Collective oscillations of Boson-Fermion mixed condensate of alkali atoms : sum rule approach

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Collective oscillation is one of the most prominent phenomena common to a variety of many-body systems. Recent realization of the Bose-Einstein condensates and the degenerate fermi system of trapped Alkali-atom gases allows one to study such phenomena under ideal conditions[1]. One may now expect to create a new type of many-body system, a cold bose-fermi mixed system, and study its dynamical structure such as collective oscillations[2].

We have calculated energies of collective monopole, dipole and quadrupole oscillations of bose-fermi mixed Alkali atoms in the sum rule method. Here the excitation energy is given by $\hbar\omega = (m_3/m_1)^{1/2}$, where $m_p = \sum_j (E_j - E_0)^p |\langle j|F|0\rangle|^2$. The relevant multipole operator F is a linear combination of the in-phase and out-of-phase operators of two types of particles

$$F(\mathbf{r};\theta) = F^+ \cos\theta + F^- \sin\theta \ , \quad F^\pm = F^{\text{boson}} \pm F^{\text{fermion}} \tag{1}$$

where θ is determined to minimize the calculated ω for each operator.

We consider the polarized boson-fermion mixed condensate of potassium isotopes in the spherically symmetric harmonic oscillator potential. Boson-boson and boson-fermion interaction strengths g and h are represented by their s-wave scattering lengths, while the fermion-fermion interaction has been neglected for the polarized system.

Following the standard calculation procedure for the monopole frequency is

$$\frac{\omega_M(\theta)}{\omega} = \sqrt{\frac{-T^+ + 5V^+ + 2(-T^- + 5V^-)\cos\theta\sin\theta - 2\Delta\sin^2\theta}{V^+ + 2V^-\cos\theta\sin\theta}} \tag{2}$$

and similary for quadpole and dipole modes.

Here we defined $T^\pm \equiv T^b \pm T^f, V^\pm \equiv V^b \pm V^f$ and $N^\pm \equiv N_b \pm N_f$, where $T^{\{b,f\}}$ and $V^{\{b,f\}}$ are respectively the expectation values of the kinetic and harmonic potential energies for boson/fermion in the ground state and we have used the stationary conditions of the ground state. The quantities Δ is given in terms of the boson/fermion densities $n_b(r), n_f(r)$ in the ground state by

$$\Delta \equiv h \int d^3r \, r^2 \frac{dn_f(r)}{dr}\frac{dn_b(r)}{dr}. \tag{3}$$

One may notice that the effects of the boson-fermion interaction come through the ground-state kinetic/potential energies and explicitly only through the quantities (3)

These excitation energies have been obtained from the ground state properties of the system calculated in the Thomas-Fermi approximation. Fig.1-a shows energies of the lower (solid lines) and the higher (dashed lines) states of the monopole oscillation as a function of h/g. For an attractive boson-fermion interaction ($h/g < 0$), the low-lying monopole mode becomes a coherent oscillation of bosons and fermions. The excitation energy shows a rapid decrease towards the instability point of the ground state into collapse. In this region the attractive boson-fermion interaction is much more effective in the excited state than in the ground state and cancels out the increase in the kinetic energies. The critical value of the interaction strength is consistent with the one obtained in the variational calculation for the instability of the ground state[3]. In contrast, the quadrupole mode (Fig.1-b) has either the bosonic or the fermionic character over a wide range of the interaction parameters.

For the dipole mode (Fig.1-c) the boson-fermion in-phase oscillation remains to be an eigenstate with energy $\hbar\omega_{\rm osc}$, while the out-of-phase oscillation is strongly dependent on the interaction strength. For a weakly repulsive h an interesting possibility arises: In the region $0 < h/g < 1$ the out-of-phase mode of the boson-fermion oscillation lies lower than the in-phase mode. This is because the boson-fermion repulsion is weaker for the out-of-phase oscillation than the in-phase one. For a larger value of h the out-of-phase mode becomes again energetically unfavorable because of the increasing repulsive interaction effect.

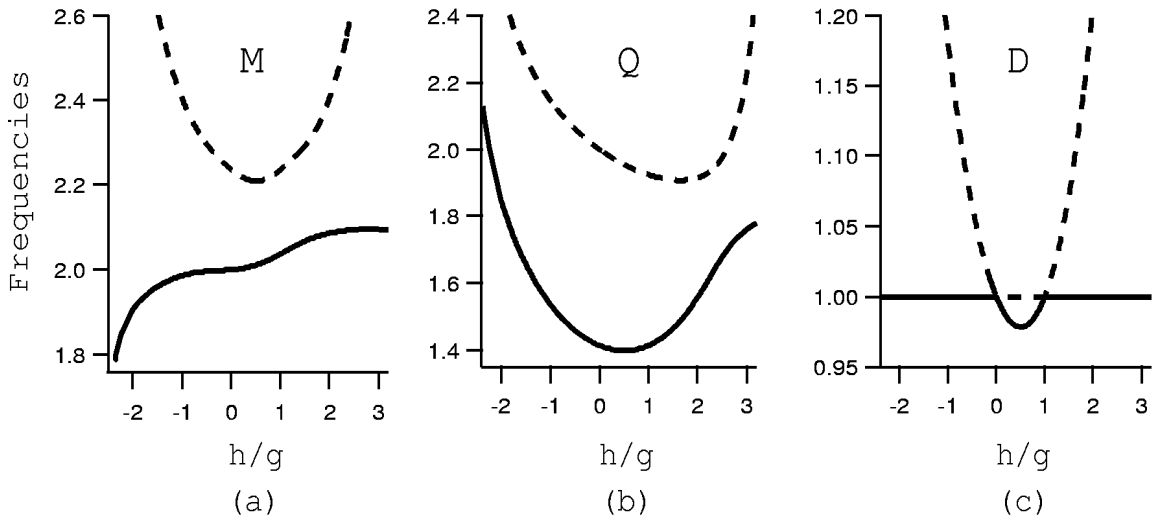

Figure 1: Excitation frequencies (a:Monopole, b:Quadrupole, c:Dipole) unit by $\omega_{\rm osc}$ as functions of h/g. The mixing angle θ has been determined so as to minimize energies.The solid (dashed) lines corresponds to the lower (higher) energy modes ($N_b = N_f = 10^6$) .

[1] B. DeMarco and D. S. Jin, Science **285**, 1703 (1999)
[2] T.Miyakawa, T.Suzuki and H.Yabu, cond-mat/0002145
[3] T.Miyakawa, T.Suzuki and H.Yabu, cond-mat/0002048