Coherence length for matter waves

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The last decade has seen the rapid development of the field of atom optics. It is now possible to design specific motional states for atoms and ions and to perform measurements with them.

Here, we introduce the concept and mathematical description of a coherence length and its conjugate property for matter states, and explain their significance via the example of the motional states of a single cold trapped ion. These parameters describe the coherence properties of matter waves in much the same way that optical coherence is determined by the coherence time and spectral width. They become visible in the interference pattern of a matter or light wave interferometer, respectively. We propose experimental methods to measure the coherence length and its conjugate quantity and determine their values for different quantum states.

In the case of a single ion, the entanglement between the motional and the electronic state may be used to displace a part of the ion's wavefunction by addressing the electronic transitions [1]. The resulting overlap of the wavefunction with its displaced form gives rise to an interference pattern in the ion's probability of being in the electronic ground state,

$$P_g = \frac{1}{2}\{1 - |\chi(\alpha_\theta)| \cos[\phi + arg(\chi(\alpha_\theta))]\},$$

which depends sinusoidally on an externally controlled phase ϕ. Here $\alpha_\theta = \frac{1}{\sqrt{2}}(\alpha e^{-i\theta} - \alpha^* e^{i\theta})$ denotes the displacement between the two matter waves and χ is the symmetrically ordered characteristic function of the initial motional state $\chi(\alpha) = Tr(\hat{\rho}\hat{D}(\alpha))$. It was recently pointed out that the motional state of the ion can thus be deduced from such an interference pattern [2]. The visibility of the interference fringes at a given displacement is determined by the coherence length L_θ. We define the coherence length as the normalized root-mean square width of the squared modulus of the characteristic function,

$$L_\theta^2 = \frac{\int d^2\alpha(\alpha_\theta - \bar{\alpha}_\theta)^2|\chi(\alpha)|^2}{2\int d^2\alpha|\chi(\alpha)|^2} = \frac{Tr\,(\hat{\rho}^2\hat{x}_\theta^2) - Tr\,(\hat{\rho}\hat{x}_\theta\hat{\rho}\hat{x}_\theta)}{Tr\,(\hat{\rho}^2)}.$$

For a pure state L_θ^2 coincides with the variance of the position but differs for example for thermal states. It is interesting to note that the parameter $Tr\,(\hat{\rho}^2)$ describes the "mixedness" of the state [3].

The coherence length and its conjugate obey an uncertainty relationship that is analogous to the Heisenberg uncertainty principle. It is also closely connected to the relationship between optical coherence time and spectral width.

We propose a new method by which the Wigner function associated with the motional state of a trapped ion can be measured. The Wigner function is related to the characteristic function by Fourier transform in the same way that the optical spectrum is related to the coherence function. The conjugate property to the coherence length is, therefore, related to the width of the Wigner function. We show how this conjugate property appears in measurements of the motional Wigner function for the trapped ion.

Similar interferometric schemes may be applied to measure the properties of other forms of matter wave. In particular, we show how our coherence length appears in measurements of the collective motional state of atoms in an optical potential.

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