1) $M = N m_g$ con $m_g = \rho \cdot \frac{4}{3}\pi r^3$ massa di 1 granello

$$N = \frac{M}{m_g} = \frac{M}{\rho \frac{4}{3}\pi r^3} = \frac{3M}{4\rho\pi r^3}$$

$$\frac{\Delta N}{N} = \sqrt{\left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta \rho}{\rho}\right)^2 + \left(\frac{\Delta r^3}{r^3}\right)} =$$

$$= \sqrt{\left(\frac{\Delta M}{M}\right)^2 + \left(\frac{\Delta \rho}{\rho}\right)^2 + \left(3\frac{\Delta r}{r}\right)^2}$$

$$\frac{\Delta N}{N} = \sqrt{\left(\frac{0,05}{2}\right)^2 + \left(\frac{0,1}{1,5}\right)^2 + 9\left(\frac{0,1}{0,5}\right)^2} = \sqrt{0,36507} \simeq 0,6042$$

$$N = \frac{3 \cdot 2}{4 \cdot 1,5 \cdot 10^{-9} \cdot \pi (0,5)^3} = 2,546 \cdot 10^9$$

$$\Delta N = 1,54 \cdot 10^9$$

$$\Rightarrow \quad \boxed{N = (2,5 \pm 1,5) \cdot 10^9}$$

2)

$22,0023 \pm 0,0012$	$(2,20023 \pm 0,00012) \cdot 10$
8248 ± 22	$(8,248 \pm 0,022) \cdot 10^3$
$527,23 \pm 0,07$	$(5,2723 \pm 0,0007) \cdot 10^2$
87600 ± 250	$(8,760 \pm 0,025) \cdot 10^4$
420 ± 40	$(4,2 \pm 0,4) \cdot 10^2$
$35,21 \pm 0,10$	$(3,521 \pm 0,010) \cdot 10$

3) $\int_{-\infty}^{+\infty} p(x)\,dx = 1$ assioma della completezza

$$\Rightarrow \int_0^1 (x^3+a)\,dx = 1$$

$$\int_0^1 x^3 dx + \int_0^1 a\,dx = \frac{1}{4}x^4\Big]_0^1 + a\,x\Big]_0^1 =$$

$$= \frac{1}{4} + a \qquad \Rightarrow \qquad \frac{1}{4} + a = 1$$

$$\boxed{a = \frac{3}{4}}$$

4) $\mu = \int_0^1 x \cdot \left(x^3 + \frac{3}{4}\right) dx =$

$$= \int_0^1 \left(x^4 + \frac{3}{4}x\right) dx = \frac{1}{5}x^5\Big]_0^1 + \frac{3}{4}\cdot\frac{1}{2}x^2\Big]_0^1 =$$

$$= \frac{1}{5} + \frac{3}{8} = \frac{8+15}{40} = \frac{23}{40} \qquad \boxed{\mu = \frac{23}{40}}$$

$$\sigma^2 = \langle x^2 \rangle - \mu^2$$

$$\langle x^2 \rangle = \int_0^1 x^2\left(x^3 + \frac{3}{4}\right) dx = \int_0^1 \left(x^5 + \frac{3}{4}x^2\right) dx =$$

$$= \frac{1}{6}x^6\Big]_0^1 + \frac{\cancel{3}^{1}}{4}\,\frac{1}{\cancel{3}_{1}}\,x^3\Big]_0^1 = \frac{1}{6} + \frac{1}{4} = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \quad \sigma^2 = \frac{1}{2} - \left(\frac{23}{40}\right)^2 = \frac{1}{2} - \frac{529}{1600} = \frac{800-529}{1600} = \frac{271}{1600}$$

$$\boxed{\sigma^2 = \frac{271}{1600}}$$

5) media $= \frac{\sum x_i}{N} = \frac{165+175+176+159+170}{5} = 169$

TABELLA ORDINATA:

159 165 170 175 176

$\downarrow$ mediana

6) Distribuzione binomiale con $N=10$ e $p=0,80$

$P_a = P(K \geq 1) = 1 - P(0)$

con $P(K) = \frac{N!}{K!(N-K)!} p^k (1-p)^{N-k}$

$P(0) = \frac{\cancel{10!}}{0!\cancel{10!}} (0,8)^0 \cdot (1-0,8)^{10} = (0,2)^{10} = 0,000000102$

$\boxed{P_a = 0,999999897}$

$P_a = P(K=10)$

$P(10) = \frac{\cancel{10!}}{\cancel{10!}\,0!} (0,8)^{10} \cdot (1-0,8)^0 = (0,8)^{10} = 0,10737$

$\boxed{P_b = 0,10737}$

7) $N = 200$

$m = \frac{48}{200} = 0,24$

$\sigma = 0,43$

$m - A\frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A\frac{\sigma}{\sqrt{N}}$

con $A = 1,96$

$1,96 \cdot \frac{\sigma}{\sqrt{N}} = 1,96 \cdot \frac{0,43}{\sqrt{200}} = 0,06$

$0,24 - 0,06 \leq \mu \leq 0,24 + 0,06$

$$\boxed{0{,}18 \leq \mu \leq 0{,}30}$$

8) A = estrarre una pallina bianca alla 1^a estrazione

B = " " " " " 2^a estrazione

$$P = P(A) \cdot P(B/A)$$

$$P(A) = \frac{\text{casi favorevoli}}{\text{casi possibili}} = \frac{2}{5}$$

$$P(B/A) = \frac{\text{casi favorevoli}}{\text{casi possibili}} = \frac{1}{4}$$

in quanto alla 2^a estrazione sono rimaste solo 4 palline, di cui, se è stata estratta una bianca alla 1^a estrazione, solo 1 è bianca.

$$\Rightarrow \quad P = \frac{\cancel{2}}{5} \cdot \frac{1}{\cancel{4}_2} \qquad \boxed{P = \frac{1}{10}}$$

9) Distribuzione dei diametri delle biglie prodotte:

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{con} \quad \mu = 0{,}8 \quad \sigma = 0{,}001$$

$$P_{\text{accettata}} = P(0{,}798 \leq x \leq 0{,}802) =$$

$$= \int_{0{,}798}^{0{,}802} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz \quad \text{con} \quad g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$z_1 = \frac{0{,}798 - 0{,}8}{0{,}001} = -2$$

$$z_2 = \frac{0{,}802 - 0{,}8}{0{,}001} = 2$$

$$P_{accettata} = \int_{-2}^{2} g(z)\,dz = 2\int_{0}^{2} g(z)\,dz =$$

$$= 2 \cdot 0{,}47725 = 0{,}9545$$

$$P_{rifiutata} = 1 - P_{accettata} = 1 - 0{,}9545 = \boxed{0{,}0455}$$

10) Metodo di fit dei minimi quadrati

	x	y	x^2	xy
	0,1	3	0,01	0,3
	0,5	3,1	0,25	1,55
	1	3,6	1	3,6
	2	4	4	8
Σ	3,6	13,7	5,26	13,45

$N = 4$

$$A = \frac{\sum y \sum x^2 - \sum x \sum xy}{\Delta}$$

$$\Delta = N\sum x^2 - \left(\sum x\right)^2$$

$$\Delta = 4 \cdot 5{,}26 - (3{,}6)^2 = 21{,}04 - 12{,}96 = 8{,}08$$

$$A = \frac{13{,}7 \cdot 5{,}26 - 3{,}6 \cdot 13{,}45}{8{,}08} = \frac{23{,}642}{8{,}08} = 2{,}92599$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x^2}{\Delta} = (0{,}1)^2 \cdot \frac{5{,}26}{8{,}08} = 0{,}0065099$$

$$\Rightarrow \Delta A = 0{,}0807 \simeq 0{,}08$$

$$\boxed{A = 2{,}93 \pm 0{,}08}$$

$$B = \frac{N\sum xy - \sum x \sum y}{\Delta}$$

$$B = \frac{4 \cdot 13{,}45 - 3{,}6 \cdot 13{,}7}{8{,}08} = \frac{4{,}48}{8{,}08} = 0{,}554455$$

$$(\Delta B)^2 = (\Delta y)^2 \cdot \frac{N}{\Delta} = (0{,}1)^2 \cdot \frac{4}{8{,}08} = 0{,}004950$$

$$\Delta B = 0{,}0703 \simeq 0{,}07$$

$$\boxed{B = 0{,}55 \pm 0{,}07}$$

11)

x	y	$A+Bx$	$(y-A-Bx_i)^2$
0,1	3	2,985	0,000225
0,5	3,1	3,205	0,011025
1	3,6	3,48	0,0144
2	4	4,03	0,0009

$$\chi_0^2 = \frac{\sum (y-A-Bx)^2}{(\Delta y)^2} = \frac{0{,}02655}{0{,}01} = 2{,}655$$

$d = 4-2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{2{,}655}{2} \cong 1{,}33$$

$$P_2\left(\tilde{\chi}^2 \geqslant 1{,}33\right) \simeq 27\%$$

L'ipotesi è quindi accettabile.