Bessel functions and optical terms of atoms

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Quantum mechanics considers all great variety of atomic spectra beginning from the simplest H-atom spectrum. In this work, we will clarify some important physical processes, taking place in H-atom (and consequently, in any atom), which were not discussed until now.

The H-atom represents a dynamic paired centrally symmetric system, the proton-electron. A central spherical component (proton) has the spherical wave field. By this radial field, proton relates with the surrounding field-space and with an orbiting electron. The orbital motion, in one's turn, is associated with the cylindrical wave field of motion-rest. Hence, both dynamic components of the proton-electron system have to be described, accordingly, by spherical and cylindrical wave functions.

Electron transitions in atoms depend on the structure of their radial shells. In the *central spherical wave field* of H-atom, amplitude of radial oscillations of a spherical shell of the proton is

$$A_s = \frac{A\hat{e}_n(kr)}{kr}\,, \tag{1}$$

where $\hat{e}_n(kr) = \sqrt{\pi kr/2}(J_{n+1/2}(kr) \pm iY_{n+1/2}(kr))$, $k = \omega_e/c = const$ is the wave vector. At that, $J(kr)$ and $Y(kr)$ are Bessel functions; ω_e is some fundamental "carrier" frequency of the spherical wave field, i. e. the frequency of oscillations of the pulsating spherical shell of the proton. Its amplitude energy takes the following form

$$E_s = \frac{M_0\omega_e^2 A_s^2}{2} = \frac{M_0\omega_e^2}{2}\left(\frac{A}{kr}\right)^2 |\hat{e}_n(kr)|^2 = \frac{M_0 c A^2}{2r^2}|\hat{e}_n(kr)|^2\,, \tag{2}$$

where M_0 is the proton mass, A is the constant equal to the oscillation amplitude at the sphere of the wave radius ($kr = 1$). Because $kr_1 = z_{n,1}$ and $kr_s = z_{n,s}$, where $z_{n,s}$ and $z_{n,1}$ are zeros of Bessel functions [1], hence, the following relation is between radial shells: $r_s = r_1 z_{n,s}/z_{n,1}$. Here, the subscript n indicates the order of Bessel functions and s the number of the root. The last defines the number of the radial shell. Zeros of Bessel functions define the radial shells with zeroth values of radial displacements (oscillations), i.e. shells of stationary states.

In the *cylindrical wave field*, the energy E_f of the orbiting electron (in the simplest case) is

$$E_f = \frac{m_e v^2}{2} = \frac{m_e\omega^2 A_f^2}{2} = \frac{m_e\omega^2}{2}\left(\frac{a}{\sqrt{kr}}\right)^2 = \frac{2\pi m_e v A_f}{2}\nu\,, \tag{3}$$

where m_e is the mass of an electron; r is the radius of its orbit; ν is the frequency and $v = \omega A_f$ is the amplitude velocity of its oscillations; $A_f = a/\sqrt{kr}$ is the amplitude of its oscillations.

Because $k = \omega/c$ (in Eq. (3)), hence $E_f = h\nu/2$, where $h = 2\pi m_e c a^2/r = 2\pi m_e v A_f$ is an elementary action. The constant a, equal to the oscillation amplitude at the Bohr orbit r_0 with the length in one wave, is

$$a = \sqrt{\frac{hr_0}{2\pi m_e c}} = 4.52050647 \cdot 10^{-10}\text{cm}\,, \tag{4}$$

where $h = 2\pi m_e v_0 r_0 = 6.626176{\cdot}10^{-27}$ erg·s is Planck constant. If $kr_0 = \omega r_0/c = v_0/c$, where v_0 is Bohr speed, then the amplitude of oscillations is equal to Bohr radius: $A_f = a/\sqrt{kr_0} = r_0$.

In a case when motion-rest exchange (interaction) between spherical and cylindrical fields takes place, the equality $E_f = \Delta E_s$ is valid. Consequently, we have

$$\frac{h\nu}{2} = \frac{M_0 c^2 A^2}{2r_0^2}\left(\frac{|\hat{e}_p(kr_m)|^2 z_{p,1}^2}{z_{p,m}^2} - \frac{|\hat{e}_q(kr_n)|^2 z_{q,1}^2}{z_{q,n}^2}\right)\,. \tag{5}$$

At $p = q$, $z_{n,s} = s\pi$ and $|\hat{e}_n(kr_s)|^2 = 1$, Eq. 5 is transformed into the spectral formula for H-atom: $1/\lambda = R(1/m^2 - 1/n^2)$, where $R = M_0 c A^2/(hr_0^2)$ is Rydberg constant.

Because $R = R_\infty/(1 + m_e/M_0) = 1.09677583{\cdot}10^5$ cm^{-1}, hence $A = r_0\sqrt{hR/(M_0 c)} = 6.370586182{\cdot}10^{-13}$ cm. Assuming in Eq. (1) that kr is equal to the first extremum of the spherical function of the zero order, unequal to zero ($kr = 4.49340946$), we find the first maximal amplitude of radial oscillations:

$$A_s = \frac{A}{kr} = 1.417762222 \cdot 10^{-13}\text{cm}\,. \tag{6}$$

The center of masses of the proton, performing such oscillations, forms a dynamic spherical volume with the radius equal to the amplitude of the oscillations and its volume can be considered as some nucleus. It should also be noted that exchange by energy between the proton and the electron in real conditions occurs on the background of exchange with the surrounding H-atom fields-spaces of a different nature. Hence, the equation of exchange (interaction) should be presented as $E_f = \Delta E_s + \delta E$, where δE takes into account the factor of external influences (perturbations).

In conclusion, since quantum numbers in spectral formulae are roots of Bessel functions, we can regard these as a mathematical variant of spectral terms. We will show also in this work that cylindrical field is a longitudinal-transversal field of motion-rest, where a field of rest (a potential field) and the field of motion (a kinetic field) are mutually perpendicular. At that the longitudinal field negates the transversal field and vice versa. It means that every component of the longitudinal-transversal field generates its opposition, so both processes are, in essence, a united process of indissolubility of rest-motion. By virtue of this, the circular motion is the optimal (equilibrium) state of the field of rest-motion, where "attraction" and "repulsion" are mutually balanced that provides for the steadiness of orbital motion in the micro- and mega-world. This is why the proton-electron, as any conjugated pair of the longitudinal-transversal field in the Universe, is a stable system.

[1] F.W.J. Olver, ed., *Royal Society Mathematical Tables*, Vol. 7, Bessel Functions, part III, Zeros and Associated Values, (Cambridge, 1960).