Cold-atom scattering: from the scattering length to the glory oscillations

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We show that in a wide range of energies, from threshold to deep into the semiclassical domain, the scattering of atoms can be described in terms of just two large parameters. They are the long-range parameter $\gamma = \sqrt{2m\alpha}$, determined by the long-range behaviour of the interatomic potential $U(r) \simeq -\alpha/r^n$ and the reduced mass m, and the "short-range" parameter $\Phi = \int_{r_0}^{\infty}[-2mU(r)]^{1/2}dr$, i.e. the semiclassical s-wave phase at zero energy (atomic units are used). The first of these parameters is known quite well ($n = 6$, and $\alpha \equiv C_6$ is just the van der Waals constant). On the other hand, the phase Φ depends on the behaviour of the potential at small distances, which is often not known sufficiently accurately. This makes reliable *ab initio* calculations of the scattering length a difficult task. To overcome this problem, experimental data on weakly-bound vibrational states or low-energy photoassociation of atoms have to be used to refine the potentials, see e.g. [1]. In the present work we extend the analysis beyond the low-energy s-wave scattering (Refs. [2, 3]) to higher partial waves. We demonstrate that γ and Φ also determine the "glory" oscillations of the total elastic scattering cross section, where many partial waves are involved. This suggests that the scattering lengths could be determined in atomic experiments performed at much higher energies than the micro-Kelvin near-threshold s-wave limit.

According to [2], the atom-atom scattering length is given by the following expression

$$a = \bar{a}\left[1 - \tan\frac{\pi}{n-2}\tan\left(\Phi - \frac{\pi}{2(n-2)}\right)\right], \quad \text{where} \quad \bar{a} = \cos\left(\frac{\pi}{n-2}\right)\left(\frac{\gamma}{n-2}\right)^{\frac{2}{n-2}}\frac{\Gamma\left(\frac{n-3}{n-2}\right)}{\Gamma\left(\frac{n-1}{n-2}\right)} \qquad (1)$$

is the mean, or "typical" scattering length, determined by the asymptotic parameter γ alone. Equation (1) shows why the scattering length is so sensitive to the potential. Even a small relative change in the potential may produce large changes in a, since the phase Φ is usually large ($\Phi \approx N\pi$, where N is number of s-wave vibrational states of the corresponding diatomic molecule; e.g., for Cs_2 in the $^3\Sigma_u$ state $N \approx 60$). The next term in the low-energy expansion of the s-wave phaseshift, $k\cot\delta_0 \simeq -a^{-1} + \frac{1}{2}r_e k^2$, depends on the effective range r_e. Generally an independent parameter, in atom-atom scattering r_e is a function of the scattering length,

$$r_e = F_n - \frac{G_n}{a} + \frac{H_n}{a^2}, \quad \text{or, for} \quad n = 6, \quad r_e = \frac{\sqrt{2\gamma}}{3}\left[\frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})} - 2\frac{\sqrt{2\gamma}}{a} + \frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\frac{4\gamma}{a^2}\right] \qquad (2)$$

where F_n, G_n, and H_n are simple functions of γ [3]. Thus, r_e is also determined by γ and Φ. The effective-range expansion is valid for atomic momenta $k < \bar{a}$. In this range elastic scattering is dominated by the s-wave.

In fact γ and Φ describe the scattering cross section in a much wider energy range $\bar{a} < k \ll \sqrt{2mU_{\rm min}}$ ($U_{\rm min}$ is the depth of the atomic potential well), as

$$\sigma = \sigma_{\rm b} + \Delta\sigma_{\rm glory}, \quad \text{where} \quad \sigma_{\rm b} = P_n \gamma^{4/(n-1)} k^{-2/(n-1)} \tag{3}$$

is the background cross section due to $U(r) \simeq -\alpha/r^n$ (see [4]), and

$$\Delta\sigma_{\rm glory} = Q_n \gamma^{3/n} k^{-(n+6)/2n} \sin\left(2\Phi - R_n \gamma^{2/n} k^{(n-2)/n} - 3\pi/4\right), \tag{4}$$

is the oscillatory "glory" contribution calculated in the present work (P_n, Q_n and R_n are dimensionless constants). This formula shows that the positions of maxima and minima in the cross section depend on the all-important phase Φ, which also determines the scattering length.

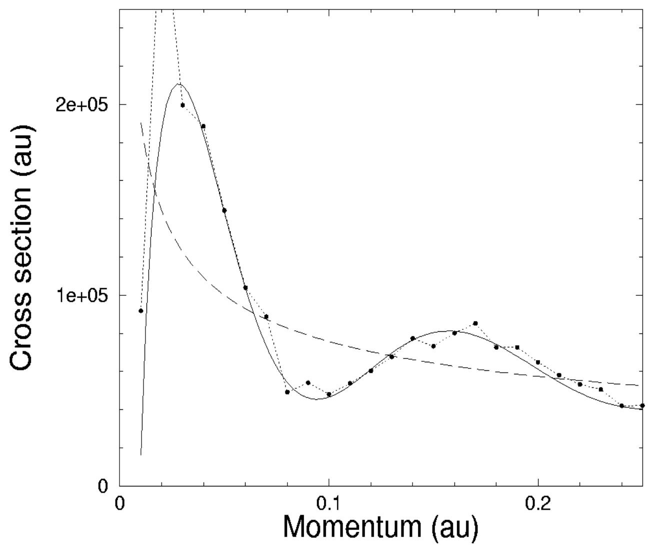

Figure 1: The elastic scattering cross section calculated for the model Cs_2 $^3\Sigma_u$ potential numerically (dots) as $\sigma = 4\pi k^{-2} \sum_l (2l+1) \sin^2 \delta_l$ ($l = 0$–25) and analytically: the dashed line is $\sigma_{\rm b}$, and the solid line is the sum $\sigma_{\rm b} + \Delta\sigma_{\rm glory}$ ($\gamma = 41234$ au, $\Phi = 182.895$).

Although we illustrate the applicability of Eqs. (3) and (4) using Cs–Cs scattering as an example, a proper application of the formulae to atoms with nonzero spins might require taking into account hyper-fine structure effects.

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