Quantal and Classical Collisional Stark Mixing at Ultralow Energies

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Both classical and quantal complete solutions for the problem of all intrashell ℓ-transitions induced by collision of a Rydberg atom with an ion are presented within the impact parameter formalism. For very low velocity of the projectile the transitions with change in quantum principal number are much less probable than the quasi - elastic angular momentum changing collisions at large impact parameters. The collisions considered are both adiabatic in energy and sudden in angular momentum. The rich symmetry of the problem allows an unified approach and is the source of the excellent agreement, beyond the usual Ehrenfest's correspondence principles, between the classical and quantal treatments. Probabilities for transition between any angular momentum states within a high Rydberg energy level are derived in exact analytic forms and are analyzed for a large number of numerical examples. Three methods for the transition probabilities are directly compared: the quantal results, Monte-Carlo classical simulations and classical transition probability.

All theoretical efforts [1] rely on the impact parameter formalism, in which the projectile is a classical particle moving in a definite and undeflected trajectory. The dipole interaction has been proven to be a good approximation for the projectile - target potential because of the long range Couloumb interactions and the importance of large impact parameters. For slow moving ions, Stark Mixing can occur without energy exchange. The dynamics of the Rydberg atom is therefore adiabatic. The orbit of the Rydberg electron can still be considered elliptical, but its shape and orientation changes slowly during the collision time which is very much greater than the orbital time. This classical mechanics picture translates into the quantal description by restricting the dynamics to the energy shell, as prescribed by the Adiabatic Perturbation Theory.

The interaction potential can be written as

$$V = -e^2\frac{\mathbf{R}\cdot\mathbf{r}}{R^3} = \frac{e^2}{vb}\frac{d\Phi}{dt}\hat{R}\cdot\mathbf{r} = \frac{e^2}{vb}\frac{d\Phi}{dt}(y\sin\Phi + z\cos\Phi) \qquad (1)$$

where b is the impact parameter, $\mathbf{R}$ is the internuclear distance and Φ is the angle between $\mathbf{R}$ and the direction of the incoming charged projectile. The angular momentum of relative motion $L_{\rm rel} = \mu R^2\dot{\Phi} = \mu v b$ (where μ is the reduced mass of the projectile - target system, remains conserved since $L_{\rm rel} \gg L$ (so that $L_{\rm rel}$ and L are decoupled).

The Stark mixing parameter α is defined as the following ratio between the Stark and

collision frequencies

$$\alpha = \frac{\omega_S}{\omega_R} = \frac{3Z_1}{2}\left(\frac{a_n v_n}{bv}\right) = \frac{3Z_1}{2}\, n\, \frac{a_0 v_0}{bv} = \frac{3Z_1}{2}\, \frac{\mu}{m_0}\, \frac{n\hbar}{L_{\rm rel}} \tag{2}$$

When $\alpha \ll 1$, ℓ-changing or **Stark sudden** transitions are favored, in contrast to **Stark adiabatic** transitions where $\alpha \gg 1$ and the electronic angular momentum does not change since the atom has sufficient time to relax to the Stark effect.

In this paper a unified theory [4] for the general solution of Collisional Stark Mixing is presented, both in the classical and quantal formulations. The exceptional rich dynamic symmetry of the hydrogen atom provides the key foundation which enables both the classical and quantal solutions to be constructed in a similar way, by using group representation theory. This classical-quantal correspondence transcends the well known Ehrenfest's theorem (as observed in the general case of weak field-atom interaction [2]) just because of the SO(4) group symmetry of the hydrogen atom. The agreement, as expected, is very good. It is shown that the present quantal solution is equivalent with the rotating coordinate frame formal result of Kazansky [3]. Also in the quantal case, an efficient alternative for practical calculation is demonstrated. A new classical solution applicable to transitions between arbitrary angular momentum states is presented. Monte Carlo simulations are also performed to yield results in agreement with the classical expression. Representative comparison of the transition probability is shown in Fig. 1.

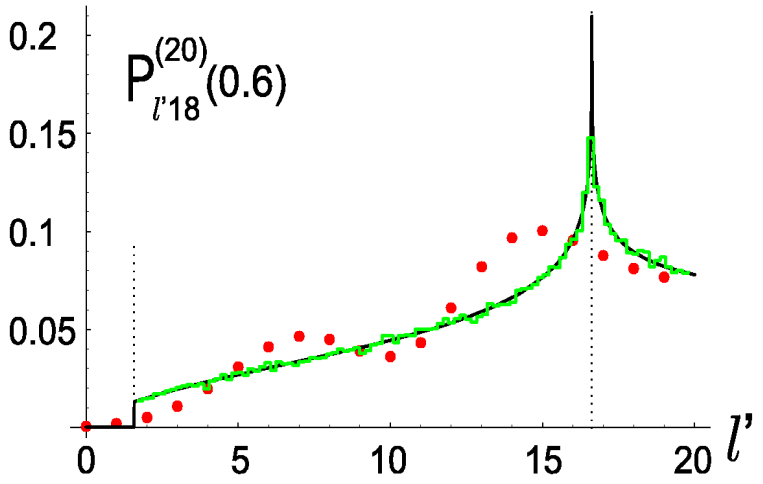

Figure 1: The Monte-Carlo simulation (step-like lines), the classical (solid line) and quantal (dots) transition probability for given Stark parameter $\alpha = 0.6$ and initial $\ell = 18$ within the $n = 20$ shell.

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[3] A. K. Kazansky and V. N. Ostrovsky, Phys. Rev. Lett **77**, 3094 (1996).
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