                   | $^{13}\mathrm{B}$ | 3.740     | 3.510     | 0.230                    |
| $^{9}C$            | $^{8}B(g.s.)$                     | <sup>7</sup> Be   | 1.433     | 1.296     | 0.137                    |
| $^{16}\mathrm{C}$  | $^{15}C(\frac{1}{2}^{+})$         | $^{14}\mathrm{C}$ | 5.469     | 4.251     | 1.218                    |
| $^{16}\mathrm{C}$  | $^{15}C(\frac{5}{2}^{+})$         | $^{14}\mathrm{C}$ | 5.469     | 4.991     | 0.478                    |

+ many others

L.D. Blokhintsev, Bull. Acad. Russ. Sci. Phys. **65** (2001), 77. N.K. Timofeyuk, L.D. Blokhintsev and J.A. Tostevin, Phys Rev C **68** (2003) 021601(R)



### Past: DR analyses with light-ions: questions?

- How important is it to take account of the loosely bound nature of the deuteron/triton/<sup>3</sup>He and three-body breakup channels in direct reactions <u>and how can one treat</u> <u>these 'practically'</u>?
- How accurate are first-order (BA, DWBA) approaches, and the <u>spectroscopic information</u> (spectroscopic factors B(E2)'s, deformations and angular momentum assignments) deduced, as a test of structure models?
- 3) How do we treat the required single-particle overlaps of many-body wave functions? (often assumed known)
- 4) How does one best deal with sensitivity of direct reaction calculations to the assumed effective interactions?



### Present: ingredients/questions with exotic beams:

- 1) It is vital to take into account <u>non-perturbatively</u> the loosely bound nature of exotic nuclei and their break-up channels in calculations of reaction observables
- 2) How accurate is the <u>spectroscopic information</u> (spectroscopic factors) deduced from approximate fewbody models as a test of structure models?
- 3) How to / can we (?) obtain 'practically' the required single-particle overlaps from realistic many-body wave functions of the best structure theory?
- How should we best choose the assumed effective interactions between reacting constituents? – we should make best use of theoretical models – sizes, densities.



Eikonal theory reveals bare requirements

Reaction mechanism complications stripped away:

$$S_{\alpha\beta}(b) = \langle \phi_{\beta} \mid S_{c}(b_{c}) S_{v}(b_{v}) \mid \phi_{\alpha} \rangle$$

#### structure

Can use overlaps from the best available few- or manybody sp wave functions <u>if can be provided/extracted</u> in a suitable form

More generally,

$$S_{\alpha\beta}(b) = \langle \phi_{\beta} \mid S_1(b_1) S_2(b_2) \dots S_n(b_n) \mid \phi_{\alpha} \rangle$$

for any choice of 1,2 ,3, .... n clusters if  $\phi$  is available



### The continuum-coupled channels methodology



### Core fragment differential cross sections - CDCC





### Adiabatic/sudden approximation – few-bodies



#### Models for transfer reactions: e.g. (d,p)



#### Large number of semi-classical methods





Few-body eikonal model – adiabatic, trajectories Modulation function after collision,  $\omega(\mathbf{r}, \mathbf{R}) = S_c(b_c) S_v(b_v)$  $\Psi_{\kappa}^{Eik}(\mathbf{r}, \mathbf{R}) \rightarrow e^{i\mathbf{K}\cdot\mathbf{R}} S_c(b_c) S_v(b_v) \phi_{\alpha}(\mathbf{r})$ 

with  $S_c$  and  $S_v$  the <code>eikonal approximations</code> to the S-matrices for the independent scattering of c and v from the target



So, inelastic amplitude (S-matrix) for the scattering of the projectile at an impact parameter b - i.e. The amplitude that it emerges in state  $\varphi_{\beta}(\mathbf{r})$  is

$$\mathbf{S}_{\alpha\beta}(\mathbf{b}) = \langle \phi_{\beta} \mid \mathbf{S}_{c}(\mathbf{b}_{c}) \mathbf{S}_{v}(\mathbf{b}_{v}) \mid \phi_{\alpha} \rangle$$



### Non-adiabatic - but trajectory based

<u>Time-dependent (finite difference) solution</u> of the valence particle motion - assuming the heavy core, or c.m., follows a trajectory: [See: Bertsch and Esbensen, Baur and Typel, Suzuki, Melezhik and Baye]





## Transfer to the continuum approximation

Related <u>transfer to the continuum model</u> is due to Angela Bonaccorso, David Brink and others (this meeting). Using additional approximations (asymptotic forms of wave function) the time-dependent finite difference solution is avoided in favour of largely analytic approach.



Do the different theories agree for <u>the same</u> <u>structure and effective interaction inputs</u>? Theorists will (sometimes/always?) argue the details but where fair tests and comparisons have been carried out and domains of approximations overlap – answer is YES

> Structure inputs – overlaps Dynamics – effective interactions



### Transfer reactions: choice of distorting potentials

It used to be thought that <u>the best procedure is to measure the</u> <u>elastic scattering by the target nucleus of the incident projectiles</u> <u>and that by the final nucleus of the outgoing particles, all at the</u> <u>proper energies</u>, and then to fit the elastic data as well as possible with optical model potentials. These potentials were then to be used as input to DWBA calculations.

Experience has shown that <u>a more sensible procedure is to use</u> <u>distorting parameters which are appropriate for a wider range of</u> <u>target nuclei and energies.</u> Emphasis on accurate fitting of data on one or two nuclei tends to optimize the fit by selecting a peculiar (and perhaps unphysical) set of parameters.

M.H. Macfarlane and J.P. Schiffer, Nucl. Spectroscopy and Reactions, Vol B, pp 169

What should we use? - appeal to theory

#### Spectroscopic factors from individual analyses



X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), in progress



### Adiabatic model for transfer reactions: e.g. (d,p)



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### Key outcomes for transfer reactions - spectroscopy



J.D. Harvey and R.C. Johnson, Phys. Rev.C 3 (1971) 636



#### Microscopic nucleon optical potentials - JLM



J.S. Petler et al. Phys. Rev. C 32 (1985), 673



### Consistent analyses of transfer reaction data



X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), submitted

#### Spectroscopic factors – consistent inputs



X. Liu, M. Famiano, B. Tsang, W. Lynch and J.A. Tostevin (2003), submitted

# One- and two-nucleon knockout reactions

Peripheral collisions (E  $\geq$  50A MeV; MSU, RIKEN, GSI)



Events contributing will be both <u>break-up</u> and <u>stripping</u> both of which leave a mass A residue in the final state



#### Absorptive cross sections - target excitation



Related equations exist for the differential cross sections, etc.



### Choice of two-body distorting interactions

- Work at MSU has used the same energy range (60 100 MeV/nucleon) and the <u>same light nuclear target</u> (<sup>9</sup>Be) combination – systematics across many data sets
- Need nucleon <sup>9</sup>Be S-matrices over limited energy. Can use JLM or other absorptive t<sub>NN</sub> effective interaction <u>consistent with n+<sup>9</sup>Be reaction cross section</u> (split between diffractive and stripping mechanisms depends on this choice – but not their sum)
- Core-target systems are black (highly absorptive).
   Calculated using 't<sub>NN</sub>ρρ' double folding model to incorporate realistic sizes and surface geometries – gives results consistent with two-body σ<sub>R</sub>(core)



#### Weakly bound states - with good statistics



#### More strongly bound states – deep hole states



P.G. Hansen and J.A.Tostevin, ARNPS 53 (2003), 219

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#### Neutron removal from the N=16 isotones



$$\sigma_{\text{strip}} = \sigma_{-2N} = \int d\mathbf{b} \langle \phi_0 || S_c |^2 (1 - |S_1|^2) (1 - |S_2|^2) |\phi_0\rangle$$



$$\sigma_{-2N} = \frac{p(p-1)}{2} \sigma_{\text{strip}}(\ell_{\alpha}\ell_{\alpha}) + \frac{q(q-1)}{2} \sigma_{\text{strip}}(\ell_{\beta}\ell_{\beta}) + pq \sigma_{\text{strip}}(\ell_{\alpha}\ell_{\beta})$$

D. Bazin et al., PRL 91 (2003) 012501



### Two proton knockout from neutron rich nuclei



### Two proton removal from n-rich – (i) uncorrelated



D. Bazin et al., PRL **91** (2003) 012501



Assuming  $(1d_{5/2})^4$  then  $\sigma_{-2N} = \frac{4(4-1)}{2} \sigma_{strip}(22) \approx 1.8 \text{ mb}$ Expt: 1.50(1)mb

 $\sigma_{\rm strip}(22) = 0.29 \,{\rm mb}$ 

There is now no factorisation!!

 $\sigma_{\rm strip}(02) = 0.32 \,{\rm mb}$ 

$$\sigma_{\rm strip}(00) = 0.35 \,{\rm mb}$$



Two proton removal from n-rich – (ii) correlated

$$\sigma_{\text{strip}} = \frac{1}{2J+1} \sum_{Mc} \int d\mathbf{b} \, \langle \Psi_{JM}^{(c)} \, || \, \mathbf{S}_{c} \, |^{2} \, (1-|\,\mathbf{S}_{1} \, |^{2})(1-|\,\mathbf{S}_{2} \, |^{2}) \, |\, \Psi_{JM}^{(c)} \rangle$$

$$\begin{array}{c}
1 & \Psi_{JM}^{(c)} = \sum_{\alpha I} C_{\alpha}^{JIc} \left[ \left[ \phi_{j_{1}\ell_{1}}(1) \otimes \phi_{j_{2}\ell_{2}}(2) \right]_{I} \otimes \phi_{c} \right]_{JM} \\
j_{1}\ell_{1} & j_{2}\ell_{2} \\
c & j_{2}\ell_{2} \\
A & = (j_{1}\ell_{1}, j_{2}\ell_{2}) \\
\begin{array}{c}
2^{8}\text{Mg} \rightarrow 2^{6}\text{Ne}(0^{+}) \\
\hline C(2s_{1/2})^{2} &= -0.305 \\
C(1d_{3/2})^{2} &= -0.301 \\
C(1d_{5/2})^{2} &= -1.05 \\
\end{array}$$

J.A. Tostevin et al., RNB6 proceedings, in press



$$^{28}Mg \rightarrow ^{26}Ne(0^+, 2^+, 4^+) S = \sigma(\text{in mb}) / 0.29$$

|    | S <sub>th</sub> | <b>S</b> <sub>exp</sub> | S <sub>th</sub> | $\sigma_{exp}$ | $\sigma_{th}$ |
|----|-----------------|-------------------------|-----------------|----------------|---------------|
|    | unc.            |                         | corr.           | (mb)           | (mb)          |
| 0+ | 1.33            | 2.4(5)                  | 1.83            | 0.70(15)       | 0.53          |
| 2+ | 1.67            | 0.3(5)                  | 0.55            | 0.09(15)       | 0.16          |
| 4+ | 3.00            | 2.0(3)                  | 1.79            | 0.58(9)        | 0.52          |
| 2+ | -               | 0.5(3)                  | 0.76            | 0.15(9)        | 0.22          |

#### Inclusive cross section (in mb) 1.50(10) 1.43

J.A. Tostevin, G. Podolyák, et al., in preparation

No suppression?



### Test case - earlier data from Berkeley (~10%)



Kidd et al., Phys Rev C **37** (1988) 2613

|    | Energy/nucleon                         | 250 MeV  | 1.05 GeV | 2.10 GeV |
|----|--|----------|----------|----------|
|    | <sup>12</sup> C→ <sup>10</sup> Be (2p) | 5.82 mb  | 5.33 mb  | 5.15 mb  |
| 00 | S(2p)=27.18 MeV                        | 5.88     | 5.30(30) | 5.81(29) |
|    | <sup>12</sup> C→ <sup>10</sup> C (2n)  | 4.26 mb  | 3.91 mb  | 3.84 mb  |
|    | S(2n)=31.84 MeV                        | 5.33(81) | 4.44(24) | 4.11(22) |

J.A. Tostevin et al., RNB6 proceedings, in press and in preparation

