Classical form factor for $nlm \to n'l'm$ transitions in the hydrogen atom

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The form factor

$$W_i^f = \left| \left\langle \psi_i \left| e^{i\mathbf{qr}} \right| \psi_f \right\rangle \right|^2 \tag{1}$$

is the probability of a transition $i \to f$ arising from any external impulsive perturbation which induces a sudden change in the electron momentum. Here $|\psi_i\rangle$ and $|\psi_f\rangle$ are the electron wave functions for the initial and final atomic states, respectively, and $\mathbf{q}$ is the momentum transferred to the electron.

In the Born approximation, the form factor (1) determines the cross sections of bound-bound, continuum-continuum and ionization transitions in atoms due to collisions with electrons, atoms and ions [1]. The probability of excitation and ionization of a Rydberg atom by a short unipolar electromagnetic field pulse (HCP) is determined , in the sudden approximation, by the same (1) transition form factor [2].

A quantal, as well as asymptotic and semiclassical, expressions for the transition form factor between two parabolic quantum states of the hydrogen atom were obtained [3]. In the spherical basis, the analytical quantal form factor $W_{nlm}^{n'l'm}$ for transitions between the initial and final hydrogenic states with quantum numbers n, l, m and n', l', m, respectively, is not available. Recently the classical form factor $W_{nl}^{n'l'}$ has been deduced [4] from the microcanonical distribution formalism of an atomic system.

We have investigated the general $nlm \to n'l'm$ transition on the basis of kinematics of classical mechanics.

A motion of a charged particle in the attractive Coulomb field is located in a plane and at the negative energy can be described by an ellipse. Orientation of the elipse is defined by the Euler angles θ, ψ and φ. Parameters n, l, m are introduced instead of energy $E = -1/2n^2$, eccentricity $\varepsilon = \sqrt{1 - l^2/n^2}$ and the Euler angle θ: $m = l\cos\theta$. Similarly for n' and l'.

The new trajectory, after impulse $\mathbf{q}$ transfer, is defined by the equations

$$\mathbf{r} = \mathbf{r}', \quad \mathbf{p} + \mathbf{q} = \mathbf{p}', \quad \mathbf{l} + \mathbf{r} \times \mathbf{q} = \mathbf{l}'. \tag{2}$$

In order to define the classical transition probability, we exploit the identity:

$$\frac{1}{T_K} \int_0^{T_K} dt \int_{-\pi}^{\pi} d\psi \int_0^{\pi} d\theta \sin\theta \equiv 1, \tag{3}$$

where T_K is the Kepler period. Using the equations for the final energy and squared angular momentum (both succeed from Eqs.(2)) and equation $m = l\cos\theta$, integrations over t, ψ and θ can be replaced by n', l' and m integrations. Then the transition form factor $W_{nlm}^{n'l'm}$ becomes equal to the Jacobian $J(t, \theta, \psi, n', l', m)$.

We have obtained analytic expression for the classical form factor for arbitrary $nlm \to n'l'm$ transition [5]. Upon summation of $W_{nlm}^{n'l'm}$ over m one can get the formula for $W_{nl}^{n'l'}$ derived recently in [4].

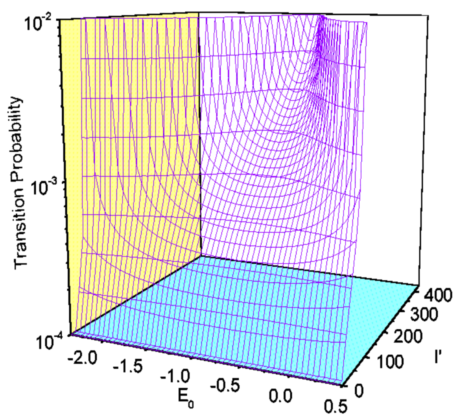

Figure 1: Transition probability from the initial state with $n = 417$ and $l = m = 0$ to different final states as a function of the final state energy $E_{n'}$ and angular momentum l'. The energy axis is scaled to the energy of the initial state, $E_0 = E_{n'}/|E_n|$. The parameter qn is equal to 0.53.

In Fig.1 the probability $W_{n00}^{n'l'0}$ is plotted as a function of l' and scaled final energy $E_0 = E_{n'}/|E_n|$ for initial $n = 417$ and $qn = 0.53$. Figure confirms the fact that the classical transition probability is singular at some combinations of parameters [3, 4]. It also shows that the transition probabilities to the states with large angular momentum l' are dominant. The mean value of the final angular momentum for chosen n and qn is equal to 285.

[1] N. F. Mott and H. S. W. Massey; *The Theory of Atomic Collisions* (Oxford University Press, London, 1965).

[2] C. D. Reinhold, J. Burgdörfer, M. T. Frey and F. B. Dunning, Phys. Rev. A**54** (1996) R33.

[3] I. Bersons and A. Kulsh, Phys. Rev. A**55** (1997) 1674.

[4] D. Vrinceanu and M. R. Flannery, Phys. Rev. Lett. **82** (1999) 3412; Phys. Rev. A**60** (1999) 1053.

[5] I.Bersons and A.Kulsh Phys. Lett. (submitted).