Semiclassical treatment of radiation energy transfer with partial frequency redistribution

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Recently, we have developed an analytical method which treats the basic equations of radiation transfer as a generalized wave (diffusion) equation and employs an original semiclassical approach for the solution, the Geometrical Quantization Technique (GQT). The method has been applied to one- and multi-dimensional geometries [1] assuming complete redistribution of the re-emitted radiation. Here, we report on the inclusion in the GQT method of partial frequency redistribution effects.

The master equation ruling the temporal evolution of the frequency-dependent density function of the excited atoms, $n^*(\vec{r},\nu,t)$, over the space coordinate $\vec{r}$ and the frequency ν is [2]

$$-\frac{\partial n^*(\vec{r},\nu,t)}{\partial t} = -\left(A_{21}+W(\vec{r})\right) n^*(\vec{r},\nu,t) + A_{21}\int_{-\infty}^{\infty} d\nu' \int_{\Omega} d^3r' \; G_{\nu\nu'}\left(\left|\vec{r}-\vec{r'}\right|\right) \, n^*(\vec{r'},\nu',t) \, , \quad (1)$$

where W and A_{21} are the quenching and radiation rate constants, respectively. To illustrate the method, pure Doppler broadening is considered as the main mechanism for formation of the partial frequency redistribution function $R_{\nu\nu'}$ and the absorption coefficient $\kappa(\nu) \equiv \kappa_0^{(D)} \cdot \exp(-\nu^2)$. We reduce the trapping equation to the form of a $4D$ generalized wave equation for some $4D$ classical system, playing the role of an associated quasiparticle. The study of the effective radiation factors and of the mode structure can be shown to be equivalent to the determination of quantized energy values and wave functions of the quasiparticle confined in the space region Ω (corresponding to the cell where excited vapor is confined). Within the frame of semiclassical approach, it turns out that space and frequency variables allows variable separation, the magnitude $p = |\vec{p}|$ of the quasiparticle momentum being the separation constant. This leads to the main three points of our approach. (i) Factorization of modes can be carried out: $\Psi_j^*(\vec{r},\nu,t) \simeq \exp(-\lambda_j t) \cdot \varphi_p^{(n)}(\nu) \cdot N_i(\vec{r})$, where the mode multiindex $j = \{n,i\}$ consists of frequency (n) and space (i) quantum numbers. (ii) The modified emission profile $\tilde{\varphi}_p^{(n)}(\nu) = \exp(\nu^2/2) \cdot \varphi_p^{(n)}(\nu)$ satisfies the wave equation for a perturbed oscillator

$$\left(-\frac{1}{2}\frac{\partial^2}{\partial \nu^2} + \frac{1}{2}\nu^2 + \frac{1}{(1-E)} \cdot V_p(\nu)\right) \tilde{\varphi}_p(\nu) = \frac{1+E}{2(1-E)}\tilde{\varphi}_p(\nu); \quad \lambda_j = E^{(n)}(p) \qquad (2)$$

$$V_p(\nu) = 1 - \frac{\kappa_0^{(D)}\exp(-\nu^2)}{p}\arctan\left[\frac{p}{\kappa_0^{(D)}}\exp(\nu^2)\right]; \quad \lambda_j(p) = \int\limits_{-\infty}^{\infty} d\nu \varphi_p^{(n)}(\nu) V_p(\nu)$$

(iii) The space-dependent part $N_i(\vec{r})$ (the total density) of the mode satisfies the Holstein trapping equation with the emission profile $\varphi_p^{(n)}(\nu)$, and is determined by GQT [1] using generalized quantization laws to evaluate the momentum value p_i. The mode radiation trapping constant $\lambda_j = E^{(n)}(p_i)$ is calculated, then, as the quasiparticle kinetic energy for the n-branch of the dispersion curve $E^{(n)}(p)$ for the wave equation Eq. (2).

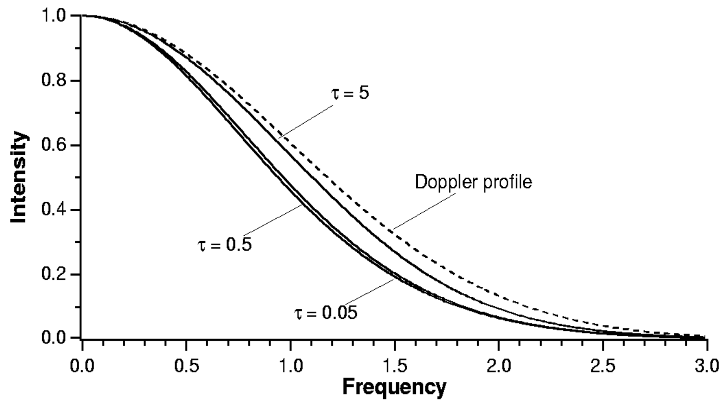

Figure 1: The modified emission profile $\tilde{\varphi}_p^{(0)}(\nu) = \exp(\nu^2/2) \cdot \varphi_p^{(n)}(\nu)$ for different reduced opacities $\tau = \kappa_0^{(D)}/p$, as reported on the graph. The Doppler profile $\exp(-\nu^2/2)$ (dashed line) corresponds to the complete frequency redistribution approach.

Figure 1, showing the behavior of the emission profile in the fundamental mode for different values of the opacity τ, demonstrates the relevance of the partial frequency redistribution effects. Data in Tab. 1 enable a comparison of escape factors derived by our method with the results of numerical calculations. Our approach turns out to be enough accurate (better than 5%) and, thus, can be considered as an efficient method for solving different radiation transfer problems in a variety of experiments dealing with thermal and cold samples.

Table 1: Fundamental mode escape factor λ_0 for a slab geometry with length $2L$.

Opacity $= \kappa_0^{(D)} L$	1.0	3.0	10.0	30.0	100
GQT	0.479	0.230	0.0710	0.0205	0.00519
Numerical [2]	0.469	0.228	0.0713	0.0204	0.0047

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