SOLUZIONE ESAME DI METODI COMPUTAZIONALI 13/11/2013

1) Chiamo $C = \frac{B+b}{2}$, quindi $A = C \cdot h$

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\Delta C}{C}\right)^2 + \left(\frac{\Delta h}{h}\right)^2}$$

$$C = \frac{575+325}{2} = 450 \text{ m}$$

$$A = 450 \cdot 20 \text{ m}^2 = 9000 \text{ m}^2$$

$$\Delta C = \frac{1}{2} \cdot \sqrt{(\Delta B)^2 + (\Delta b)^2}$$

$$\Delta C = \frac{1}{2}\sqrt{4+4} \text{ m}^2 = \frac{\sqrt{8}}{2} \text{ m}^2 = \sqrt{2} \text{ m}^2$$

$$\frac{\Delta A}{A} = \sqrt{\left(\frac{\sqrt{2}}{450}\right)^2 + \left(\frac{1}{20}\right)^2} = 0{,}050$$

$$\Rightarrow \Delta A = 0{,}050 \cdot 9000 = 450 \text{ m}^2 \simeq 4 \cdot 10^2 \text{ m}^2$$

$$\boxed{A} = (9000 \pm 400) \text{ m}^2 = \boxed{(9{,}0 \pm 0{,}4) \cdot 10^3 \text{ m}^2}$$

2) $246{,}59 \pm 0{,}07$

1790 ± 250

960 ± 90

23970 ± 140

$72{,}84 \pm 0{,}08$

$32{,}60 \pm 0{,}06$

6584 ± 22

$325{,}338 \pm 0{,}015$

3) $\int_{-\infty}^{+\infty} p(x)\,dx = 1$ assioma della completezza

$$\int_1^2 \left(\frac{a}{x+3} + x^2\right) dx = a\Big[\ln(x+3)\Big]_1^2 + \frac{1}{3}\Big[x^3\Big]_1^2 =$$

$$= a\left(\ln 5 - \ln 4\right) + \frac{1}{3}(8-1) =$$

$$= a\left(\ln 5 - \ln 4\right) + \frac{7}{3}$$

deve essere $a(\ln 5 - \ln 4) + \frac{7}{4} = 1$

$\Rightarrow a(\ln 5 - \ln 4) = 1 - 7/3$

$$\boxed{a = \frac{4}{3} \cdot \frac{1}{\ln 4 - \ln 5}} \qquad a \simeq -6$$

4) $P = P(x > 43) = \int_{43}^{+\infty} p(x)\,dx$

con $p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ distribuzione gaussiana

$P = \int_{z_1}^{+\infty} g(z)\,dz$ con $g(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ gaussiana standard

$$z_1 = \frac{43-\mu}{\sigma} = \frac{43-40}{5} = \frac{3}{5} = 0{,}6$$

$$P = \int_{0,6}^{+\infty} g(z)\,dz = \int_{0}^{+\infty} g(z)\,dz - \int_{0}^{0,6} g(z)\,dz =$$

$= 0{,}5 - 0{,}2257$ guardando le tavole della gaussiana standard

$$\boxed{P = 0{,}2743}$$

5) $N = 10$ $m = 12$
$s = 2$

$$m - t_{crit} \frac{s}{\sqrt{N}} \leq \mu \leq m + t_{crit} \frac{s}{\sqrt{N}}$$

con 9 gradi di libertà
e livello di confidenza al 99% $\Big\} \rightarrow t_{crit} = 3{,}250$

$$t_{crit} \cdot \frac{s}{\sqrt{N}} = 3,25 \cdot \frac{2}{\sqrt{10}} = 2,0$$

$$12 - 2 \leqslant \mu \leqslant 12 + 2$$

$$\boxed{10 \leqslant \mu \leqslant 12}$$

6) Considerando 3 ore $\mu = 9$

$$P(K) = \frac{\mu^k}{k!} e^{-\mu}$$

$$P(K \geqslant 2) = 1 - P(0) - P(1)$$

$$P(0) = \frac{9^0}{0!} e^{-9} = e^{-9}$$

$$P(1) = \frac{9^1}{1!} e^{-9} = 9 e^{-9}$$

$$P(K \geqslant 2) = 1 - e^{-9} - 9e^{-9} = 1 - 10e^{-9} \cong 1 - 0,001$$

$$\boxed{P(K \geqslant 2) \cong 0,999}$$

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7)

	x_i	y_i	x_i^2	$x_i y_i$
	0	10	0	0
	1	8	1	8
	2	6,5	4	13
	5	-3	25	-15
$\sum$	8	21,5	30	6

Metodo di fit dei minimi quadrati

$$y = A + Bx$$

$$N = 4$$

$$\Delta y = 0,4$$

$$\Delta = N \sum x_i^2 - \left(\sum x_i\right)^2$$

$$\Delta = 4 \cdot 30 - (8)^2 = 56$$

$$A = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{\Delta}$$

$$A = \frac{21,5 \cdot 30 - 8 \cdot 6}{56} = \frac{597}{56} = 10,6607$$

$$(\Delta A)^2 = (\Delta y)^2 \cdot \frac{\sum x_i^2}{\Delta} = (0,4)^2 \cdot \frac{30}{56} = 0,08571$$

$$\Rightarrow \Delta A = 0,293 \simeq 0,29$$

$$\boxed{A = 10,66 \pm 0,29}$$

$$B = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{\Delta}$$

$$B = \frac{4 \cdot 6 - 8 \cdot 21,5}{56} = \frac{-148}{56} = -2,64286$$

$$(\Delta B)^2 = (\Delta y)^2 \cdot \frac{N}{\Delta} = (0,4)^2 \cdot \frac{4}{56} = 0,01428$$

$$\Rightarrow \Delta B = 0,107 \simeq 0,11$$

$$\boxed{B = -2,64 \pm 0,11}$$

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x_i	y_i	$f(x_i)$	$[y_i - f(x_i)]^2$
0	10	10,66	0,4356
1	8	8,02	0,0004
2	6,5	5,38	1,2544
5	-3	-2,54	0,2116

$$f(x) = A + Bx$$
$$f(x) = 10,66 - 2,64\,x$$

$$\chi_0^2 = \frac{\sum [y_i - f(x_i)]^2}{(\Delta y)^2} = \frac{1,902}{(0,4)^2} = 11,89$$

$d = 4 - 2 = 2$ gradi di libertà

$$\tilde{\chi}_0^2 = \frac{\chi_0^2}{d} = \frac{11,89}{2} = 5,945$$

$$P_2\left(\tilde{\chi}^2 \geq 5,945\right) \simeq 0,2\,\%$$

L'ipotesi **non** è accettabile (poiché $0,2\% < 5\%$)