Soluzioni esame di Metodi computazionali 18/9/2013

(1)

1) $\rho = \rho_{H_2O} \dfrac{m_r}{m_1 - m_2}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m_r}{m_r} + \frac{\Delta(m_1 - m_2)}{m_1 - m_2}$$

nota che in questo caso non viene fatta la somma in quadratura poiché m_1 e m_2 dipendono da m_r

$$\Delta(m_1 - m_2) = \Delta m_1 + \Delta m_2$$

$$\Rightarrow \frac{\Delta\rho}{\rho} = \frac{\Delta m_r}{m_r} + \frac{\Delta m_1 + \Delta m_2}{m_1 - m_2}$$

$$\frac{\Delta\rho}{\rho} = \frac{0,1}{10} + \frac{0,1 + 0,1}{260 - 256,5}$$

$$\frac{\Delta\rho}{\rho} = 0,01 + \frac{0,2}{3,5} = 0,01 + 0,057 = 0,067$$

$$\rho = 1000 \cdot \frac{10}{260 - 256,5} \ kg/m^3 = 2857,1 \ kg/m^3$$

$$\Delta\rho = 2857,1 \cdot 0,067 \ kg/m^3 = 191,4 \ kg/m^3$$

$$\Delta\rho \simeq 190 \ kg/m^3 = 1,9 \cdot 10^2 \ kg/m^3$$

$$\Rightarrow \boxed{\begin{array}{l} \rho = (2860 \pm 190) \ kg/m^3 \\ \rho = (2,86 \pm 0,19) \cdot 10^3 \ kg/m^3 \end{array}}$$

2) 790 ± 250 $\qquad (7,9 \pm 2,5) \cdot 10^2$

$25,55 \pm 0,09$ $\qquad (2,555 \pm 0,009) \cdot 10$

$45{,}001 \pm 0{,}011$ $\quad (4{,}5001 \pm 0{,}0011) \cdot 10$ 2

$82{,}89 \pm 0{,}09$ $\quad (8{,}289 \pm 0{,}009) \cdot 10$

$75{,}67 \pm 0{,}10$ $\quad (7{,}567 \pm 0{,}010) \cdot 10$

4214 ± 21 $\quad (4{,}214 \pm 0{,}021) \cdot 10^3$

$325{,}23 \pm 0{,}07$ $\quad (3{,}2523 \pm 0{,}0007) \cdot 10^2$

26590 ± 250 $\quad (2{,}659 \pm 0{,}025) \cdot 10^4$

3) $\displaystyle\int_{-\infty}^{+\infty} p(x)\, dx = 1$ ossia la completezza

$$\rightarrow \int_1^2 \left(\frac{1}{x^2} + a x\right) dx = \left[-\frac{1}{x}\right]_1^2 + a\left[\frac{x^2}{2}\right]_1^2 =$$

$$= -\frac{1}{2} + 1 + \frac{a}{2}(4-1) = \frac{1}{2} + \frac{3}{2}a$$

$$\Rightarrow \quad \frac{1}{2} + \frac{3}{2}a = 1$$

$$\frac{3}{2}a = \frac{1}{2}$$

$$3a = 1 \quad \Rightarrow \quad \boxed{a = 1/3}$$

4) $\displaystyle\mu = E[x] = \int_{-\infty}^{+\infty} x\, p(x)\, dx$

$$\mu = \int_1^2 x \cdot \left(\frac{1}{x^2} + \frac{1}{3}x\right) dx = \int_1^2 \left(\frac{1}{x} + \frac{1}{3}x^2\right) dx =$$

$$= \left[\ln x\right]_1^2 + \frac{1}{3}\left[\frac{x^3}{3}\right]_1^2 = \ln 2 - \ln 1 + \frac{1}{9}(8-1) \Rightarrow$$

$$\boxed{\mu = \ln 2 + \frac{7}{9}}$$

5) $P(A) = 1/6$ $\quad P(\overline{A}) = 1 - P(A) = 5/6$

$P(B) = 1/4$ $\quad P(\overline{B}) = 1 - P(B) = 3/4$

$P(C) = 1/3$ $\quad P(\overline{C}) = 1 - P(C) = 2/3$

$$P = P(A \text{ e } \overline{B} \text{ e } \overline{C}) + P(\overline{A} \text{ e } B \text{ e } \overline{C}) + P(\overline{A} \text{ e } \overline{B} \text{ e } C) =$$

$$= P(A) \cdot P(\overline{B}) P(\overline{C}) + P(\overline{A}) P(B) P(\overline{C}) + P(\overline{A}) P(\overline{B}) P(C) =$$

$$= \frac{1}{6} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{3} =$$

$$= \frac{6}{72} + \frac{10}{72} + \frac{15}{72} = \frac{31}{72}$$

$$\boxed{P = \frac{31}{72} \cong 0,43}$$

6) Nota che $\quad P(A \text{ e } B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$

$$\Rightarrow \quad P(A) \cdot P(B/A) = P(B) P(A/B)$$

$$\frac{1}{2} \cdot \cancel{\frac{1}{4}} = P(B) \cdot \cancel{\frac{1}{4}} \quad \Rightarrow \quad \boxed{P(B) = 1/2}$$

$$P(\overline{A}/\overline{B}) = 1 - P(A/\overline{B})$$

sapendo che $\quad P(A \text{ e } \overline{B}) = P(\overline{B}) \cdot P(A/\overline{B})$

$$\Rightarrow \quad P(A/\overline{B}) = \frac{P(A \text{ e } \overline{B})}{P(\overline{B})}$$

Inoltre $\quad P(A \text{ e } \overline{B}) = P(A) \cdot P(\overline{B}/A)$

Quindi $\quad P(\overline{A}/\overline{B}) = 1 - \dfrac{P(A) \cdot P(\overline{B}/A)}{P(\overline{B})} =$

$$= 1 - \frac{P(A) \cdot [1 - P(B/A)]}{1 - P(B)} = 1 - \frac{\cancel{1/2}\,[1 - 1/4]}{\cancel{1/2}}$$

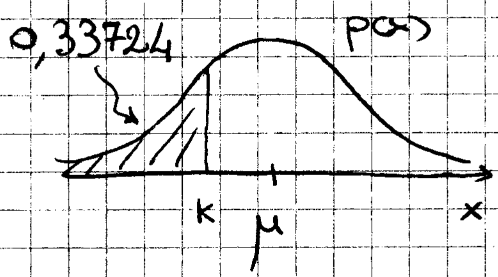
$$\Rightarrow P(\overline{A}/\overline{B}) = 1 - (1 - 1/4) = 1/4$$

$$\boxed{P(\overline{A}/\overline{B}) = 1/4}$$

7) $\mu = 1$

$\sigma^2 = 9 \rightarrow \sigma = 3$

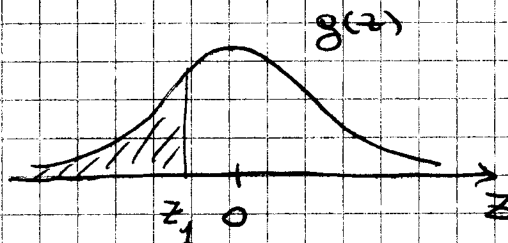
$$P(x \leq K) = \int_{-\infty}^{K} p(x)\,dx \quad \text{con} \quad p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Nota che poiché $P(x \leq K)$ è $< 0,5$ allora $K < \mu$

$$z_1 = \frac{K - \mu}{\sigma} = \frac{K - 1}{3}$$

$$\int_{-\infty}^{z_1} g(z)\,dz = 0,33724 \quad \text{con} \quad g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$



poiché $K < \mu$ allora $z_1 < 0$

$$P = \int_{z_1}^{+\infty} g(z)\,dz = 0,5 - \int_{0}^{z_1} g(z)\,dz = 0,33724$$

$$\Rightarrow \int_{0}^{z_1} g(z)\,dz = 0,5 - 0,33724 = 0,16276$$

Dalle tavole si ottiene che $z_1 = 0,42$

$$\frac{K-1}{3} = 0{,}42 \rightarrow K-1 = 3 \cdot 0{,}42$$

$$K = 1 + 1{,}26 \qquad \boxed{K = 2{,}26}$$

8) $N = 70$

$m = 91{,}3$

$s = 8{,}9$

Poiché $N \gg 1$ $\quad \sigma \cong s$

$$m - A\frac{\sigma}{\sqrt{N}} \leq \mu \leq m + A\frac{\sigma}{\sqrt{N}}$$

$A = 2{,}576$

$$A \cdot \frac{\sigma}{\sqrt{N}} = 2{,}576 \cdot \frac{8{,}9}{\sqrt{70}} = 2{,}7$$

$$91{,}3 - 2{,}7 \leq \mu \leq 91{,}3 + 2{,}7$$

$$\boxed{88{,}6 \leq \mu \leq 94}$$ in decibel

9) $N = 10$

$m = 91{,}3$

$s = 8{,}9$

$$m - t_{crit} \cdot \frac{s}{\sqrt{N}} \leq \mu \leq m + t_{crit} \cdot \frac{s}{\sqrt{N}}$$

$t_{crit} = 3{,}250$ con 9 gradi di lib.

$$t_{crit} \cdot \frac{s}{\sqrt{N}} = 3{,}25 \cdot \frac{8{,}9}{\sqrt{10}} = 9{,}1$$

$$91{,}3 - 9{,}1 \leq \mu \leq 91{,}3 + 9{,}1$$

$$\boxed{82{,}2 \leq \mu \leq 100{,}4}$$ in decibel

10) Metodo di fit dei minimi quadrati

$y = A + Bx$

	x_i	y_i	x_i^2	$x_i y_i$
	-1	0	1	0
	0	3	0	0
	1	4	1	4
	2	5	4	10
Σ	2	12	6	14

$N = 4$

$\Delta = N \sum x_i^2 - \left(\sum x_i \right)^2$

$\Delta = 4 \cdot 6 - (2)^2 = 24 - 4 = 20$

$$A = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{\Delta}$$

$$A = \frac{12 \cdot 6 - 2 \cdot 14}{20} = \frac{44}{20} = 2,2$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x_i^2}{\Delta} = (0,3)^2 \cdot \frac{6}{20} = 0,027 \rightarrow \Delta A = 0,16$$

$$\boxed{A = 2,20 \pm 0,16}$$

$$B = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta}$$

$$B = \frac{4 \cdot 14 - 2 \cdot 12}{20} = \frac{56 - 24}{20} = \frac{32}{20} = 1,6$$

$$(\Delta B)^2 = (\Delta y)^2 \cdot \frac{N}{\Delta} = (0,3)^2 \cdot \frac{4}{20} = 0,018 \rightarrow \Delta B = 0,13$$

$$\boxed{B = 1,60 \pm 0,13}$$

11)

x_i	y_i	$f(x_i)$	$y_i - f(x_i)$	$[y_i - f(x_i)]^2$
-1	0	0,6	-0,6	0,36
0	3	2,2	0,8	0,64
1	4	3,8	0,2	0,04
2	5	5,4	-0,4	0,16

$f(x_i) = A + B x_i$

$f(x_i) = 2,2 + 1,6\, x_i$

$$\chi_0^2 = \frac{\sum (y_i - f(x_i))^2}{(\Delta y)^2} = \frac{1,2}{(0,3)^2} = 13,3$$

$d = 4 - 2 = 2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{\chi_0^2}{d} = \frac{13,3}{2} = 6,66$$

$$\boxed{P_2(\tilde{\chi}^2 \geq 6,66) < 0,2\%}$$

Quindi l'ipotesi **NON è accettabile**, perché la probabilità trovata P_2 è inferiore al 5%.

NOTA: Probabilmente ciò si verifica poiché sono stati sottostimati gli errori Δy. Utilizzando $\Delta y = 0,5$ si avrebbe $\tilde{\chi}_0^2 = 2,4$

$P_2(\tilde{\chi}^2 \geq 2,4) \simeq 8\%$ quindi l'ipotesi in questo caso sarebbe stata accettabile.