SOLUZIONI ESAME DI METODI COMPUTAZIONALI

25/9/2012

1) $\ell = 10.0 \pm 0,2$ cm
$L = 500 \pm 2$ cm

n° di mattonelle per lato $= n$

$n = \frac{L}{\ell}$ con $n \pm \Delta n$ $\qquad n = \frac{500}{10} = 50$

$$\frac{\Delta n}{n} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta \ell}{\ell}\right)^2}$$

$$\frac{\Delta n}{n} = \sqrt{\left(\frac{2}{500}\right)^2 + \left(\frac{0,2}{10}\right)^2} = 0,020$$

$$\Delta n = 50 \cdot 0,020 = 1,0$$

$$n = 50 \pm 1,0$$

il numero di mattonelle necessario è quindi

$N = n^2$ con $N \pm \Delta N$ $\qquad N = 50^2 = 2500$

$$\frac{\Delta N}{N} = 2\,\frac{\Delta n}{n}$$

$$\frac{\Delta N}{N} = 2 \cdot 0,020 = 0,04$$

$$\Delta N = 2500 \cdot 0,04 = 100$$

$$\Rightarrow \boxed{2500 \pm 100}$$

2) Area stanza $= A_s = L^2$

$$\frac{\Delta A_s}{A_s} = 2\,\frac{\Delta L}{L} = 2 \cdot \frac{2}{500} = 0,008 \qquad \boxed{0,8\%}$$

Area mattonella $= A_m = \ell^2$ (2)

$$\frac{\Delta A_m}{A_m} = 2\frac{\Delta \ell}{\ell} = 2 \cdot \frac{0{,}2}{10} = 0{,}04 \quad \boxed{4\%}$$

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3) $$T_{best} = \frac{\sum T_i}{5} = 134{,}48 \ °C = T_{medio}$$

$$\Delta T = \frac{\sigma}{\sqrt{N}}$$ deviazione standard della media

$$\sigma = \sqrt{\frac{\sum (T_i - T_{medio})^2}{N-1}} = 0{,}2588$$

$$\Delta T = \frac{0{,}2588}{\sqrt{5}} = 0{,}1157 \simeq 0{,}12 \ °C$$

$$\boxed{(134{,}48 \pm 0{,}12)\ °C}$$

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4)

$74{,}847 \pm 0{,}004$

512 ± 17

$2{,}13500 \pm 0{,}00008$

55700 ± 1300

$61{,}48 \pm 0{,}23$

$35{,}13 \pm 0{,}05$

5690 ± 290

815 ± 2

5) $p = 0,3$ binomiale $N = 3$

$K = 1, 2, 3$

$$1 - P(N=3\,,\,k=0) =$$

$$= 1 - \frac{3!}{0!\,3!}(0,3)^0 \cdot (0,7)^3 = 1 - (0,7)^3 =$$

$$= \boxed{0,657} \qquad \approx 66\%$$

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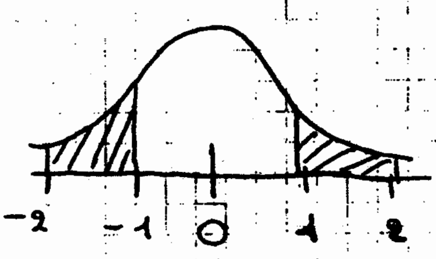
6) $\begin{cases} \mu = 172,5 \\ \sigma = 5 \end{cases}$ $\qquad p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$$P(162,5 < x < 167,5) = \int_{162,5}^{167,5} p(x)\,dx = \int_{z_1}^{z_2} g(z)\,dz$$

$$g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \qquad z_1 = \frac{162,5 - 172,5}{5} = -2$$

$$z_2 = \frac{167,5 - 172,5}{5} = -1$$

$$P = \int_{-2}^{-1} g(z)\,dz = \int_{1}^{2} g(z)\,dz = \int_{0}^{2} g(z)\,dz - \int_{0}^{1} g(z)\,dz =$$



$$= 0,4772 - 0,3413 = 0,1359$$

$$\boxed{P \simeq 0,136}$$

(4)

$$P(x > 187{,}4) = \int_{187{,}4}^{+\infty} p(x)\,dx = \int_{z_1}^{+\infty} g(z)\,dz$$

$$z_1 = \frac{187{,}4 - 172{,}5}{5} = 2{,}98$$

$$= \int_{2{,}98}^{+\infty} g(z)\,dz = 0{,}5 - \int_{0}^{2{,}98} g(z)\,dz = 0{,}5 - 0{,}4986 = 0{,}0014$$

$$\boxed{P = 0{,}0014}$$

7) $$f(x) = \begin{cases} \frac{x}{3} + a & 0 \le x \le 2 \\ 0 & x < 0, \; x > 2 \end{cases}$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$ assioma della completezza

$$\Rightarrow \int_0^2 \left(\frac{x}{3} + a\right) dx = 1$$

$$\frac{1}{3}\left[\frac{x^2}{2}\right]_0^2 + a\left[x\right]_0^2 = \frac{1}{3} \cdot 2 + 2a = \frac{2}{3} + 2a$$

$$\frac{2}{3} + 2a = 1$$

$$2a = \frac{1}{3}$$

$$\boxed{a = 1/6}$$

8) $$m = \frac{\sum x_i}{N} = 1{,}21429 \simeq 1{,}214$$ $N = 7$

$$s = \sqrt{\frac{\sum_i (x_i - m)^2}{6}} \simeq 0{,}195$$

$N-1=6$ gradi di libertà

(5)

$$t_{crit} = 2,447$$

$$m - t_{crit}\frac{s}{\sqrt{N}} \leq \mu \leq m + t_{crit}\frac{s}{\sqrt{N}}$$

$$t_{crit}\cdot\frac{s}{\sqrt{N}} = 2,447\cdot\frac{0,195}{\sqrt{7}} = 0,180$$

$$\boxed{1,034 \leq \mu \leq 1,394}$$

9) Fit dei minimi quadrati:

	x_i	y_i	Δy_i	x_i^2	$x_i y_i$
	0	-2,8	0,3	0	0
	1	-5,7	0,3	1	-5,7
	3	-9	0,3	9	-27
$\sum_i$	4	-17,5		10	-32,7

$N=3$

$$A = \frac{(\sum y_i)(\sum x_i^2) - (\sum x_i)(\sum x_i y_i)}{\Delta}$$

con $\Delta = N(\sum x_i^2) - (\sum x_i)^2$

$$\Delta = 3\cdot 10 - (4)^2 = 30 - 16 = 14$$

$$A = \frac{-17,5\cdot 10 - 4\cdot(-32,7)}{14} = \frac{-175 + 130,8}{14} = -3,1571$$

$$(\Delta A)^2 = (\Delta y)^2\frac{(\sum x_i^2)}{\Delta} = (0,3)^2\cdot\frac{10}{14} = 0,06428$$

$\Rightarrow \Delta A = 0{,}25$

$$\boxed{A = -3{,}16 \pm 0{,}25}$$

$$B = \frac{N(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{\Delta}$$

$$B = \frac{3\cdot(-32{,}7) - 4\cdot(-17{,}5)}{14} = \frac{-98{,}1 + 70}{14} = -2{,}007$$

$$(\Delta B)^2 = (\Delta y)^2 \frac{N}{\Delta} = (0{,}3)^2 \cdot \frac{3}{14} = 0{,}01928$$

$\Rightarrow \Delta B = 0{,}139 \simeq 0{,}14$

$$\boxed{B = -2{,}01 \pm 0{,}14}$$

11)

x_i	y_i	$f(x_i)$	$y_i - f(x_i)$
0	−2,8	−3,16	0,36
1	−5,7	−5,17	−0,53
3	−9	−9,19	0,19

$f(x) = A + Bx$

$f(x) = -3{,}16 - 2{,}01\,x$

$$\chi_0^2 = \sum_i \frac{(y_i - f(x_i))^2}{(\Delta y)^2} = \frac{(0{,}36)^2 + (-0{,}53)^2 + (0{,}19)^2}{(0{,}3)^2}$$

$$\Rightarrow \chi_0^2 = 4{,}962$$

Per calcolare il chi quadro ridotto, devo dividere per il n° di gradi di libertà

$$d = 3 - 2 = 1$$

n° di punti

ho calcolato A e B dai dati

(7)

$$\tilde{\chi}_0^2 = \frac{1}{d}\chi_0^2 = 4{,}962$$

Per giudicare la bontà dell' ipotesi fatta devo calcolare

$$P_1\left(\tilde{\chi}^2 \geq 4{,}962\right) \cong 2{,}5\,\%$$

guardo sulle tavole

Avendo posto la soglia del valore di accettazione al 5% si ha quindi che:

l'ipotesi <u>non è accettabile</u>.