Final-State Effects in the Electromagnetic Breakup of Exotic Nuclei

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1. Coulomb dissociation method
2. Final-state interactions
3. Examples: n+core and p+core nuclei
4. Summary and conclusions

Workshop on Spectroscopic Factors
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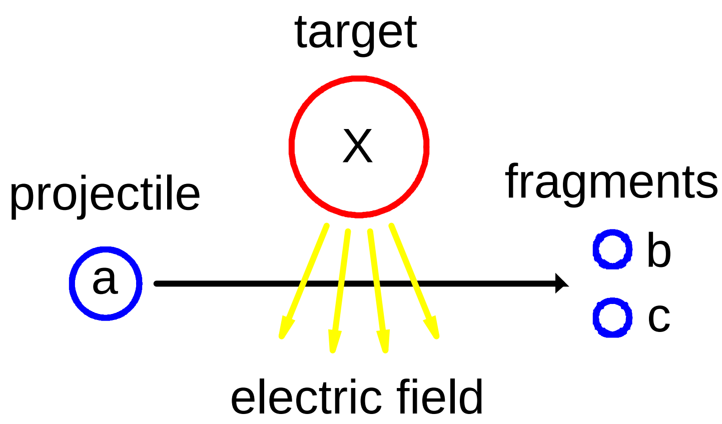
Coulomb Excitation I



idea

study properties of nucleus a by excitation with well-known electromagnetic interaction

photon source

Coulomb field of highly charged nucleus X during scattering of projectile a (virtual photons)

application to

nuclear structure physics

comparison of experimentally measured cross sections with theoretical cross sections in single-particle model

$\Rightarrow$ spectroscopic factors of ground state configurations

nuclear astrophysics

relate experimentally measured cross sections to cross sections of inverse process

$\Rightarrow$ extraction of radiative capture cross sections/astrophysical S factors at low energies

Coulomb Excitation II

adiabaticity parameter

$$\xi = \frac{\omega b}{\gamma v}$$

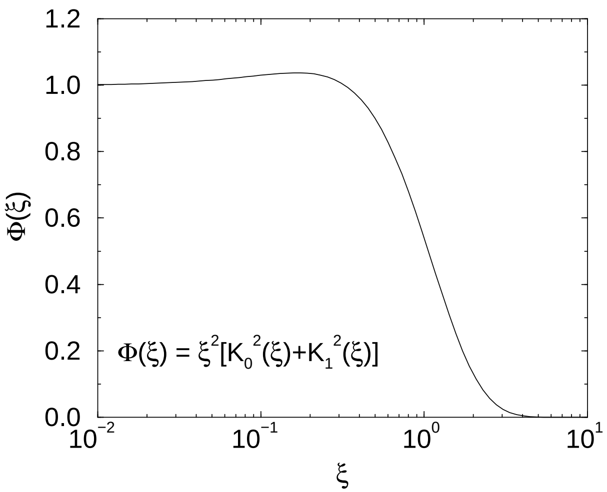
$\hbar\omega$	excitation energy
b	impact parameter
v	projectile velocity

$\xi = 0$: sudden excitation

$\xi \gg 1$: adiabatic excitation

$\xi \approx 1 \Rightarrow E_{\text{exc}}^{\max} \approx \gamma v \hbar / b$

photon spectrum (E1)



strength parameter

$$\chi = \frac{Z_X e \langle f || \mathcal{M}(\pi\lambda) || i \rangle}{\hbar v b^\lambda}$$

with target charge number Z_X
and multipole operator $\mathcal{M}(\pi\lambda)$

χ small $\Rightarrow$ first order perturbation theory sufficient

χ large $\Rightarrow$ higher order effects

structure of nucleus a:
nucleon $(b = n, p)$ + core (c)

$$\Rightarrow \langle f || \mathcal{M}(E\lambda) || i \rangle \propto Z_{\text{eff}}^{(\lambda)} e$$

with effective charge number

$$Z_{\text{eff}}^{(\lambda)} = Z_b \left(\frac{m_c}{m_b + m_c} \right)^\lambda + Z_c \left(- \frac{m_b}{m_b + m_c} \right)^\lambda$$

$\Rightarrow$ p+core: E1-E2 interference

$\Rightarrow$ n+core: E2 suppression $\propto A^{-1}$

Theory of Coulomb Excitation I

three-body final state in the continuum
$\Rightarrow$ only approximate treatment

semiclassical methods

- classical description of projectile-target relative motion (heavy targets, $\eta_{aX} = Z_a Z_X e^2/(\hbar v) \gg 1$)
- time-dependent perturbation $V(t)$ of projectile system

$\Rightarrow$ excitation amplitude a_{fi}

quantal methods

- all projectile/target combinations, energies
- time-independent scattering theory

$\Rightarrow$ T-matrix element T_{fi}

$a + X \to b + c + X$ in first order theory
$\Rightarrow$ breakup cross section

$$\frac{d^2\sigma}{dE_{bc}d\Omega_{aX}} = \frac{1}{E_\gamma}\sum_{\pi\lambda} \sigma_{\pi\lambda}(a+\gamma \to b+c)\frac{dn_{\pi\lambda}}{d\Omega_{aX}}$$

with photo absorption cross section

$$\sigma_{\pi\lambda} = \frac{\lambda+1}{\lambda}\frac{(2\pi)^3}{[(2\lambda+1)!!]^2}\left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1}\frac{dB(\pi\lambda)}{dE_{bc}}$$

$\Rightarrow$ reduced transition probability $\frac{dB(\pi\lambda)}{dE_{bc}}$
$\Rightarrow$ information on nuclear structure

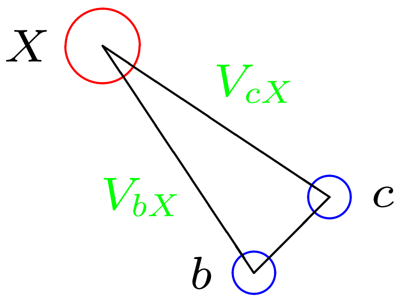
theory: virtual photon number $\frac{dn_{\pi\lambda}}{d\Omega_{aX}}$

E2 enhancement $\frac{dn_{E2}}{d\Omega_{aX}} \Big/ \frac{dn_{E1}}{d\Omega_{aX}} \approx \frac{4\hbar^2c^2}{E_\gamma^2 b^2}$

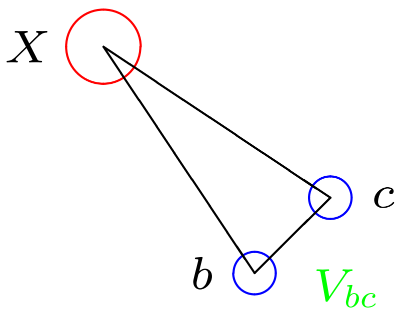
M1 suppression $\frac{dn_{M1}}{d\Omega_{aX}} \Big/ \frac{dn_{E1}}{d\Omega_{aX}} \approx \frac{v^2}{c^2}$

Final-State Interaction

interaction between
target X and fragments b, c



interaction between
fragments b and c



Coulomb interaction

- change of fragment momenta in final state
  - — small effect on c.m. momentum
  - — large effect on relative momentum

  classical picture: "post-acceleration"
- internal excitation of fragments/target (not considered here)

nuclear interaction

- breakup, stripping, absorption, . . .
- only relevant for small impact parameters (not considered here)

Coulomb interaction

- only relevant if both b and c are charged (e.g. p+core)
- always considered fully in semiclassical methods

  and prior-form quantal methods

nuclear interaction

- responsible for binding of projectile a
- determines structure of continuum (resonances)
  - — effects on excitation function
- often neglected in n-core case for continuum states

Theory of Coulomb Excitation II

final-state interaction

- between target and fragments V_{bX} and V_{cX} (Coulomb interaction ⇒ "post-acceleration", "higher-order effects")
- between fragments V_{bc} (nuclear contribution often neglected, independent of excitation process)

semiclassical methods

- higher-order perturbation theory

  (valid for small χ, small ξ)

  $a_{fi} = a_{fi}^{(1)} + a_{fi}^{(2)} + a_{fi}^{(3)} + \ldots$
- sudden approximation

  (valid for all χ, but $\xi = 0$)

  $a_{fi} = \langle f | \exp\left[\frac{1}{i\hbar}\int dt\, V(t)\right] |i\rangle$
- full treatment of projectile evolution in time

  (valid for all χ and all ξ)

– a_{fi} from coupled-channel calculation with discretized continuum

– $a_{fi} = \langle f|U|i\rangle$ with time-evolved wave function $U|i\rangle$ from solution of time-dependent Schrödinger equation (dynamical calculation)

quantal methods

- prior-form description (all orders in V_{bc})

– $T_{fi} = \langle \chi^{(-)}_{(bc)X} \chi^{(-)}_{bc} \phi_b \phi_c | V_{bX} + V_{cX} - U_{(bc)X} | \Psi^{(+)}_i \rangle$

– distorted-wave Born approximation $\Psi^{(+)}_i \to \chi^{(+)}_{aX} \phi_a$

⇒ quantal first order theory (Coulomb distorted waves)

– CDCC method $\Psi^{(+)}_i \to$ coupled-channel wave function with discretized continuum for $b + c$ states

- post-form description (all orders in V_{bX} and V_{cX})

– $T_{fi} = \langle \chi^{(-)}_{bX} \chi^{(-)}_{cX} \phi_b \phi_c | V_{bc} | \chi^{(+)}_{aX} \phi_a \rangle$ in DWBA

factorization of T_{fi} in local momentum approximation, Coulomb distorted waves ⇒ Bremsstrahlung integrals

– $T_{fi} = \langle \chi^{(-)}_{bX} \chi^{(-)}_{cX} \phi_b \phi_c | V_{bc} | \Psi^{(+)}_i \rangle$ with adiabatic approximation for $\Psi^{(+)}_i$ (energetic degeneracy of states in $b + c$ system), factorization of T_{fi} in neutron-core case

Coulomb Dissociation of Neutron-Halo Nuclei I

higher order effects in excitation from V_{bX} and V_{cX} (Coulomb only)
characteristic dependence on relevant quantities from analytical model

- **zero-range model** for excitation from s-wave ground state
  - ground state wave function $\phi_0 = \sqrt{\frac{q}{2\pi}}\frac{\exp(-qr)}{r}$ with $q = \sqrt{2\mu S_n}/\hbar$
  - scattering wave function $\phi_k^{(+)} = e^{i\vec{k}\cdot\vec{r}} - \frac{1}{q+ik}\frac{\exp(ikr)}{r}$ with momentum $\hbar\vec{k}$
- E1 excitation amplitude in sudden approximation vs. first order calculation
  - correct to all orders but $\xi = 0 \Rightarrow$ finite ξ correction from first order calculation
  - expansion in effective strength parameter $\chi_{\text{eff}} = 2Z_X Z_{\text{eff}}^{(1)} e^2/(\hbar v)$
- ratio of cross sections (LO: leading order, NLO: next-to-leading order)

$$\frac{\frac{d^2\sigma(NLO)}{dE_{\text{rel}}d\Omega}}{\frac{d^2\sigma(LO)}{dE_{\text{rel}}d\Omega}} = \left(\frac{\chi_{\text{eff}}}{qb}\right)^2 \frac{5-55x^2+28x^4}{15x^2(1+x^2)^2} \qquad \frac{\sigma(LO+NLO)}{\sigma(LO)} = 1 - \left(\frac{\chi_{\text{eff}}}{qb_{\text{min}}}\right)^2 \frac{\frac{23}{40}+18\xi_{\text{min}}^2}{1-6\ln(4\xi_{\text{min}})}$$

with $x = \frac{k}{q} = \sqrt{\frac{E_{\text{rel}}}{S_n}}$, $\xi_{\text{min}} = \frac{S_n b_{\text{min}}}{\hbar v}$ and adiabatic cut-off ($\xi_{\text{max}} = 1$)

$\Rightarrow$ reduction of differential cross section for $0.309 < x < 1.367$

$\Rightarrow$ reduction of total excitation cross section

Coulomb Dissociation of Neutron-Halo Nuclei II

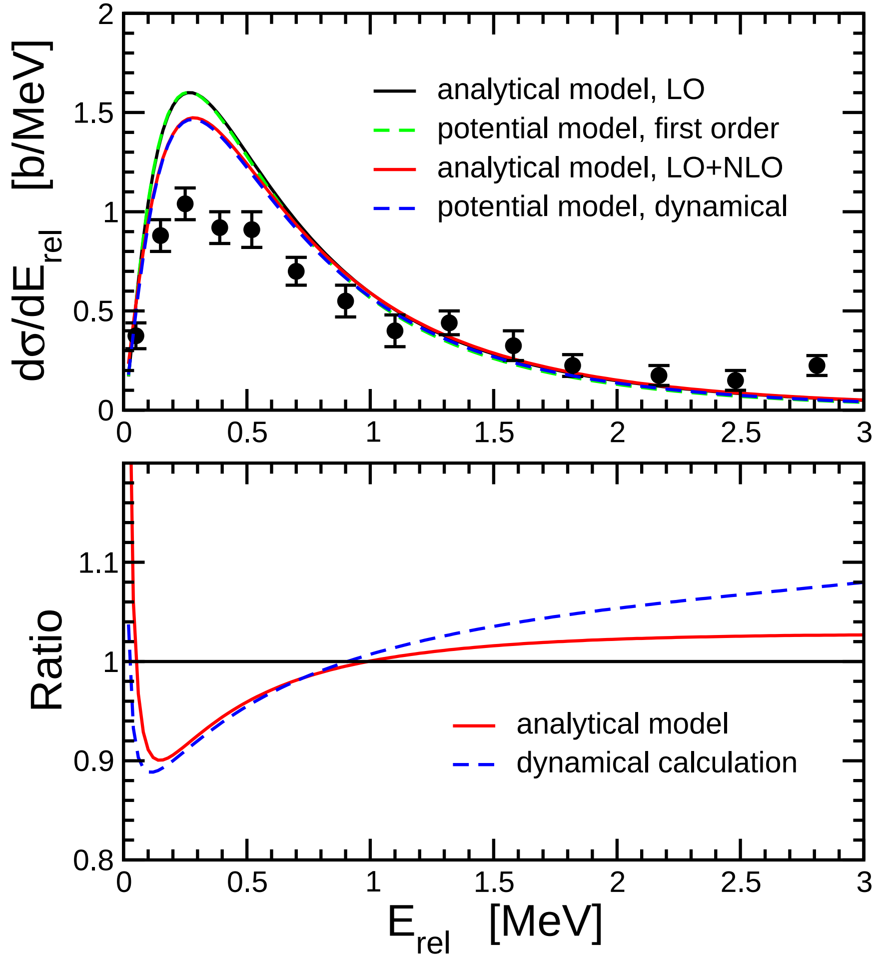
Example: ^{19}C Coulomb breakup

- experiment with 67 A MeV ^{19}C beam with Pb target at RIKEN (T. Nakamura et al., Phys. Rev. Lett. 83 (1999) 1112)
- $2s_{1/2}$ neutron with $S_n = (530 \pm 130)$ keV
- comparison of first-order result with higher-order/dynamical calculation
- finite ξ correction in analytical model
- simple potential model in dynamical calculation with adjusted potential depth for correct S_n

⇒ breakup cross section $d\sigma/dE_{\mathrm{rel}}$ in first-order/ LO and dynamical/LO+NLO calculations

⇒ ratio of dynamical/LO+NLO to first-order/LO cross sections

⇒ reduction of total cross section
analytical model: -3.3%
dynamical calculation: -3.2%

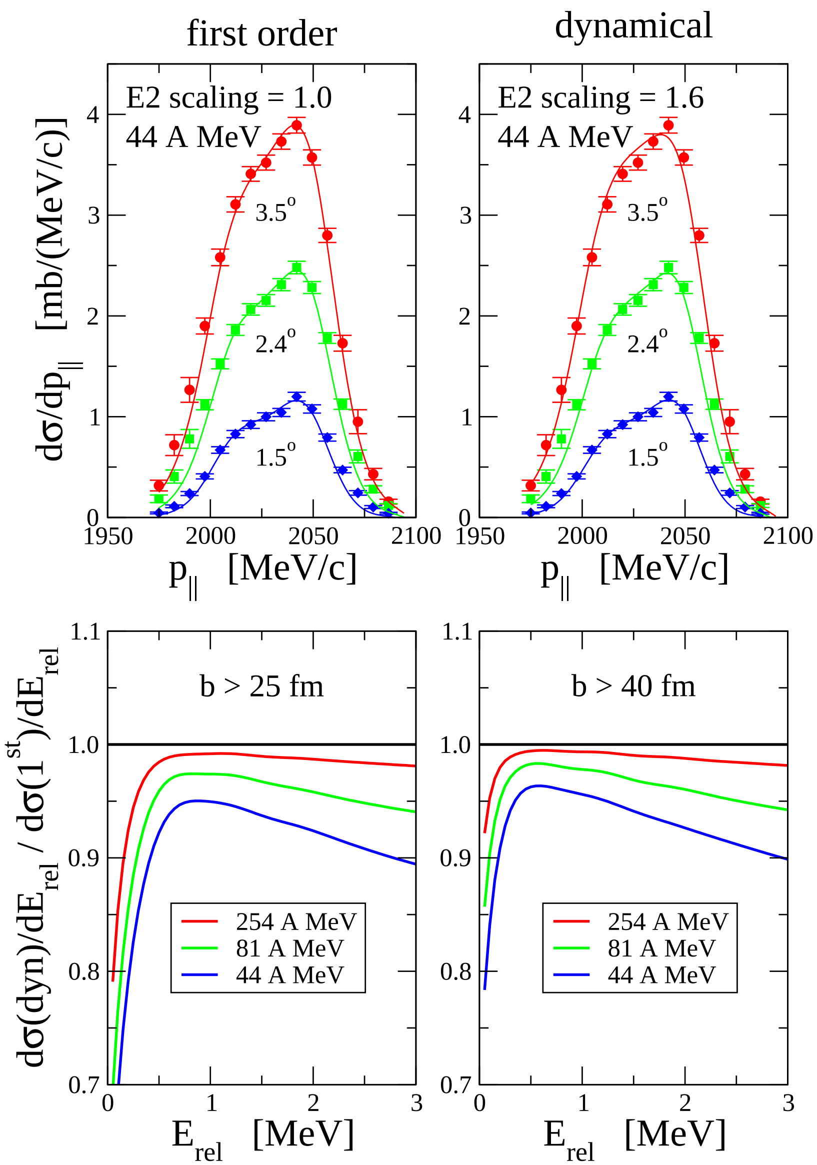


experiment vs. theory ⇒ spectroscopic factor (≈ 0.67)

Coulomb Dissociation of Proton-Halo Nuclei

Example: ^{8}B Coulomb breakup

- ground state: p-wave proton, $S_p = 137$ keV
- E1+E2 contributions to continuum breakup, interference ⇒ asymmetry in
  ^{7}Be longitudinal momentum distributions,
  ^{7}Be-p c.m. angular distributions
- experiments with ^{8}B beam on Pb target
  44/81 A MeV (MSU), 254 A MeV (GSI)
  (B. Davids et al., Phys. Rev. C 63 (2001) 065806)
  (F. Schümann et al., Phys. Rev. Lett. 90 (2003) 232501)
- calculations with simple potential model
- higher-order effects (V_{bX}, V_{cX})
  - reduction of asymmetry in longitudinal momentum distribution
    ⇒ larger E2 contribution required
  - reduction of cross section: dependence on projectile energy, impact parameter, relative energy



Reduced Transition Probabilities

- Eλ transitions at low relative energies
  $\Rightarrow$ matrix elements determined by asymptotic of wave functions $(r > R)$

- radial ground state wave function
  $f_{l_i}(r) \to C_{l_i} W_{-\eta_i, l_i+1/2}(2qr)$
  – Whittaker function $W_{-\eta_i, l_i+1/2}$
  – inverse binding length $q = \sqrt{2\mu S_b}/\hbar$
  – asymptotic normalization coefficient C_{l_i} (ANC)

- radial continuum wave function
  $f_{l_f}(r) \to \cos(\delta_{l_f}) F_{l_f}(kr) + \sin(\delta_{l_f}) G_{l_f}(kr)$
  – Coulomb wave functions F_{l_f}, G_{l_f}
  – momentum $\hbar k = \sqrt{2\mu E_{\mathrm{rel}}}$
  – nuclear phase shifts δ_{l_f} (effects of V_{bc})

$$\Rightarrow \qquad \frac{dB(E\lambda)}{dE_{\mathrm{rel}}} = \left[Z_{\mathrm{eff}}^{(\lambda)} e \right]^2 \frac{2\mu D_s}{\pi \hbar^2} \frac{|C_{l_i}|^2}{q^{2\lambda+3}} \mathcal{S}_{l_i}^{l_f}(\lambda)$$

with effective charge number $Z_{\mathrm{eff}}^{(\lambda)} = Z_b \left(\frac{m_c}{m_b+m_c}\right)^\lambda + Z_c \left(-\frac{m_b}{m_b+m_c}\right)^\lambda$, spin factor D_s and dimensionless universal shape functions

$$\mathcal{S}_{l_i}^{l_f}(\lambda) = \frac{q}{k} \left| \mathcal{I}_{l_i}^{l_f}(\lambda) \right|$$

with reduced radial integrals

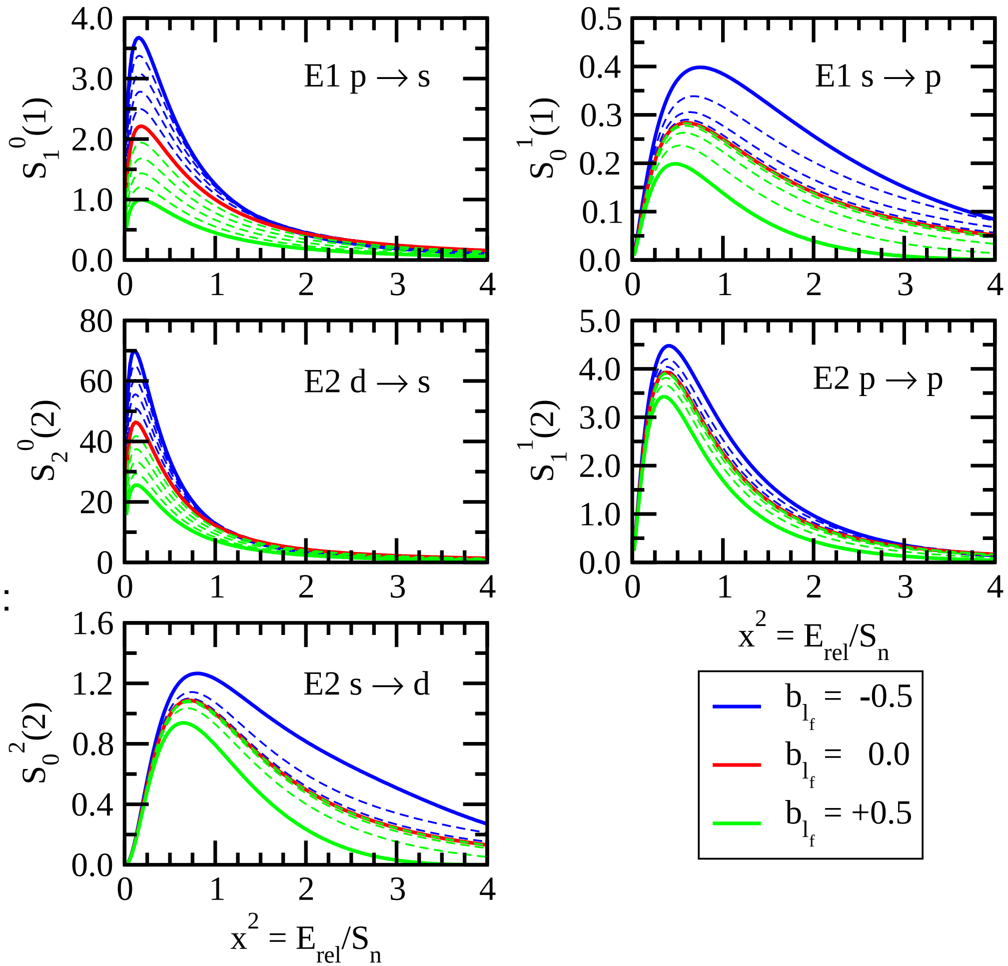
$$\mathcal{I}_{l_i}^{l_f}(\lambda) = q^{\lambda+1} \int_R^\infty dr\, r^\lambda \left[\cos(\delta_{l_f}) F_{l_f}(kr) + \sin(\delta_{l_f}) G_{l_f}(kr) \right] W_{-\eta_i, l_i+1/2}(2qr)$$

Shape Functions for Neutron+Core Nuclei

- analytical results for $\mathcal{S}_{l_i}^{l_f}(\lambda)$ depending on characteristic parameters
$$\eta = qR \text{ and } \kappa = kR$$

- effects of potential V_{bc} in continuum: reduced scattering length b_l from
$$\tan\delta_l = -(b_l x)^{2l+1}$$
with $x = \kappa/\eta = k/q = \sqrt{E_{\text{rel}}/S_n}$ (cf. effective-range expansion)

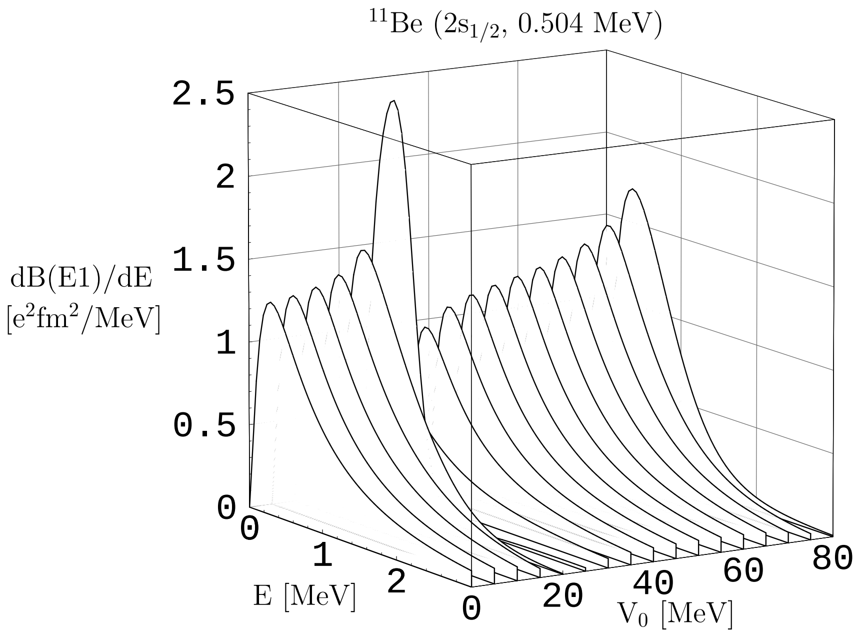
- limit $\eta \longrightarrow 0$ (reasonable for halo nuclei): $\mathcal{S}_{l_i}^{l_f}(\lambda)$ only depends on x and b_{l_f}, e.g.
$$\mathcal{S}_1^0(1) = \frac{x}{(1+x^2)^4}\frac{(3+x^2-2b_0)^2}{1+x^2 b_0^2}$$
$$\mathcal{S}_0^1(1) = \frac{x^3}{(1+x^2)^4}\frac{[2-b_1^3(1+3x^2)]^2}{1+x^6 b_1^6}$$



Reduced Transition Probabilities of Neutron+Core Nuclei I

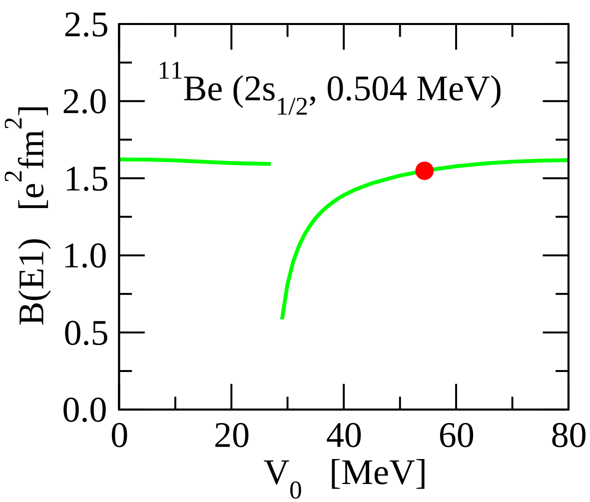
single particle model for n+core system

- wave functions of bound and scattering states calculated from Woods-Saxon potential with radius $R = r_0 A^{1/3}$, $r_0 = 1.25$ fm, diffuseness $a = 0.65$ fm and adjustable depth V_0 (spin-orbit potential neglected for simplicity)
- V_0 adjusted for ground state wave function to obtain correct neutron separation energy S_n
- $dB(E1)/dE$ for varying V_0 in continuum



examples with p-wave final state

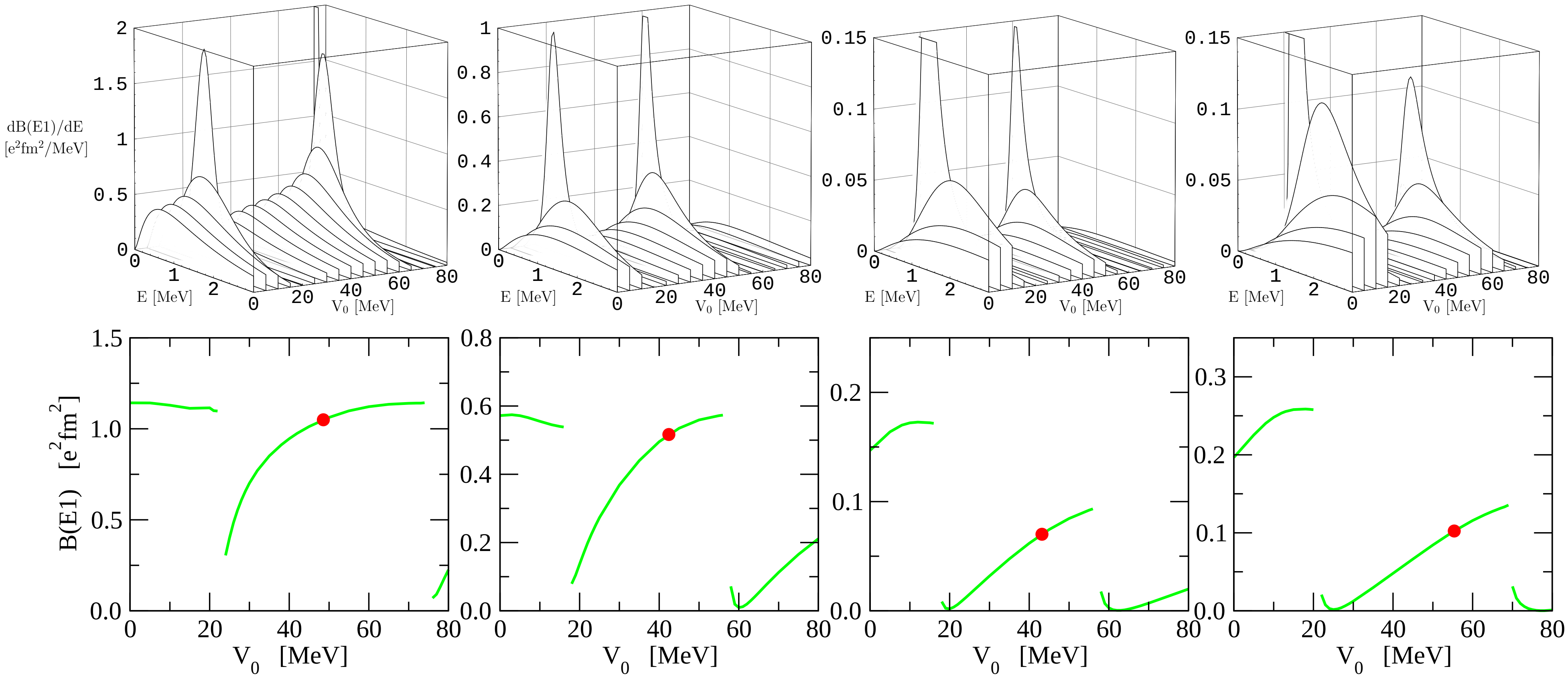
- nucleus ^{11}Be ($S_n = 0.504$ MeV) with $2s_{1/2}$ ground state (neutron halo)
- comparison with other neutron+core nuclei
  - assuming $2s_{1/2}$ ground state for ^{15}C (1.218 MeV), ^{23}O (2.74 MeV)
  - assuming $1d_{5/2}$ ground state for ^{23}O (2.74 MeV), ^{17}O (4.14 MeV)



Reduced Transition Probabilities of Neutron+Core Nuclei II

E1 transitions to p-wave final state in the continuum

^{15}C ($2s_{1/2}$, 1.218 MeV) ^{23}O ($2s_{1/2}$, 2.74 MeV) ^{23}O ($1d_{5/2}$, 2.74 MeV) ^{17}O ($1d_{5/2}$, 4.14 MeV)



s-wave ground state d-wave ground state

extraction of spectroscopic factors ?

Effect of V_{bc} on Astrophysical S Factors I

- Coulomb breakup for nuclear astrophysics

  Coulomb dissociation $X(a,bc)X$

  virtual photons $\Updownarrow$

  photo absorption $a(\gamma,b)c$

  detailed balance $\Updownarrow$

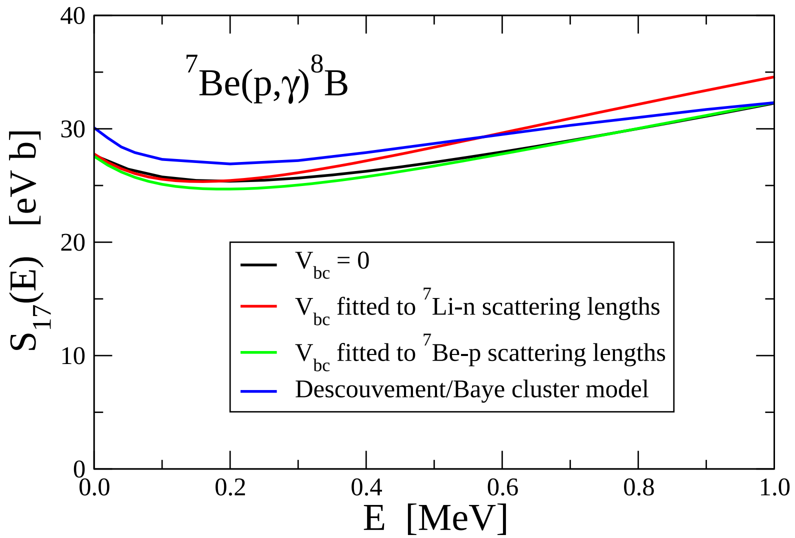
  radiative capture reaction $c(b,\gamma)a$

  $\Rightarrow$ astrophysical S factor

  $S(E) = \sigma_{\mathrm{capt}}(E)\ E\ \exp(2\pi\eta)$

- example: ^{7}Be(p,γ)^{8}B
  dominated by $E1$ transitions at low energies,
  s-wave continuum to p-wave bound state,
  contributions from two channel spins
  $S = 1,2$ with scattering lengths a_S

- effect of V_{bc} in continuum:
  change of energy dependence
  $\Rightarrow$ extrapolation of exp. data to $E = 0$



experimental scattering lengths:

- ^{7}Be+p : $a_2 = (-7 \pm 3)$ fm
  $a_1 = (25 \pm 9)$ fm

  (C. Angulo et al., Nucl. Phys. A 716 (2003) 211)

- ^{7}Li+n : $a_2 = (-3.63 \pm 0.05)$ fm
  $a_1 = (0.87 \pm 0.07)$ fm

  (L. Koester et al., Z. Phys. A 312 (1983) 81)

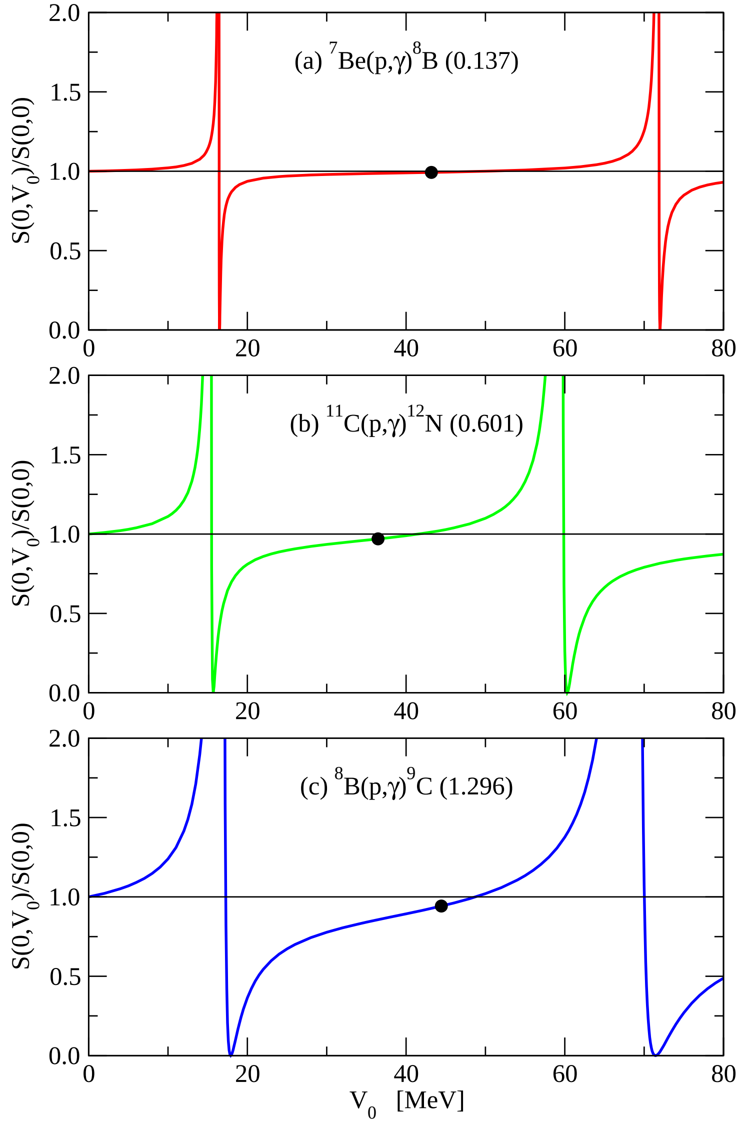
Effect of V_{bc} on Astrophysical S Factors II

- method of asymptotic normalization coefficients:
  extract ground state ANC (C_l) from transfer/nucleon-removal reactions
  $\Rightarrow$ analytical calculation of $S(0)$

- study dependence of $S(0)$ on depth V_0 of s-wave continuum potential for p+core nuclei with p-wave ground state for different proton-separation energies S_p and fixed ANC, e.g. ^{8}B, ^{12}N, ^{9}C

- absolute value of $S(E)$ depends on V_{bc} even at zero energy, sensitivity increases with S_p

- $S(0)$ not uniquely determined by ANC



Summary and Conclusions

Coulomb excitation method

- established tool for nuclear structure and astrophysics
- theory well developed and understood
- exotic nuclei ideally suited for investigation
- n+core and p+core halo nuclei are prime examples with large E1 strength at low energies
- extraction of astrophysical S factors (indirect method)

final-state effects

- various methods available for calculation
- dependence on characteristic parameters from analytical models
- higher-order effects (from target-fragment interactions) can be reduced for suitable experimental conditions (high projectile energies, large impact parameters)
- interaction between fragments has to be considered in order to extract reliable information on nuclear structure (e.g. spectroscopic factors)

References:
S. Typel and G. Baur, Nucl. Phys. A573 (1994) 486, Phys. Rev. C 49 (1994) 379, Phys. Rev. C 50 (1994) 2104, Phys. Lett. B 356 (1995) 186, Phys. Rev. C 64 (2001) 024601; S. Typel, H.H. Wolter and G. Baur, Nucl. Phys. A613 (1997) 147; S. Typel and H. H. Wolter, Z. Naturforsch. 54a (1999) 63; S. Typel and R. Shyam, Phys. Rev. C 64 (2001) 024605; B. Davids and S. Typel, Phys. Rev. C 68 (2003) 045802; S. Typel, in preparation