Off-shell effects in laser-assisted scattering?

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The theory of few-body interactions is often represented in terms of sequences of two-body interactions, treated, in principle, off the energy shell since energy conservation is a global property, not necessarily fulfilled in intermediate propagation of subsystems. Atomic collisions may often be represented by intermediate propagation on the energy shell although important exceptions, like ionization and charge exchange in energetic ion-atom collisions, do exist. If scattering processes are embedded in a laser field, off-shell effects due to exchange of photons may come into play. In fact, laser-assisted electron-atom scattering performed under so-called classically forbidden conditions offers a special possibility to study off-shell effects as discussed in this contribution.

The impulse approximation for laser-assisted electron-atom scattering under exchange of n photons reads

$$f_n(\mathbf{q}_f, \mathbf{q}_i) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \exp\left[-i\left(n\omega t - \frac{A_0}{\omega}\epsilon\mathbf{Q}\sin\omega t\right)\right] \times f(\mathbf{p}_f(t), \mathbf{p}_i(t)), \tag{1}$$

with ϵ the linear polarization of the laser, A_0 the amplitude of the vector potential, $\mathbf{Q} = \mathbf{q_f} - \mathbf{q_i}$ the momentum transfer and $f(\mathbf{p_f}(t), \mathbf{p_i}(t))$ the field-free *off-shell* scattering amplitude, evaluated at instantaneous kinematical momenta $\mathbf{p_j} = \mathbf{q_j} + \mathbf{A}(t)$. In the classically allowed region where $|n|$ is less than the classical cut-off in the number of exchanged photons

$$n_c = |(A_0/\omega)\epsilon\mathbf{Q}|, \tag{2}$$

equation (1) may be evaluated in the stationary-phase approximation to derive the standard Kroll-Watson approximation

$$f_n(\mathbf{q_f}, \mathbf{q_i}) = J_n(n_c) f(\mathbf{q_f} + \boldsymbol{\gamma}, \mathbf{q_i} + \boldsymbol{\gamma}), \tag{3}$$

where $\boldsymbol{\gamma} = n\omega\epsilon/(\epsilon\mathbf{Q})$ is exactly the momentum displacement needed to keep the field-free scattering amplitude *on the energy shell*, $|\mathbf{q_f} + \boldsymbol{\gamma}| = |\mathbf{q_i} + \boldsymbol{\gamma}|$.

Detailed experimental tests by Wallbank and Holmes have been done in the regime where the momentum transfer in the direction of the laser field, $\epsilon\mathbf{Q}$, is so small that exchange of the considered number of photons is classically forbidden, $|n| > n_c$. When this is the case, the stationary phase argument no longer applies and the Kroll-Watson approximation is not well founded. Instead one has to consider the full expression of equation (1). The off-shell scattering amplitude is accordingly needed. This is a rather difficult task for realistic atomic potentials but, the δ-shell potential

$$V(r) = V_0 a \delta(r - a) \tag{4}$$

provides an exactly solvable model where the off-shell scattering amplitude is readily identified. The result is

$$f(\mathbf{k_f}, \mathbf{k_i}) = \frac{1}{k_i} \sum_l (2l+1) e^{i\delta_l(k_i)} \sin \delta_l(k_i) P_l(\cos\theta) \xi_l(k_f, k_i) \tag{5}$$

where the phaseshift δ_l and the off-shell factor ξ_l are given by

$$\tan \delta_l(k) = ka^3 V_0 j_l(ka)^2 / (ka^3 V_0 j_l(ka) n_l(ka) - 1/2) \tag{6}$$

and

$$\xi_l(k_f, k_i) = j_l(k_f a)/j_l(k_i a), \tag{7}$$

respectively. Note that equation (5) reduces to the standard on-shell form for potential scattering when the off-shell factor, ξ_l, is unity. Equation (5) embodies quantitatively and transparently the off-shell effects. The on-shell approximation is clearly fine as long as $j_l(k_f a) \simeq j_l(k_i a)$ for all relevant l-values, i.e., for $\Delta k = |k_i - k_f| \ll 2\pi/a$, or

$$|n| \; \nu \; t_c \ll 1 \tag{8}$$

where ν is the frequency of the laser field and $t_c = a/k$ is the collision time. Although equation (8) is derived for the special case of the δ-shell potential we expect it to be valid in a more general context. If we insert the experimental values used by Wallbank and Holmes the condition in (8) reads $a \ll 1000/|n|$. Typical values of $|n|$ ranges from 1 to 4 in the experiments. Clearly this condition is expected to be fulfilled even for long-range polarization potentials, and we do, accordingly, not expect any importance of off-shell propagation in the Wallbank and Holmes experiments. This conclusion supports a conjecture in earlier work by us.

It is our conclusion that the on-shell approximation is valid for low-frequency, few photon-exchange laser-assisted scattering. When, on the other hand, the number of exchanged photons and/or the frequency of the laser are significantly increased, off-shell effects cannot be ruled out. Although representing bound-free and free-bound transitions instead of free-free transitions as considered in ordinary laser-assisted scattering, it is indicative that off-shell effects might play an essential role in multiphoton (above threshold) ionization processes and in high harmonic generation where rescattering models provide a qualitative description of the physical process and where the number of exchanged photons and the frequency of the applied laser are typically much larger than in the Wallbank-Holmes experiments.