Vortex dynamics in Bose condensates

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The observation of vortices in Bose-Einstein condensates (BEC's) [1, 2] has opened a new way to study topological properties of quantum liquids and gases. So far the vortices have been created either by stirring the condensate with external perturbation or by printing an appropriate phase pattern onto the condensate [1, 2, 3, 4]. We propose the use of a different method that may also enable studies about vortex dynamics and interactions: the method utilises the dynamical instability of the soliton-like solutions.

Solitons in a 2D system can be dynamically unstable [5, 6, 7]. In non-linear optics solitons have been studied extensively and their dynamical instability is an experimental fact [6]. We study how a soliton in BEC decays into vortices in a quasi-2D ring trap. The created vortices move rapidly, and we can study the interaction between a vortex-antivortex pair by looking into the vortex dynamics. Gross-Pitaevskii (GP) equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi + NU_0|\Psi|^2\Psi \qquad (1)$$

forms the basis for our studies.

In our proposal the condensate is shaken first in x-direction, thus splitting it in half [8]. After this we apply an appropriate phase shift to the other half and add shaking in y-direction. The time-averaged final result is a trap with toroidal symmetry and two colliding condensates with a phase step across the collision region, which appears as a soliton. The dynamical instability will cause the soliton to decay into vortices and during the subsequent time evolution the created vortices can collide.

The instability of the soliton is maximised by sinusoidal disturbancies with wavenumbers $k = \sqrt{1/2}$ (in units of correlation length) [7]. This is manifested by the creation of vortices at roughly equal distances with separation $d \approx \pi\sqrt{2}$. With the parameters we use, each soliton breaks into only one vortex.

In contrast to a homogeneous 2D system, vortices of opposite circulation repel and bounce from each other, move around the ring, bounce off again and so on. We explain the observed behavior with inhomogeneous density distribution and Magnus-force. The importance of the attractive Coulomb-type force is also discussed. In fig. (1) we show the vortex dynamics calculated with classical equations of motion which corresponds quite closely to the observed behavior with the GP-equation.

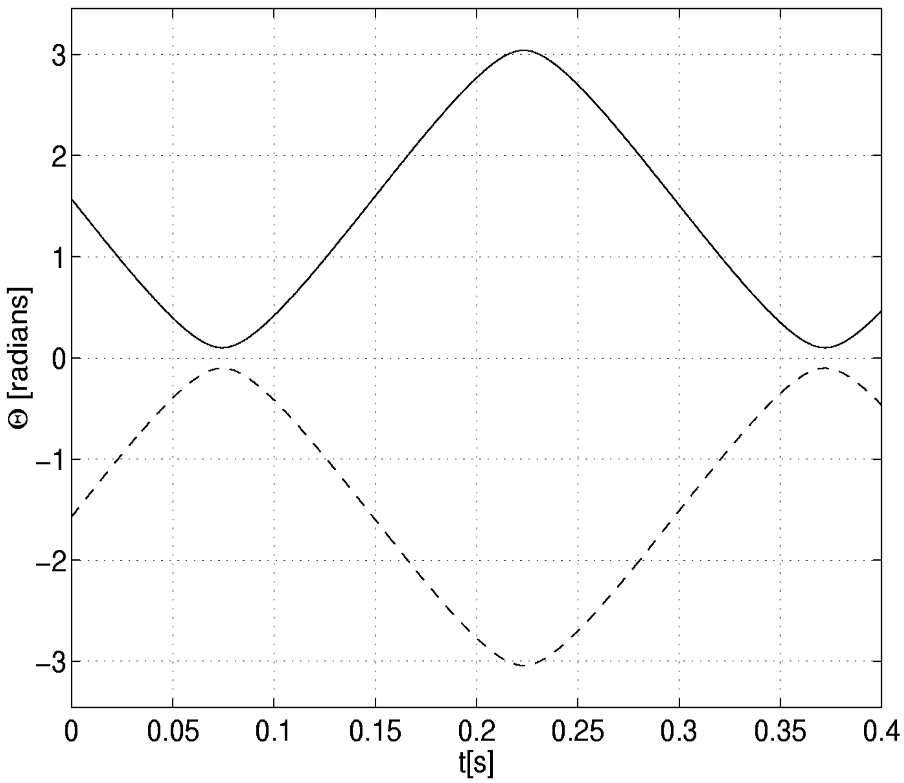

Figure 1: Vortex dynamics in a ring for a vortex-antivortex pair according to the classical equations of motion, with Magnus-force and radial confinement. Initial conditions were taken from the simulations with GP-equation and Θ is the angle from the origin to the vortex.

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