SOLUZIONI PRIMA PROVA IN ITINERE METODI COMPUTAZIONALI 15/11/2012

1) $V = \frac{4}{3}\pi a^2 c$

$$\frac{\Delta V}{V} = \sqrt{\left(\frac{\Delta a^2}{a^2}\right)^2 + \left(\frac{\Delta c}{c}\right)^2} = \sqrt{\left(\frac{2\Delta a}{a}\right)^2 + \left(\frac{\Delta c}{c}\right)^2} =$$

$$= \sqrt{\left(\frac{2 \cdot 0{,}1}{2}\right)^2 + \left(\frac{0{,}2}{1}\right)^2} = \sqrt{0{,}01 + 0{,}04} = 0{,}2236$$

$$V = \frac{4}{3}\pi \cdot 4 \cdot 1 \ \text{cm}^3 = 16{,}755 \ \text{cm}^3$$

$$\Delta V = 0{,}2236 \cdot V = 3{,}746 \ \text{cm}^3 \simeq 4 \ \text{cm}^3$$

$$\boxed{V = (17 \pm 4)\ \text{cm}^3}$$

[con $V = \frac{3}{4}\pi a^2 c^2$
$V = (9{,}4 \pm 2{,}1)\ \text{cm}^3$]

2)

$43{,}33 \pm 0{,}09$	$(4{,}333 \pm 0{,}009) \cdot 10^1$
75500 ± 400	$(7{,}55 \pm 0{,}04) \cdot 10^4$
$475{,}22 \pm 0{,}06$	$(4{,}7522 \pm 0{,}0006) \cdot 10^2$
333 ± 17	$(3{,}33 \pm 0{,}17) \cdot 10^2$
253 ± 22	$(2{,}53 \pm 0{,}22) \cdot 10^2$
$72{,}448 \pm 0{,}022$	$(7{,}2448 \pm 0{,}0022) \cdot 10^1$
430 ± 60	$(4{,}3 \pm 0{,}6) \cdot 10^2$
$25{,}55 \pm 0{,}09$	$(2{,}555 \pm 0{,}009) \cdot 10^1$
$43{,}002 \pm 0{,}012$	$(4{,}3002 \pm 0{,}0012) \cdot 10^1$

3) $\int_{-\infty}^{+\infty} p(x)\,dx = 1$ assioma della completezza

$$\int_{\pi/2}^{\pi} \left(\frac{1}{x} + a \sin x\right) dx = \left[\ln x\right]_{\pi/2}^{\pi} - a\left[\cos x\right]_{\pi/2}^{\pi} =$$

$$= \ln \pi - \ln \frac{\pi}{2} - a(-1 - 0) =$$

$= \cancel{\ln}\pi - \cancel{\ln}\pi + \ln 2 + a = \ln 2 + a$

Deve essere $\ln 2 + a = 1$

$$\boxed{a = 1 - \ln 2}$$

4) $\mu = E[x] = \int_{-\infty}^{+\infty} x\, p(x)\, dx = \int_1^2 x \cdot \left(\frac{1}{x^2} + \frac{1}{3}x\right) dx =$

$= \int_1^2 \left(\frac{1}{x} + \frac{1}{3}x^2\right) dx = \left[\ln x\right]_1^2 + \frac{1}{3} \cdot \left[\frac{x^3}{3}\right]_1^2 =$

$= \ln 2 - \underset{\overset{\shortparallel}{0}}{\ln 1} + \frac{1}{9}(8-1) = \ln 2 + \frac{7}{9}$

$\boxed{\mu = \ln 2 + \frac{7}{9}}$ $\quad \mu \simeq 1{,}47$

$\sigma^2 = E[x^2] - \mu^2$

$E[x^2] = \int_{-\infty}^{+\infty} x^2 p(x)\, dx = \int_1^2 x^2 \left(\frac{1}{x^2} + \frac{1}{3}x\right) dx =$

$= \int_1^2 \left(1 + \frac{1}{3}x^3\right) dx = \left[x\right]_1^2 + \frac{1}{3}\left[\frac{x^4}{4}\right]_1^2 =$

$= 2 - 1 + \frac{1}{12}(16-1) = 1 + \frac{15}{12} = \frac{27}{12} = \frac{9}{4}$

$\boxed{\sigma^2 = \frac{9}{4} - \left(\ln 2 + \frac{7}{9}\right)^2}$ $\quad \sigma^2 \simeq 0{,}086$

5) A = viene estratto un dado equilibrato

B = viene estratto un dado truccato

A e B sono due eventi incompatibili (mutuamente esclusivi)

a) "3" = esce la faccia n° 3

$P(3) = P(3/A) \cdot P(A) + P(3/B) \cdot P(B)$

$P(A) = \frac{56}{112} = \frac{1}{2}$ $\qquad$ $P(3/A) = \frac{1}{6}$

$P(B) = \frac{56}{112} = \frac{1}{2}$ $\qquad$ $P(3/B) = \frac{1}{10}$

Quindi

$$P(3) = \frac{1}{6} \cdot \frac{1}{2} + \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{12} + \frac{1}{20} = \frac{\cancel{8}^{\,2}}{\cancel{60}_{\,15}}$$

$$\boxed{P(3) = \frac{2}{15}} \qquad P(3) \simeq 13\%$$

b) $E[x] = \sum_{i=1}^{6} x_i P(x_i/A) P(A) + \sum_{i=1}^{6} x_i P(x_i/B) P(B) =$

$$= \frac{1}{2} \left(\sum x_i P(x_i/A) + \sum x_i P(x_i/B) \right) =$$

$$= \frac{1}{2} \left(\frac{1}{6}(1+2+3+4+5+6) + 1 \cdot \frac{1}{2} + \frac{1}{10}(2+3+4+5+6) \right) =$$

$$= \frac{1}{2} \left(3,5 + \left(\frac{1}{2} + \frac{20}{10} \right) \right) = \frac{1}{2}(3,5 + 2,5) = \frac{\cancel{6}^{\,3}}{\cancel{2}_{\,1}}$$

$$\boxed{\mu = E[x] = 3}$$

c) no $\qquad$ $P(x) = P(y)$ se si verifica A

$P(x) \neq P(y)$ in generale se si verifica B

6) $m = \frac{\sum x_i}{N}$ $\qquad$ $N = 5$

$$m = \frac{89,7 + 88 + 92,2 + 91 + 90,5}{5} = 90,28 \text{ s}$$

$$\sigma = \sqrt{\frac{\sum (x_i - m)^2}{N-1}} = 1,56 \text{ s} \qquad \frac{\sigma}{\sqrt{N}} = 0,697$$

$$\Rightarrow \boxed{T = (90,3 \pm 0,7) \text{ s}}$$

7) $\left(\frac{\Delta T}{T}\right)_{sist} = 0{,}01 \Rightarrow (\Delta T)_{sist} = 0{,}90$

$$\Delta T_{TOT} = \sqrt{(\Delta T)^2_{stat} + (\Delta T)^2_{sist}} = \sqrt{(0{,}7)^2 + (0{,}9)^2} = 1{,}1\ s$$

$$\boxed{T = (90{,}3 \pm 1{,}1)\ s}$$

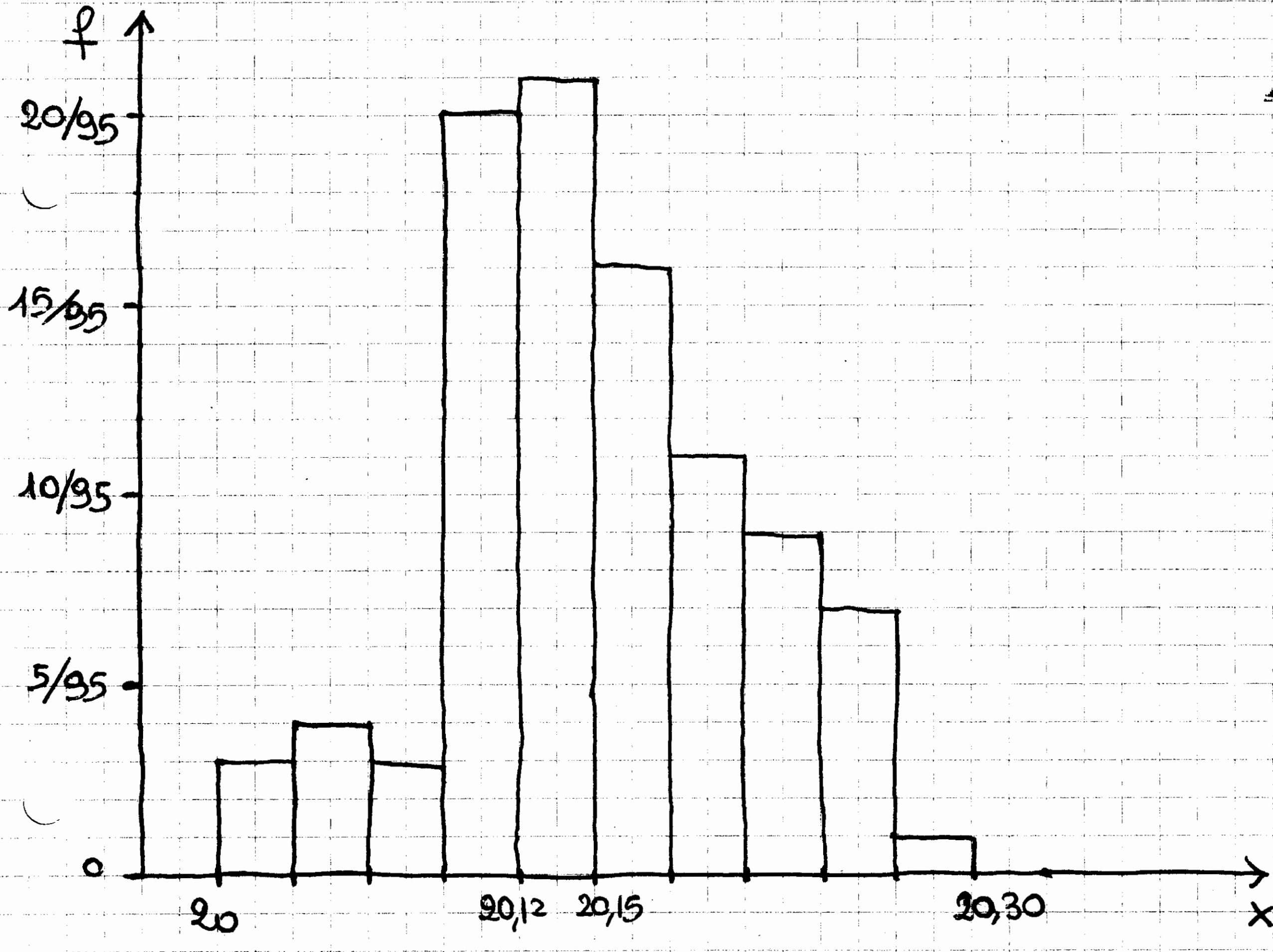
8) $x_{max} = 20{,}28$ $\quad \Delta x \simeq \frac{x_{max} - x_{min}}{\sqrt{N}} = \frac{0{,}28}{\sqrt{95}} = 0{,}029$

$x_{min} = 20$

Scelgo $\Delta x = 0{,}03$. Il n° di bins è 10

$[x_1\ ,\ x_2[$		occorrenze	frequenze f
20	20,03	3	3/95
20,03	20,06	4	4/95
20,06	20,09	3	3/95
20,09	20,12	20	20/95
20,12	20,15	21	21/95
20,15	20,18	16	16/95
20,18	20,21	11	11/95
20,21	20,24	9	9/95
20,24	20,27	7	7/95
20,27	20,30	1	1/95

Somma: 95 1



moda: $\boxed{x = 20,13}$ (è il valore con la frequenza max)

cade nell' intervallo $x \in [20,12\ ,\ 20,15[$ con $f = \frac{21}{95}$

$\frac{20,12+20,15}{2} = \boxed{20,135}$ centro dell'intervallo

mediana $m_{1/2}$ = valore di x nella tabella a metà, ovvero nella posizione 48

$\Rightarrow m_{1/2} = 20,14$