Quantum Münchhausen effect: radiative corrections increase tunneling probability

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Famous baron von Münchhausen saved himself from a swamp pulling his hairs with his own hands. According to classical physics, such a feat seems to be impossible. However, we live in a quantum world. In the tunneling of a charged particle, the head of the particle wave function can send a photon to the tail which absorbs this photon and penetrates the barrier with enhanced probability.

Formally speaking, we are looking for the effects of radiative corrections to single-particle tunneling. These effects can be described by the Schrödinger equation with the self-energy operator:

$$\hat{H}\Psi(\mathbf{r}) + \int \Sigma(\mathbf{r}, \mathbf{r}'; E)\Psi(\mathbf{r}')d^3r' = E\Psi(\mathbf{r}) \tag{1}$$

where $\hat{H}$ is the unperturbed particle hamiltonian, which includes a barrier potential, and $\Sigma = M - i\Gamma/2$ is the complex nonlocal and energy-dependent operator determined by the coupling to virtual photons and the possibility of real photon emission. The "photon hand" here connects two points $\mathbf{r}$ and $\mathbf{r}'$ of the same wave function. As a result of our calculation we obtained the negative correction $\delta U(\mathbf{r})$ to the potential barrier which leads to a conclusion that jiggling of the photon increases the tunneling amplitude of the particle [1],

$$\delta U(\mathbf{r}) = \frac{Z^2\alpha}{3\pi m^2} \ln \frac{m}{|U_p(r) - E|} \nabla^2 U(\mathbf{r}). \tag{2}$$

As usual, this semiclassical expression is not valid near the turning points where $U_p(r) = E$. However, a very weak logarithmic singularity does not produce any practical limitations on the applicabilty of eq. (2).

Our conclusion that the tunneling probability is enhanced seems to contradict common sense: radiation should cause energy losses and reduce the tunneling amplitude of the charged particle. However, such an argument may be valid only for real photon emission. Real radiation would be impossible if there were no tunneling. Hence, the radiation width must vanish together with the tunneling width. On the contrary, the real part of Σ under the barrier would be present even if the tunneling probability vanishes.

Note that the semiclassical expression (2) for the self-energy operator can be used to estimate a contribution of the electron-electron interaction to the Lamb shift in many-electron atoms.

[1] V.V. Flambaum and V.G. Zelevinsky,*Phys. Rev. Lett.***83** 3108 (1999).