Multistate generalization of Landau-Zener model

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The famous *two-state* Landau-Zener model [1] received numerous and fruitful applications in various fields, in particular in atomic and molecular physics. It is natural to seek for *multistate* generalizations of the model which retain its basic feature, namely, linear dependence of a Hamiltonian on time. One of such exactly solvable generalizations, Demkov-Osherov model [2], describes set of parallel diabatic potential curves crossed by a slanted curve. The other generalization, the so-called *bow-tie model*, was considered by a number of authors [3]; its complete and rigorous solution for an arbitrary number of states N was recently obtained [4]. However, the physical interpretation of the result remained unclear, because the model implies simultaneous strong interaction of all the states apparently irreducible to any simpler pattern.

We introduce a *generalized bow-tie model* that contains an additional parameter ε and an additional state. Generally the N-state Landau-Zener-type models are characterized by the Hamiltonian matrix H depending linearly on time t,

$$H(t) = Bt + A \ , \tag{1}$$

where A and B are time-independent Hermitian $N \times N$ matrices that generally do not commute. One can always choose the basis of states in which the matrix B is diagonal, $B_{jk} = \beta_j \, \delta_{jk}$;

$$H_{jj} = \beta_j t + A_{jj} \ , \qquad H_{jk} = A_{jk} \equiv V_{jk} \qquad (j \neq k) \ . \tag{2}$$

The diagonal elements H_{jj} of the Hamiltonian matrix (2) are named *diabatic potential curves.* They are slanted straight lines with the slopes β_j. The non-diagonal elements H_{jk} describe the *coupling* between the diabatic basis states; these couplings are time-independent.

In the generalized bow-tie model an arbitrary number of diabatic potential curves cross at the same point, i.e. $A_{jj} \equiv 0$. There are also two particular parallel potential curves, denoted as 0^+ and 0^-, which lie symmetrically, see Fig. 1. They always could be chosen horizontal:

$$H_{0^+ 0^+} = \varepsilon/2 \ , \qquad H_{0^- 0^-} = -\varepsilon/2 \ . \tag{3}$$

All the slanted potential curves are coupled only with the horizontal curves, so that

$$H_{0^+ j} = H_{j \, 0^+} = H_{0^- j} = H_{j \, 0^-} = V_j/\sqrt{2} \ , \qquad H_{0^+ 0^-} = H_{0^- 0^+} = 0 \ . \tag{4}$$

Thus the model contains a large number of parameters suitable for matching: β_j, V_j, ε.

We show that in the generalized bow-tie model all the state-to-state transitions are reduced to the sequence of pairwise transitions. The two-state Landau-Zener model is applied to each of them. Several paths connect initial and final states; the contributions of different paths are summed up coherently to obtain the overall transition probability. The special ET (energy and time)-symmetry intrinsic for the generalized bow-tie model results in a particular property of interference phases: only purely constructive or purely destructive interference is operative. The complete set of transition probabilities is obtained in a closed form. Importantly, the results do not depend on the parameter ε. The conventional bow-tie model is obtained in the limit $\varepsilon \to 0$ where all previously derived results [4] are reproduced. This amounts to rationalization of the bow-tie model by its interpretation in terms of multipath successive two-state transitions.

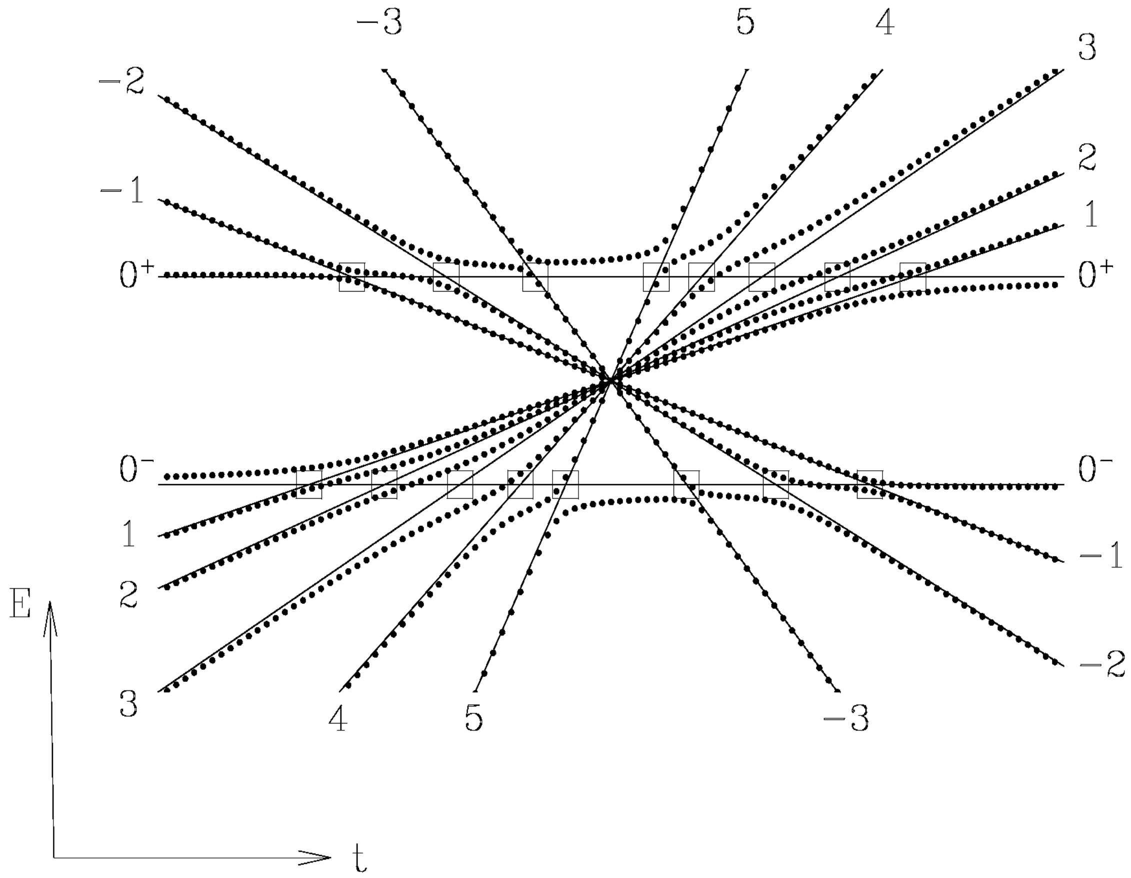

Figure 1: Diabatic (solid lines) and adiabatic (dotted curves) potential curves for some particular realization of the generalized bow-tie model. The blocks show domains of pairwise transitions. The slanted diabatic curves are labeled according to the convention of Ref. [4].

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