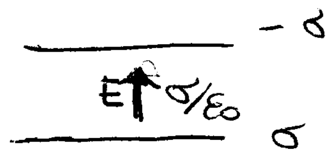
Soluzione III^a Verifica in itinere 28/5/09

ESERCIZIO 1

1) Utilizzando il principio di sovrapposizione

$$\vec{E}_0 = \vec{E}_{fogli} + \vec{E}_{anello}$$

$$\vec{E}_{fogli} = \frac{\sigma}{\varepsilon_0}\hat{z}$$



campo all'interno di un condensatore

$\vec{E}_{anello} = 0$ perché il punto O è nel centro dell'anello

$$\Rightarrow \boxed{\vec{E}_0 = \frac{\sigma}{\varepsilon_0}\hat{z}}$$

$$\frac{\sigma}{\varepsilon_0} = \frac{7\times\cancel{10^{-12}}}{8{,}85\cdot\cancel{10^{-12}}}\ \frac{V}{m} = 0{,}79\ \frac{V}{m}$$

$$\vec{E}_0 = \left(0{,}79\ \frac{V}{m}\right)\hat{z} \qquad \text{(2 cifre significative)}$$

Nel punto A $\quad \vec{E}_{fogli} = \frac{\sigma}{\varepsilon_0}\hat{z}$

mentre $\quad \vec{E}_{anello}(z) = \dfrac{Q z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}\hat{z}$

con $z = d/4$ nel punto A

$$\boxed{\vec{E}_A = \left[\frac{\sigma}{\varepsilon_0} + \frac{\frac{d}{4} Q}{4\pi\varepsilon_0\left(\frac{d^2}{16}+R^2\right)^{3/2}}\right]\hat{z}}$$

$$\vec{E}_A = (0{,}79 + 0{,}35)\frac{V}{m}\hat{z} = 1{,}14\ \frac{V}{m}\hat{z}$$

$$\vec{E}_A = \left(1{,}1\ \frac{V}{m}\right)\hat{z} \qquad \text{(2 cifre signific.)}$$

2) $\Delta V = V(B) - V(A)$

Principio di sovrapposizione:

$$\Delta V = \Delta V_{fogli} + \Delta U_{anello}$$

$$\Delta V_{fogli} = \int_B^A \frac{\sigma}{\varepsilon_0}\, dz = \frac{\sigma}{\varepsilon_0} \int_{-d/4}^{+d/4} dz = \frac{\sigma}{\varepsilon_0} \frac{d}{2}$$

$\Delta V_{anello} = 0$ per simmetria

verifica:

$$V_{anello}(z) = \frac{Q}{4\pi \varepsilon_0 \sqrt{z^2 + R^2}}$$

dipende da z^2

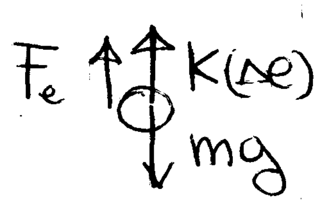
$A \equiv (0, 0, +d/4)$

$B \equiv (0, 0, -d/4)$

$$\Rightarrow \boxed{\Delta V = V(B) - V(A) = \frac{\sigma d}{2\varepsilon_0}}$$

$\Delta V = 7{,}9 \times 10^{-3}\ V$ (2 cifre signif.)

3) All'equilibrio, la somma delle forze sulla massa $= \phi$.



F_e = modulo della forza elettrostatica

$F_e = |\vec{F}_e|$

$\vec{F}_e = q\vec{E}_A$

Δe = allungamento della molla

$\Delta e = d/4$

$$\Rightarrow \quad qE_A + \frac{Kd}{4} - mg = 0$$

$$\Rightarrow \quad K = \frac{4mg}{d} - \frac{4qE_A}{d}$$

$$\boxed{K = \frac{4mg}{d} - \frac{4q}{d}\left[\frac{\sigma}{\varepsilon_0} + \frac{d/4\,Q}{4\pi\varepsilon_0\left(\frac{d^2}{16}+R^2\right)^{3/2}}\right]}$$

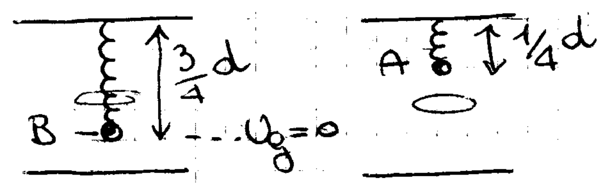
$$K = \left(19{,}62 - \frac{4\cdot 3\cdot 10^{-4}}{2\cdot 10^{-2}}\cdot[6{,}49]\right)\frac{N}{m}$$

$$K = (19{,}62 - 0{,}39)\frac{N}{m} = 19{,}23\,\frac{N}{m}$$

$$K = 19\,\frac{N}{m}$$ (2 cifre signif.)

4) Conservazione dell'energia meccanica + elettromagnetica

$$E_B = E_A$$



$$E_B = qV(B) + \frac{1}{2}K\left(\frac{3}{4}d\right)^2$$

$$E_A = \frac{1}{2}mv^2 + qV(A) + \frac{1}{2}K\left(\frac{d}{4}\right)^2 + mgh$$

con $h = \frac{d}{2}$

$$\Rightarrow \frac{1}{2}mv^2 + qV(A) + \frac{1}{2}K\frac{d^2}{16} + \frac{mgd}{2} = qV(B) + \frac{1}{2}K\frac{9d^2}{16}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}K\frac{8d^2}{16} + q\left(V(B)-V(A)\right) - mg\frac{d}{2}$$

$$v = \sqrt{\frac{K}{m}\frac{d^2}{2} + \frac{2q}{m}\left(V(B)-V(A)\right) - gd}$$

$$v = \sqrt{\frac{Kd^2}{2m} + \frac{\cancel{2}q}{m}\cdot\frac{\sigma d}{\cancel{2}\varepsilon_0} - gd}$$

$$\boxed{v=\sqrt{\frac{kd^2}{2m}+\frac{\sigma q d}{\varepsilon_0 m}-gd}}$$

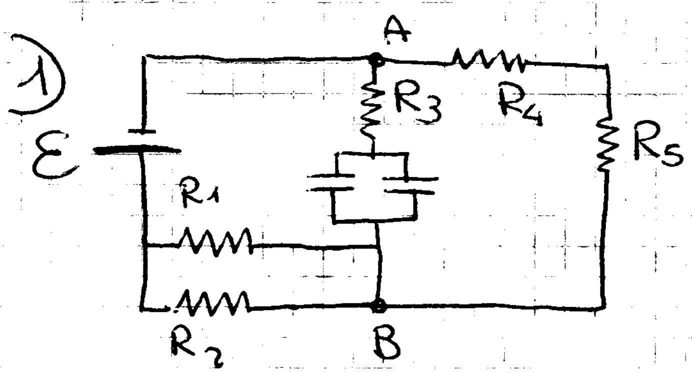
$$v=\sqrt{\frac{19{,}23\cdot 4\cdot 10^{-4}}{2\cdot 10^{-2}}+\frac{7\cdot \cancel{10^{-12}}\cdot 3\cdot 10^{-4}\cdot 2\cancel{\cdot 10^{-2}}}{8{,}85\cdot \cancel{10^{-12}}\cdot \cancel{10^{-2}}}-9{,}81\cdot 2\cdot 10^{-2}}\ \frac{m}{s}$$

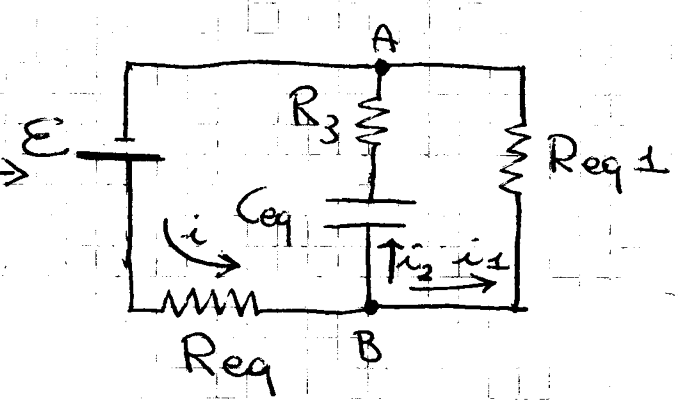
$$v=\sqrt{\left(38{,}46\cdot 10^{-2}+4{,}74\cdot 10^{-4}-19{,}62\cdot 10^{-2}\right)}\ \frac{m}{s}$$

$$v=43{,}3\cdot 10^{-2}\ \frac{m}{s}$$

$$v=43\ \frac{cm}{s}$$ (2 cifre signif.)

ESERCIZIO 2

1) 

→ 

con $R_{eq}=R_1 /\!/ R_2$

$$\rightarrow \frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$$

$$\frac{1}{R_{eq}}=\frac{1}{2}+\frac{1}{6}=\frac{2}{3} \rightarrow R_{eq}=\frac{3}{2}\,\Omega$$

$$\boxed{R_{eq}=1{,}5\,\Omega}$$

$$R_{eq_1}=R_4+R_5 \qquad \boxed{R_{eq_1}=2{,}0\,\Omega}$$

eq1) $i = i_1 + i_2$

eq2) $\mathcal{E} - R_{eq}\, i - R_{eq_1} \cdot i_1 = 0$ considerando la maglia più esterna

$V_B - V_A = i_2 R_3 + \underbrace{\Delta V}_{\text{ai capi di } C_{eq}}$

$V_B - V_A = i_1 R_{eq_1}$

nota che $\Delta V = 0$ perché a $t=0$ il condensatore è scarico

$\Rightarrow V_B - V_A = i_2 R_3$

da cui $i_2 R_3 = i_1 R_{eq_1}$ eq 3)

$\Rightarrow i_2 = \frac{R_{eq_1}}{R_3} i_1$ sostituisco in eq2)

$$\mathcal{E} - R_{eq}\left(i_1 + \frac{R_{eq_1}}{R_3} i_1\right) - R_{eq_1} i_1 = 0$$

da cui ricavo i_1

$$\mathcal{E} - R_{eq}\left(i_1 + \frac{2}{6} i_1\right) - R_{eq_1} i_1 = 0$$

$$\mathcal{E} - \left(\frac{4}{3} R_{eq} + R_{eq_1}\right) i_1 = 0$$

$$i_1 = \frac{6}{\frac{4}{3} \cdot 1{,}5 + 2} \text{ A}$$

$i_1 = \frac{6}{4}$ A

$i_1 = 1{,}5$ A

e quindi $i_2 = \frac{1}{3} i_1 = \frac{1}{3} \cdot 1{,}5\text{A} = 0{,}50$ A

La potenza dissipata a $t=0$ su R_3 è data da: $P_3 = R_3 \cdot (i_2)^2$

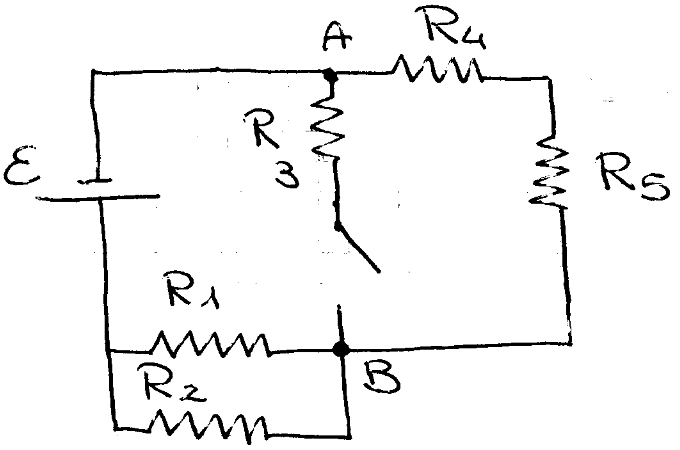
$\rightarrow$ corrente che scorre in R_3

$P_3 = 6 \cdot (0{,}5)^2$ W

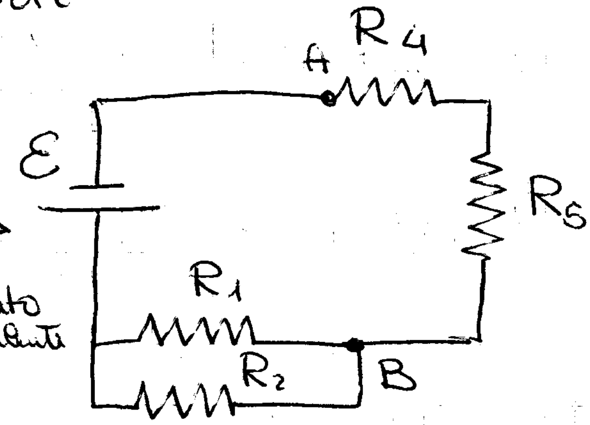
NOTA che $i = i_1 + i_2 = 2{,}0$ A

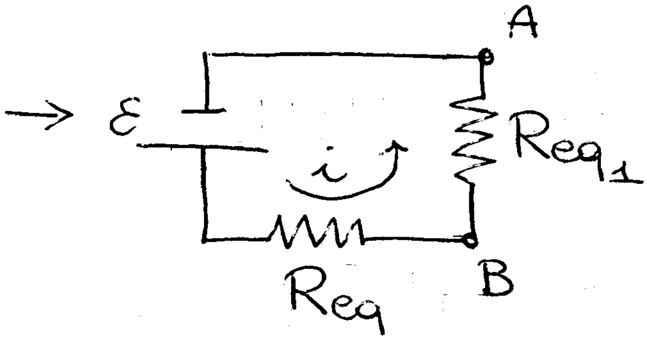
$\boxed{P_3 = 1,5\ W}$ 2 cifre significative

2) Stato stazionario: i condensatori corrispondono a interruttori aperti



circuito equivalente





con $\frac{1}{Req} = \frac{1}{R_1} + \frac{1}{R_2}$

$Req = 1,5\ \Omega$

e con $Req_1 = R_4 + R_5 = 2,0\ \Omega$

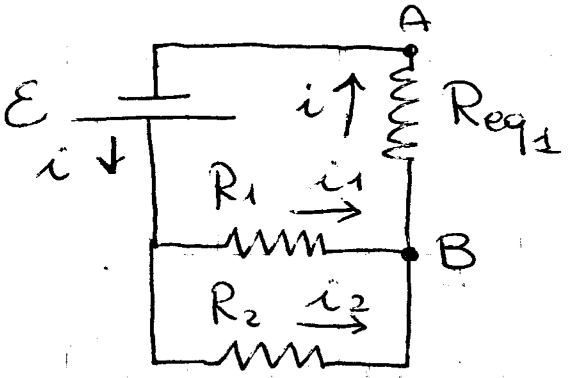
$$\mathcal{E} - Req\, i - Req_1\, i = 0$$

$$\Rightarrow \quad i = \frac{\mathcal{E}}{Req + Req_1} \qquad i = \frac{6}{1,5+2}\ A$$

$$i = \frac{6}{3,5}\ A = 1,714\ A$$

nota che allo stato stazionario la corrente totale i è $\neq$ dall'istante $t=0$

Considero questo circuito equivalente:



$$i = i_1 + i_2$$

$$R_1 i_1 = R_2 i_2$$

essendo R_1 e R_2 in parallelo

$$\rightarrow \quad i_1 = \frac{R_2}{R_1} i_2 \qquad \rightarrow \quad i_1 = \frac{6}{2} i_2 = 3 i_2$$

Quindi

$$i = i_2 + 3 i_2$$
$$i = 4 i_2$$

$$\rightarrow \quad i_2 = \frac{1}{4} i \qquad \text{ovvero} \qquad i_2 = \frac{1{,}714}{4} \text{ A}$$

$$i_2 = 0{,}428 \text{ A} \qquad \boxed{i_2 = 0{,}43 \text{ A}}$$ 2 cifre significative

3)

$$V_B - V_A = Req_1 \cdot i$$

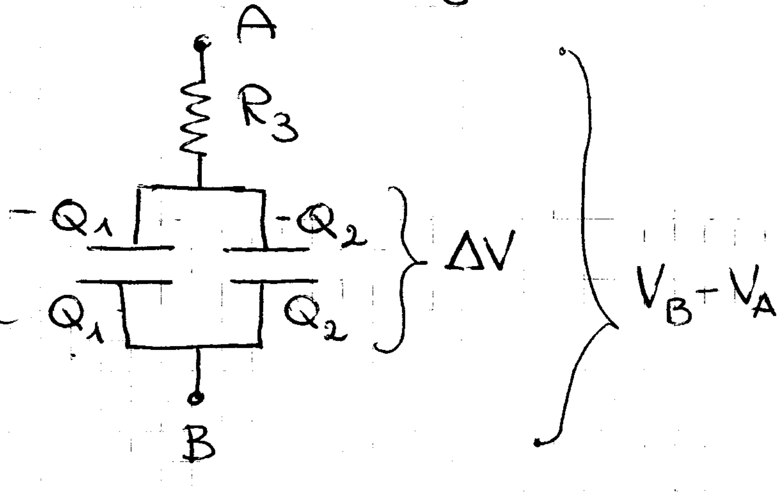
$$V_B - V_A = 2 \cdot 1{,}714 \text{ V} = 3{,}429 \text{ V}$$

$$\boxed{V_B - V_A = 3{,}4 \text{ V}}$$

4) Poiché C_1 e C_2 sono in parallelo hanno la stessa d.d.p. tra le armature pari a ΔV

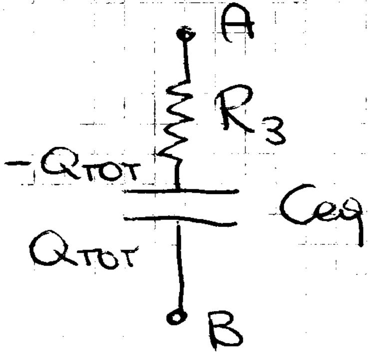
$$C_1 = \frac{Q_1}{\Delta V} \qquad C_2 = \frac{Q_2}{\Delta V}$$

Inoltre, poiché nello stato stazionario non scorre corrente in R_3, si ha che $\Delta V = V_B - V_A$



Quindi
$$\begin{cases} \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \\ C_1 = \dfrac{Q_1}{(V_B - V_A)} \longrightarrow Q_1 = C_1(V_B - V_A) \end{cases}$$

<u>Osservazione</u>: considerando il circuito equiv.



con $C_{eq} = C_1 + C_2$

$C_{eq} = 5{,}0\,\mu F$

e $Q_{TOT} = Q_1 + Q_2$

$$\Rightarrow C_{eq} = \frac{Q_{TOT}}{(V_B - V_A)} = \frac{Q_1 + Q_2}{(V_B - V_A)}$$

$$Q_1 = C_1(V_B - V_A)$$
$$Q_1 = (2 \cdot 10^{-6} \cdot 3{,}429)\,C = 6{,}858\,\mu C$$

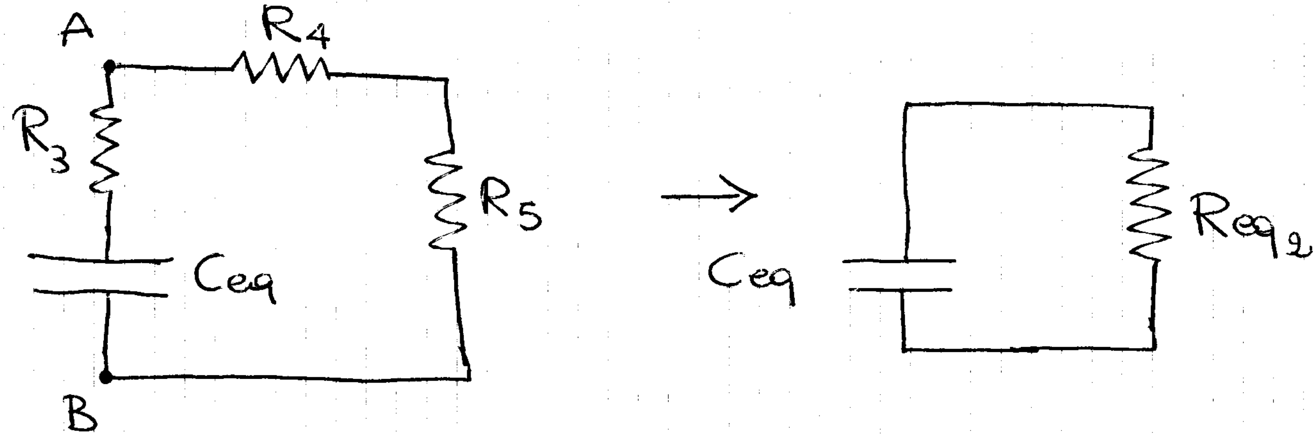
$$\boxed{Q_1 = 6{,}9\,\mu C}$$

da cui $Q_2 = \dfrac{C_2}{C_1} Q_1$

$$Q_2 = \frac{3 \cdot \cancel{10^{-6}}}{2 \cdot \cancel{10^{-6}}} \cdot 6{,}858\,\mu C = 10{,}287\,\mu C$$

$$\boxed{Q_2 = 10\,\mu C}$$

5) Quando l'interruttore T viene aperto il circuito equivalente è dato da:



Con $R_{eq_2} = R_3 + R_4 + R_5$

$R_{eq_2} = (6 + 1 + 1)\,\Omega$ $\qquad R_{eq_2} = 8{,}0\,\Omega$

$\tau = C_{eq} R_{eq_2}$

$\tau = 5 \cdot 10^{-6} \cdot 8 \;\; s = 40 \cdot 10^{-6}\, s$

$\boxed{\tau = 40 \mu s}$