ESAME DI METODI COMPUTAZIONALI DEL 2/4/2012

1) $S = S_1 + S_2$

con $S_1 = a^2$ e $S_2 = \frac{b \cdot h}{2} = \frac{\sqrt{3}}{4} b^2$

$h = \sqrt{b^2 - \left(\frac{b}{2}\right)^2} = \frac{\sqrt{3}}{2} b$

$\frac{\Delta S_1}{S_1} = 2 \frac{\Delta a}{a} \rightarrow \Delta S_1 = 2 \frac{\Delta a}{a} \cdot S_1 = 2 \frac{\Delta a}{\cancel{a}} a^{\cancel{2}} = 2 a \Delta a$

$\frac{\Delta S_2}{S_2} = 2 \frac{\Delta b}{b} \qquad \Delta S_2 = 2 \frac{\Delta b}{b} S_2 = \frac{\sqrt{3}}{2} b^{\cancel{2}} \cdot \frac{\Delta b}{\cancel{b}} = \frac{\sqrt{3}}{2} b \Delta b$

$\Delta S = \sqrt{(\Delta S_1)^2 + (\Delta S_2)^2}$

$S = a^2 + \frac{\sqrt{3}}{4} b^2 \qquad S = 331{,}24 + 270{,}63 \ m^2$

$S = 601{,}87 \ m^2$

$\Delta S_1 = 2 \cdot 18{,}2 \cdot 0{,}5 \ m^2 \rightarrow \Delta S_1 = 18{,}2 \ m^2$

$\Delta S_2 = \frac{\sqrt{3}}{2} \cdot 25 \cdot 0{,}5 \ m^2 \rightarrow \Delta S_2 = 10{,}8 \ m^2$

$\Delta S = \sqrt{(18{,}2)^2 + (10{,}8)^2} \ m^2 = 21{,}2 \ m^2 \simeq 21 \ m^2$

$\Rightarrow \boxed{S = (602 \pm 21) \ m^2}$

2)

$32{,}346 \pm 0{,}025$	$(3{,}2346 \pm 0{,}0025) \cdot 10$
$246{,}32 \pm 0{,}05$	$(2{,}4632 \pm 0{,}0005) \cdot 10^2$
25980 ± 240	$(2{,}598 \pm 0{,}024) \cdot 10^4$
480 ± 80	$(4{,}8 \pm 0{,}8) \cdot 10^2$
$52{,}53 \pm 0{,}08$	$(5{,}253 \pm 0{,}008) \cdot 10$
444 ± 17	$(4{,}44 \pm 0{,}17) \cdot 10^2$

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3) $\int_{-\infty}^{+\infty} p(x)\,dx = 1$

$\rightarrow \int_{-\infty}^{+\infty}\left(ax^2 + \frac{a}{2}x\right)dx = a\int_0^1\left(x^2 + \frac{x}{2}\right)dx =$

$= a\left[\frac{x^3}{3}\right]_0^1 + \frac{a}{2}\left[\frac{x^2}{2}\right]_0^1 = \frac{a}{3} + \frac{a}{4}$

$\frac{a}{3} + \frac{a}{4} = 1$

$a\,\frac{4+3}{12} = 1 \qquad a\,\frac{7}{12} = 1 \qquad \boxed{a = \frac{12}{7}}$

4) $\mu = \int_{-\infty}^{+\infty} x\,p(x)\,dx$

$\mu = \int_0^1 \frac{12}{7}x\left(x^2 + \frac{x}{2}\right)dx = \frac{12}{7}\int_0^1\left(x^3 + \frac{x^2}{2}\right)dx =$

$= \frac{12}{7}\left[\frac{x^4}{4}\right]_0^1 + \frac{\cancel{12}^{6}}{7}\cdot\frac{1}{\cancel{2}_1}\left[\frac{x^3}{3}\right]_0^1 = \frac{\cancel{12}^{6}}{\cancel{28}_{14}} + \frac{\cancel{6}^{2}}{7}\cdot\frac{1}{\cancel{3}} =$

$= \frac{3}{7} + \frac{2}{7} = \frac{5}{7} \qquad \boxed{\mu = \frac{5}{7}}$

5) $m = 1 \qquad N = 10$

$\sigma = 0{,}05$

→ di tutta la popolazione

$m - A\frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A\frac{\sigma}{\sqrt{N}}$

con $A = 2{,}576$

$1 - 2{,}576\cdot\frac{0{,}05}{\sqrt{10}} \leq \mu \leq 1 + 2{,}576\cdot\frac{0{,}05}{\sqrt{10}}$

$1-0{,}04 \leq \mu \leq 1+0{,}04$

$\boxed{0{,}96 \leq \mu \leq 1{,}04}$

6) $p = 0{,}25$

$N = 5$ Distribuzione binomiale

K = n° di giocatori che centrano il bersaglio

$P(K \leq 3) = P(0) + P(1) + P(2) + P(3) = 1 - P(4) - P(5)$

$P(4) = \dfrac{5!}{4!\,1!}(0{,}25)^4 \cdot (1-0{,}25)^1 = 5(0{,}25)^4 \cdot 0{,}75$

$P(5) = \dfrac{5!}{5!\,0!}(0{,}25)^5 \cdot (1-0{,}25)^0 = (0{,}25)^5$

$P = 1 - 0{,}0124 - 0{,}0010 = \boxed{0{,}9866}$

7) $p = 0{,}007$
$N = 200$ $\mu = Np = 0{,}007 \cdot 200 = 1{,}4$

Possiamo approssimare la distribuzione binomiale con una poissoniana di media $\mu = 1{,}4$

$P(k) = \dfrac{\mu^k}{k!} e^{-\mu}$

$P(A) = P(k=3) = \dfrac{(1{,}4)^3}{3!} e^{-1{,}4} = \boxed{0{,}113}$

$P(B) = P(k>2) = 1 - P(0) - P(1) - P(2)$

$P(0) = \dfrac{1{,}4^0}{0!} e^{-1{,}4} = e^{-1{,}4} = 0{,}2466$

$P(1) = 1{,}4 \cdot e^{-1{,}4} = 0{,}3452$

$P(2) = \dfrac{(1{,}4)^2}{2!} e^{-1{,}4} = 0{,}2417$

$P(B) = 1 - 0,2466 - 0,3452 - 0,2417 = \boxed{0,1665}$

8) $p = \frac{10}{30} = \frac{1}{3}$ prob. di estrarre una pallina rossa

$N = 10$ binomiale

$\mu = Np = 10 \cdot \frac{1}{3}$ $\boxed{\mu = \frac{10}{3}}$

$\sigma = \sqrt{Np(1-p)} = \sqrt{10 \cdot \frac{1}{3}\left(1-\frac{1}{3}\right)} = \sqrt{\frac{20}{9}}$ $\boxed{\sigma = \frac{\sqrt{20}}{3}}$

9) $\left.\begin{array}{l} P(\text{pari}) = \frac{1}{2} \\ P(\text{dispari}) = \frac{1}{2} \end{array}\right\}$ su un lancio

$P = P(K=3) + P(K=4)$

$N = 4$

K = numero di volte che esce pari

$P(K)$ è binomiale con $p = \frac{1}{2} = p(\text{pari})$

$P(3) = \frac{4!}{3!\,1!}(0,5)^3 \cdot (0,5) = 4 \cdot (0,5)^4$

$P(4) = \frac{4!}{4!\,0!}(0,5)^4 \cdot (0,5)^0 = (0,5)^4$

$P = 4(0,5)^4 + (0,5)^4 = 5 \cdot (0,5)^4 = \boxed{0,3125}$

10) Metodo di fit dei minimi quadrati

x	y	x^2	xy
0	0,1	0	0
1	-0,2	1	-0,2
5	-6	25	-30
10	-10	100	-100
Σ 16	-16,1	126	-130,2

$N = 4$

$$A = \frac{\sum y \sum x^2 - \sum x \sum xy}{\Delta}$$

car $\Delta = N \sum x^2 - \left(\sum x\right)^2 = 4 \cdot 126 - (16)^2 = 248$

$$A = \frac{-16,1 \cdot 126 - 16 \cdot (-130,2)}{248} = \frac{54,6}{248} = 0,22016$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x^2}{\Delta} = (0,5)^2 \cdot \frac{126}{248} = 0,127016$$

$\Rightarrow \Delta A = 0,356 \simeq 0,4$

$$\boxed{A = 0,2 \pm 0,4}$$

$$B = \frac{N \sum xy - \sum x \sum y}{\Delta}$$

$$B = \frac{4 \cdot (-130,2) - 16 \cdot (-16,1)}{248} = \frac{-263,2}{248} = -1,0613$$

$$(\Delta B)^2 = (\Delta y)^2 \frac{N}{\Delta} = (0,5)^2 \cdot \frac{4}{248} = 0,00403$$

$\Rightarrow \Delta B = 0,0635 \simeq 0,06$

$$\boxed{B = -1,06 \pm 0,06}$$

11)

x	y	$f(x)$	$y - f(x)$	$(y - f(x))^2$
0	0,1	0,2	−0,1	0,01
1	−0,2	−0,86	0,66	0,4356
5	−6	−5,1	−0,9	0,81
10	−10	−10,4	0,4	0,16

$f(x) = A + Bx = 0,2 - 1,06 \cdot x$

$f(0) = 0,2$ $\qquad f(1) = -0,86$

$f(5) = -5,1$ $\qquad f(10) = -10,4$

$$\chi_0^2 = \frac{\sum (y_i - f(x_i))^2}{(\Delta y)^2} = \frac{1,4156}{(0,5)^2} = 5,6624$$

$d = 4 - 2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{5,6624}{2} = 2,8312$$

$$P_2\left(\tilde{\chi}^2 \geq 2,8312\right) \simeq 6,1\%$$

L'ipotesi è accettabile