Schrödinger-Cat Entangled State Reconstruction in the Penning Trap

Mauro Fortunato

INFM and Dipartimento di Matematica e Fisica, Università di Camerino

A single electron trapped in a Penning trap [1] is a unique quantum system in that it allows the measurement of fundamental physical constants with striking accuracy. Penning traps can be also used to trap and store protons and ions. It is therefore apparent that the determination of the genuine quantum state of the trapped electron is an important issue, with implications in the very foundations of physics, and in particular of quantum mechanics. After the pioneering work of Vogel and Risken [2], several methods have been proposed in order to reconstruct the quantum state of light and matter, which range from quantum tomography [3] through quantum state endoscopy [4], to Wigner function determination from outcome probabilities [5].

In the present work we propose to reconstruct the full state (combined cyclotron and spin state) of an electron in a Penning trap by using a modified version of quantum state tomography. Previous proposals [6] need the *a priori* knowledge of the spin state and therefore are not able to deal with entangled states. Our method, on the contrary, has the ability of measuring the full (entangled) pure state of the two relevant degrees of freedom of the electron.

As an example of application of our method, we show in Fig. 1 a Monte-Carlo simulation of the faithful reconstruction of the total density matrix and of the Wigner function of the electron in the case of a Schrödinger-Cat-like entangled state of the form

$$|\Psi\rangle = c_1|\alpha\rangle|\uparrow\rangle + c_2|-\alpha\rangle|\downarrow\rangle \,, \tag{1}$$

where $|\alpha\rangle$ is a coherent state for the cyclotron degree of freedom and $|\uparrow\rangle$, $|\downarrow\rangle$ are the eigenstates of the z component of the spin degree of freedom. The results are shown in terms of the Wigner matrix associated to the state of Eq. (1) in the spin representation.

[1] L. S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 233 (1986).

[2] K. Vogel and H. Risken, Phys. Rev. A **40**, R2847 (1989).

[3] D. T. Smithey, M. Beck, M. G. Raymer and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993).

[4] P. J. Bardroff, E. Mayr and W. P. Schleich, Phys. Rev. A **51**, 4963 (1995).

[5] L. G. Lutterbach and L. Davidovich, Phys. Rev. Lett. **78**, 2547 (1997).

[6] S. Mancini and P. Tombesi, Phys. Rev. A **56**, R1679 (1997); S. Mancini and P. Tombesi, Phys. Rev. A **56**, 3060 (1997).

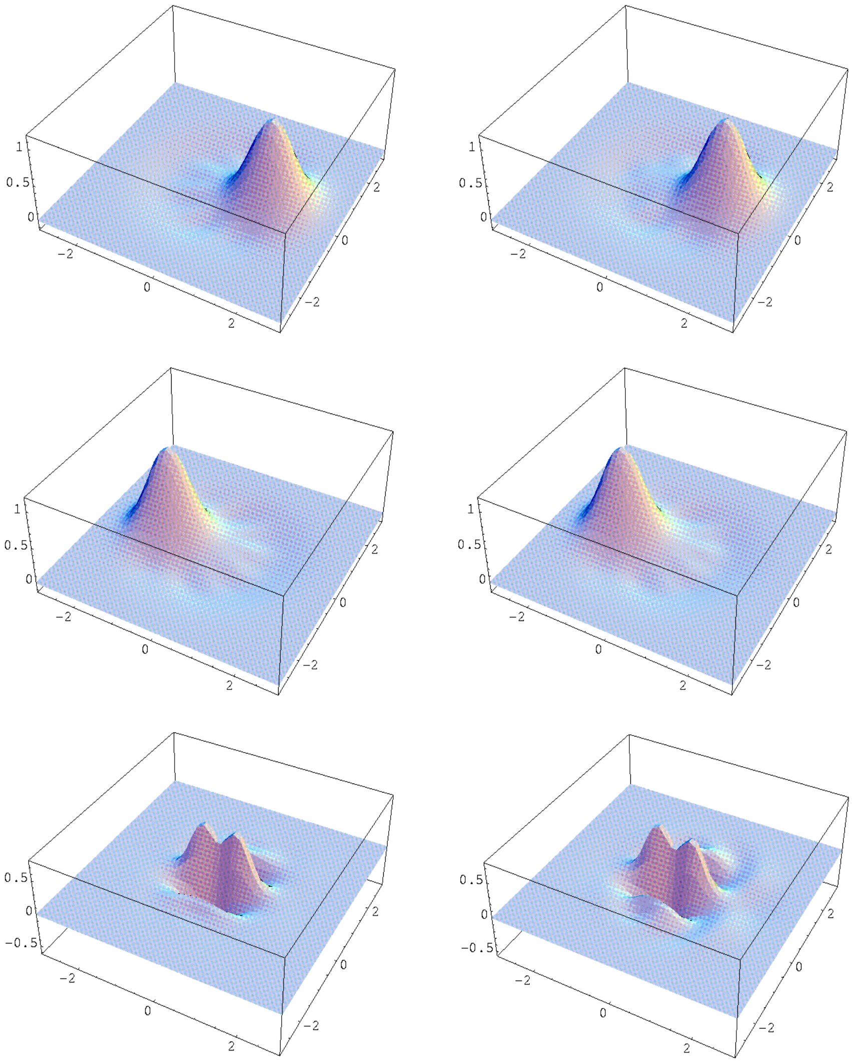

Figure 1: Numerical simulation showing an example of entangled-state reconstruction for the state of Eq. (1). Here $\alpha = 1.3$ and $c_1 = c_2 = 2^{-1/2}$. Left: true Wigner functions. Right: reconstructed Wigner functions. In a), b), and c) different elements of the Wigner matrix are represented.