Soluzioni II^a prova in itinere Metodi Computaz.

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pag 1.

1) $$\int_{3-a}^{3+a} p(x)\,dx = 0{,}925$$

con $$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\mu = 3$

$\sigma = 1/2$

$$\int_{3-a}^{3+a} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz \qquad \text{con } g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

gaussiana standard

$$z_1 = \frac{3-a-\mu}{\sigma} = \frac{\cancel{3}-a-\cancel{3}}{1/2} = -2a$$

$$z_2 = \frac{3+a-\mu}{\sigma} = \frac{\cancel{3}+a-\cancel{3}}{1/2} = 2a$$

$$\int_{-2a}^{2a} g(z)\,dz = 2\int_0^{2a} g(z)\,dz = 0{,}925$$

$$\Rightarrow \int_0^{2a} g(z)\,dz = \frac{0{,}925}{2} = 0{,}4625$$

Guardando le tavole $2a = 1{,}78$

$$\Rightarrow \boxed{a = 0{,}89}$$

2) Applichiamo l'approssimazione gaussiana della binomiale. Infatti:

$\mu = Np = 14$

$\sigma^2 = Np(1-p) = 4{,}2$ $\rightarrow$ $1-p = \frac{4{,}2}{14} = 0{,}3 \Rightarrow p = 0{,}7$

$N \cdot 0{,}7 = 14 \quad \Rightarrow \quad N = 20$

La condizione per poter approssimare la binomiale con la gaussiana è che $\quad Np \gtrsim 5$

$$N(1-p) \gtrsim 5$$

In questo caso $\quad Np = 14$

$$N(1-p) = 20 \cdot (1-0{,}7) = 6$$

quindi è ok.

$\mu = 14$

$\sigma = \sqrt{4{,}2} = 2{,}05$

$$P(x \geq 13) = \int_{12{,}5}^{+\infty} p(x)\,dx = \quad \text{con } p(x) \text{ gaussiana}$$

$$= \int_{z_1}^{+\infty} g(z)\,dz = \int_{-0{,}73}^{+\infty} g(z)\,dz = \int_{-0{,}73}^{0} g(z)\,dz + \int_{0}^{+\infty} g(z)\,dz =$$

$$z_1 = \frac{12{,}5 - 14}{2{,}05} = -0{,}73$$

$$= \int_{0}^{0{,}73} g(z)\,dz + 0{,}5 = 0{,}2673 + 0{,}5 = 0{,}7673$$

$$\boxed{P(x \geq 13) = 0{,}7673}$$

3) Assioma della completezza:

$$\int_{-\infty}^{+\infty} p(x)\,dx = 1$$

$$\Rightarrow \int_{0}^{1} \left(ax^2 + \frac{1}{3}\right) dx = 1$$

$$\int_0^1 \left(a x^2 + \frac{1}{3}\right) dx = a\left[\frac{x^3}{3}\right]_0^1 + \frac{1}{3}\left[x\right]_0^1 =$$

$$= \frac{a}{3} + \frac{1}{3} \Rightarrow \frac{a}{3} + \frac{1}{3} = 1$$

$$\frac{a}{3} = 1 - \frac{1}{3}$$

$$\frac{a}{3} = \frac{2}{3}$$

$$\boxed{a = 2}$$

$$\mu = \int_{-\infty}^{+\infty} x\, p(x)\, dx = \int_0^1 x \left(2x^2 + \frac{1}{3}\right) dx = \int_0^1 \left(2x^3 + \frac{x}{3}\right) dx =$$

$$= 2\left[\frac{x^4}{4}\right]_0^1 + \frac{1}{3}\left[\frac{x^2}{2}\right]_0^1 = 2 \cdot \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{2} + \frac{1}{6} = \frac{3+1}{6}$$

$$\boxed{\mu = \frac{2}{3}}$$

$\sigma^2 = E[x^2] - \mu^2$ con $E[x^2] = \int_{-\infty}^{+\infty} x^2 p(x)\, dx =$

$$= \int_0^1 x^2 \left(2x^2 + \frac{1}{3}\right) dx = \int_0^1 \left(2x^4 + \frac{x^2}{3}\right) dx =$$

$$= 2 \cdot \left[\frac{x^5}{5}\right]_0^1 + \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{5} + \frac{1}{9} = \frac{18+5}{45} = \frac{23}{45}$$

$$\sigma^2 = \frac{23}{45} - \left(\frac{2}{3}\right)^2 = \frac{23}{45} - \frac{4}{9} = \frac{23-20}{45} \qquad \boxed{\sigma^2 = \frac{1}{15}}$$

4) Distribuzione binomiale con $p = 0,05$ $N = 4$

$P_a = P(K=1)$ con $P(k) = \frac{N!}{k!\,(N-k)!} p^k (1-p)^{N-k}$

$$P_a = \frac{4!}{1!\,3!} (0,05)^1 \cdot (1-0,05)^3 = 4 \cdot 0,05 \cdot (0,95)^3$$

$$\boxed{P_a = 0,1715}$$

$$P_b = P(k=0) = \frac{4!}{0!\,4!}(0,05)^0 \cdot (1-0,05)^4 = (0,95)^4$$

$$\boxed{P_b = 0,8145}$$

$$P_c = P(k<2) = P(k=0) + P(k=1) = 0,8145 + 0,1715$$

$$\boxed{P_c = 0,9860}$$

$N = 400 \qquad \mu = Np = 400 \cdot 0,05 = 20 \qquad \boxed{\mu = 20}$

$\sigma = \sqrt{Np(1-p)} = \sqrt{400 \cdot 0,05 \cdot (1-0,05)} \qquad \boxed{\sigma \doteq 4,36}$

5) Distribuzione di Poisson con $\mu = 1,8$

$P(k) = \frac{\mu^k}{k!} e^{-\mu}$ $\qquad k$ = n° di operai assenti in un giorno qualsiasi

$$P = P(k=3) = \frac{1,8^3}{3!} e^{-1,8} = 0,1607 \qquad \boxed{P = 0,1607}$$

6) $N = 1000 \qquad m = 21,6$

$s = 5,1$

ma poiché $N \geq 25$ allora $\sigma = s = 5,1$

$$m - A\frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A\frac{\sigma}{\sqrt{N}}$$

con $A = 2,576$

$$21,6 - 2,576 \cdot \frac{5,1}{\sqrt{1000}} \leq \mu \leq 21,6 + 2,576 \cdot \frac{5,1}{\sqrt{1000}}$$

$$21,6 - 0,4 \leq \mu \leq 21,6 + 0,4$$

$$\boxed{21,2 \leq \mu \leq 22,0}$$

7) $N = 10$ $\quad m = 21{,}6$

$s = 5{,}1$

$$m - t_{crit} \cdot \frac{s}{\sqrt{N}} \leq \mu \leq m + t_{crit} \cdot \frac{s}{\sqrt{N}}$$

$t_{crit} = 3{,}250$ (9 gradi di libertà)

$$21{,}6 - 3{,}250 \cdot \frac{5{,}1}{\sqrt{10}} \leq \mu \leq 21{,}6 + 3{,}250 \cdot \frac{5{,}1}{\sqrt{10}}$$

$$21{,}6 - 5{,}2 \leq \mu \leq 21{,}6 + 5{,}2$$

$$\boxed{16{,}4 \leq \mu \leq 26{,}8}$$

8) Metodo di fit dei minimi quadrati

	x_i	y_i	x_i^2	$x_i y_i$
	0	3,5	0	0
	1	0,5	1	0,5
	2	-2,3	4	-4,6
	3	-3,5	9	-10,5
Σ	6	-1,8	14	-14,6

$\Delta y = 0{,}5$

$N = 4$

$$A = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{\Delta}$$

$$\Delta = N \sum x_i^2 - \left(\sum x_i\right)^2$$

$$\Delta = 4 \cdot 14 - (6)^2 = 56 - 36 = 20$$

$$A = \frac{-1{,}8 \cdot 14 - 6 \cdot (-14{,}6)}{20} = \frac{-25{,}2 + 87{,}6}{20} = \frac{62{,}4}{20} = 3{,}12$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x_i^2}{\Delta} = (0{,}5)^2 \cdot \frac{14}{20} = 0{,}175$$

$$\Delta A = \sqrt{0{,}175} = 0{,}418 \simeq 0{,}4$$

$$\boxed{A = 3{,}1 \pm 0{,}4}$$

$$B = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta}$$

$$B = \frac{4 \cdot (-14,6) - 6 \cdot (-1,8)}{20} = \frac{-58,4 + 10,8}{20} = \frac{-47,6}{20}$$

$$B = -2,38$$

$$(\Delta B)^2 = (\Delta y)^2 \cdot \frac{N}{\Delta} = (0,5)^2 \cdot \frac{4}{20} = 0,05$$

$$\Delta B = \sqrt{0,05} = 0,224 \simeq 0,22$$

$$\boxed{B = -2,38 \pm 0,22}$$

9)

x_i	y_i	$f(x_i)$	$y_i - f(x_i)$
0	3,5	3,1	0,4
1	0,5	0,72	-0,22
2	-2,3	-1,66	-0,64
3	-3,5	-4,04	0,54

con $f(x_i) = A + Bx_i$

$f(x_i) = 3,1 - 2,38 \cdot x_i$

$$\chi_0^2 = \frac{\sum (y_i - f(x_i))^2}{(\Delta y)^2} = \frac{(0,4)^2 + (-0,22)^2 + (-0,64)^2 + (0,54)^2}{(0,5)^2}$$

$$\chi_0^2 = \frac{0,9096}{0,25} = 3,6384$$

$d = 4 - 2 = 2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{3,6384}{2} \simeq 1,82$$

$$P_2(\tilde{\chi}^2 \geq 1,82) \simeq 17\%$$

Quindi l'ipotesi è accettabile.