Separability and classicality of states generated with a $SU(1,1)$ interferometer

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The statistical properties of two-mode light generated in several well-known processes as nondegenerate parametric amplification and degenerate four-wave mixing were intensely studied in order to find evidence for the quantum nature of light. The two-mode radiation resulting from these processes has nonclassical properties such as squeezing and strong correlations between the two beams. A unified treatment of four-port devices having as active elements nondegenerate parametric amplifiers or degenerate four-wave mixers has been initiated by Yurke, McCall and Klauder [1] as a "$SU(1,1)$ interferometer", namely a device which realizes in fact Bogoliubov transformations of the amplitude operators.

In this work we study the way in which a two-mode $SU(1,1)$-interferometer acts on mixed squeezed-states inputs. This problem meets the recent interest in quantum information processing of Gaussian field states [2, 3]. Quantum communication experiments in the near-infrared and optical domain are possible using Gaussian states at the input ports of active devices modelled by a $SU(1,1)$ interferometer. Moreover, it has recently been understood that testing for the preservation of nonlocal entanglement during teleportation involves the need to formulate separability criteria for two-mode Gaussian states [4].

Our work encompasses three main parts, as follows.

1. First we give a comprehensive description of the mixed two-mode Gaussian states based on the characteristic function of the density operator,

$$\chi_G(x) = \exp\{-\frac{1}{2}x^T\mathcal{V}x\}, \tag{1}$$

with x^T the row vector $(x_1\ x_2\ x_3\ x_4)$. $\mathcal{V}$ is the real, symmetric and positive-definite 4×4 covariance matrix. It has the following block structure

$$\mathcal{V} = \left(\begin{array}{c|c} \mathcal{V}_1 & \mathcal{E} \\ \hline \mathcal{E}^T & \mathcal{V}_2 \end{array}\right), \tag{2}$$

where $\mathcal{V}_1$, $\mathcal{V}_2$, and $\mathcal{E}$ are 2×2 matrices: $\mathcal{V}_1$ and $\mathcal{V}_2$ are symmetric covariance matrices for the reduced individual modes. We call $\mathcal{E}$ the *entanglement* matrix. It contains the correlations between coordinates and impulses. The properties of the two-mode covariance matrix (2) studied in our paper are determined by the generalized form of the Robertson-Schrödinger uncertainty relation [4].

2. We determine the $SU(1,1)$ transformation of the covariance matrix of the two-mode input state. The action of the $SU(1,1)$ interferometer imposes a squeezing operation with real parameter Θ and a common phase shifting for the two reduced modes.

3. We analyze the conditions for classicality (=existence of the P representation) and separability for the output of a $SU(1,1)$ interferometer. The condition

$$\mathcal{V} - \frac{1}{2}I \geq 0, \tag{3}$$

defines the classicality of a quantum two-mode Gaussian state. In Eq. (3) I is the 4×4 unity matrix. The semipositiveness condition (3) is also *sufficient* to ensure separability of the two-mode Gaussian state.

Finally, we apply the separability criterion of positivity of the density matrix under partial transposition [5], to the case of the two-mode output states of a $SU(1,1)$ interferometer. Recently, Simon has proved that this criterion is *a necessary and sufficient condition* for separability of two-mode Gaussian states.

We have found the following properties.

- A squeezed (nonclassical) input of the $SU(1,1)$ interferometer generates a nonclassical two-mode output state.
- A classical input at one of the ports leads to a classical two-mode output provided that the gain of the interferometer does not exceed the threshold

$$\left((\cosh\frac{\Theta}{2})^2\right)_P = \frac{(\sigma^{(0)}_{p_2p_2} + \frac{1}{2})(\sigma^{(0)}_{p_1p_1} + \frac{1}{2})}{\sigma^{(0)}_{p_1p_1} + \sigma^{(0)}_{p_2p_2}}. \tag{4}$$

  We have denoted by $\sigma^{(0)}$ the variances of the input single-mode states.
- The range of gain for which the output state is separable does not coincide with that allowing for its classicality. Only for purely thermal input do classicality and separability have the same threshold gain.

[1] B. Yurke, S. L. McCall, and J. R. Klauder, *Phys. Rev. A* **33** 4033 (1986).
[2] S. L. Braunstein and H. J. Kimble, *Phys. Rev. Lett.* **80** 869 (1998).
[3] S. L. Braunstein, *Nature* **394** 47 (1998).
[4] R. Simon, *quant-ph*/9909044.
[5] A. Peres, *Phys. Rev. Lett.* **77** 1413 (1996).