Electroweak Theory

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1 Introduction

The discovery of the electroweak theory crowned long years of investigation on weak interactions. The key earlier developments included Fermi's phenomenological four-fermion interactions for the beta decay, discovery of parity violation and establishment of $V - A$ structure of the weak currents, the Feynman-Gell-Mann CVC (conserved vector current) hypothesis, current algebra and its beautiful applications in the sixties, Cabibbo mixing and lepton-hadron universality, and finally, the proposal of intermediate vector bosons (IVB) to mitigate the high-energy behavior of the pointlike Fermi's interaction theory.

It turned out that the scattering amplitudes in IVB theory still generally violated unitarity, due to the massive vector boson propagator,

$$\frac{-g^{\mu\nu} + q^{\mu}\, q^{\nu}/M^2}{q^2 - M^2 + i\,\epsilon}.$$

The electroweak theory, known as Glashow-Weinberg-Salam theory [1], was born through the attempts to make the hypothesis of intermediate vector bosons for the weak interactions such that it is consistent with unitarity.

The Glashow-Weinberg-Salam (GWS) theory contains, and is in a sense a generalization of, quantum electrodynamics which was earlier successfully established as the quantum theory of electromagnetism in interaction with matter. GWS theory describes the weak and electromagnetic interactions in a single, unified gauge theory with gauge group

$$SU_L(2) \times U(1). \tag{1}$$

Part of this gauge symmetry is realized in the so-called "spontaneously broken" mode; only a $U_{EM}(1) \subset SU_L(2) \times U(1)$ subgroup, corresponding to the usual local gauge symmetry of the electromagnetism, remains manifest at low energies, with a massless gauge boson (photon). The other three gauge bosons $W^{\pm}, Z$, are massive, with masses approximately 80.4 GeV and 91.2 GeV, respectively.

The theory is renormalizable, as conjectured by S. Weinberg and by A. Salam, and subsequently proven by G. 't Hooft (1971), and makes well-defined predictions order by order in perturbation theory.

Since the experimental observation of neutral currents (a characteristic feature of the Weinberg-Salam theory which predicts an extra, neutral massive vector boson, Z, as compared to the naïve IVB hypothesis) at Gargamelle bubble chamber at CERN (1973), the theory has passed a large number of experimental tests. The first basic confirmation also included the discovery of various new particles required by the theory: the charm quark (SLAC, BNL, 1974), the bottom quark (Fermilab, 1977) and the tau (τ) lepton (SLAC, 1975). The heaviest top quark, having mass about two hundred times that of the proton, was found later (Fermilab, 1995). The direct observation of W and Z vector bosons was first made by UA1 and UA2 experiments at CERN (1983).

The GWS theory is today one of the most precise and successful theories in physics. Even more important, perhaps, together with quantum chromodynamics (QCD), which is a $SU(3)$ (color) gauge theory describing the strong interactions (which bind quarks into protons and neutrons, and the latter two into atomic nuclei), it describes correctly - within the present experimental and theoretical uncertainties - all the presently known fundamental forces in Nature, except gravity. The $SU(3)_{QCD} \times (SU_L(2) \times U(1))_{GWS}$ theory is known as the standard model (SM).

Both the electroweak (GSW) theory and quantum chromodynamics are gauge theories with a nonabelian (non-commutative) gauge group. This type of theories, known as Yang-Mills theories, can be constructed by generalizing the well-known gauge principle of quantum electrodynamics to more general group transformations. It is a truly remarkable fact that all of the fundamental forces known today (apart from gravity) are described by Yang-Mills theories, and in this sense a very nontrivial unification can be said to underlie the basic laws of Nature (G. 't Hooft).

There are further deep and remarkable conditions (anomaly cancellations), satisfied by the structure of the theory and by the charges of experimentally known spin $\frac{1}{2}$ elementary particles (see Tables 1 and 2), which guarantees the consistency of the theory as a quantum theory.

It should be mentioned, however, that the recent discovery of neutrino oscillations (SuperKamiokande (1998), SNO, KamLAND, K2K experiments), which proved the neutrinos to possess nonvanishing masses, clearly indicates that the standard Glashow-Weinberg-Salam theory must be extended, in an as yet unknown way.

The following is a brief summary of the GWS theory, its characteristic features, its implications to the symmetries of Nature, the status of the precision tests, and its possible extensions.

2 Glashow-Weinberg-Salam theory

All the presently known elementary particles (except for the gauge bosons $W^{\pm}$, Z, γ, the gluons, the graviton, possibly right-handed neutrinos) are listed in Table 1, Table 2 and Table 3 together with their charges with respect to the $SU_L(2) \times U(1)$ gauge group.

A doublet of Higgs scalar particles is included even though the physical component (which should appear as an ordinary scalar particle) has not yet been experimentally observed.

The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{quarks} + \mathcal{L}_{leptons} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{g.f.} + \mathcal{L}_{ghosts}.$$

The gauge kinetic terms are

$$\mathcal{L}_{gauge} = -\frac{1}{4}\sum_{a=1}^{3} F^a_{\mu\nu}\, F^{a\,\mu\nu} - \frac{1}{4} G_{\mu\nu}\, G^{\mu\nu};$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\epsilon^{abc} A^b_\mu\, A^c_\nu, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

are $SU_L(2)\times U(1)$ gauge field tensors; $\mathcal{L}_{g.f.}$ and $\mathcal{L}_{FP}$ are the so-called gauge-fixing term and Faddeev-Popov ghost term, needed to define the gauge boson propagators appropriately and to eliminate certain unphysical contributions. The gauge invariance of the theory is ensured by a set of identities (A. Slavnov, J.C. Taylor). The quark kinetic terms have the form,

$$\mathcal{L}_{quarks} = \sum_{quarks} \bar{\psi}\, i\, \gamma^\mu\, \mathcal{D}_\mu \psi$$

where $\mathcal{D}_\mu$ are appropriate covariant derivatives,

$$\mathcal{D}_\mu\, q_L = (\partial_\mu - \frac{i\,g}{2} \tau \cdot A_\mu - \frac{i\,g'}{6} B_\mu)\, q_L$$

for the lefthanded quark doublets,

$$\mathcal{D}_\mu\, u_R = (\partial_\mu - \frac{2\,i\,g'}{3} B_\mu)\, u_R; \qquad \mathcal{D}_\mu\, d_R = (\partial_\mu + \frac{i\,g'}{3} B_\mu)\, d_R;$$

and similarly for other "up" quarks c_R (charm) and t_R (top), and "down" quarks, s_R (strange) and b_R (bottom). Analogously, the lepton kinetic terms are given by

$$\begin{aligned} \mathcal{L}_{leptons} &= \sum_{i=1}^{3} \bar{\psi}^i\, i\, \gamma^\mu\, \mathcal{D}_\mu \psi^i = \sum_{i=1}^{3} \bar{\psi}^i_L\, i\, \gamma^\mu (\partial_\mu - ig \frac{\tau^a A^a_\mu}{2} + \frac{i\,g'}{2} B_\mu\,)\, \psi^i_L \\ &+ \sum_{i=1}^{3} \bar{\psi}^i_R\, i\, \gamma^\mu (\partial_\mu + i\, g'\, B_\mu)\, \psi^i_R; \end{aligned}$$

where $\psi^i\, (i = 1, 2, 3)$ indicate the e, μ, τ lepton families; finally the part involving the Higgs fields are

$$\mathcal{L}_{Higgs} = \mathcal{D}_\mu \phi * \mathcal{D}^\mu \phi + V(\phi, \phi^\dagger), \quad V(\phi, \phi^\dagger) = -\mu^2\, \phi^\dagger \phi - \lambda\, (\phi^\dagger \phi)^2,$$

and

$$\mathcal{L}_{Yukawa} = \sum_{i,j=1}^{3} [\, g_d^{ij}\, \bar{q}_L^i \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R^j + g_u^{ij}\, \bar{q}_L^i \begin{pmatrix} \phi^{0\,*} \\ -\phi^- \end{pmatrix} u_R^j \,] + h.c.$$
$$+ \sum_{i,j=1}^{3} [\, g_e^i\, \bar{\psi}_L^i \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \psi_R^i \,] + h.c.. \qquad (2)$$

For $\mu^2 < 0$ the Higgs potential has a minimum at

$$\langle\, \phi^\dagger\, \phi \,\rangle = \langle\, |\phi^+|^2 + |\phi^0|^2 \,\rangle = -\frac{\mu^2}{2\,\lambda} \equiv \frac{v^2}{2} \neq 0.$$

By choosing conveniently the direction of the Higgs field, its vacuum expectation value (VEV) is expressed as

$$\langle \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \qquad v \equiv \sqrt{-\frac{\mu^2}{\lambda}}. \qquad (3)$$

The physical properties of Higgs and gauge bosons are best seen by choosing the so-called unitary gauge,

$$\Phi(x) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = e^{i\,\zeta^a(x)\,\tau^2/v} \begin{pmatrix} 0 \\ (v + \eta(x))/\sqrt{2} \end{pmatrix} \equiv U(\zeta)\, \Phi'(x),$$
$$\psi_L = U(\zeta)\, \psi'_L; \qquad \psi_R = \psi'_R,$$
$$A_\mu = U(\zeta)\, (A'_\mu + \frac{i}{g}\partial_\mu)\, U^{-1}(\zeta), \qquad A_\mu \equiv \frac{\tau^a A_\mu^a}{2},$$

and expressing everything in terms of primed variables. It is easy to see that

(i) there is one physical scalar (Higgs) particle with mass,

$$m_\eta = \sqrt{-2\,\mu^2}; \qquad (4)$$

(ii) the Higgs kinetic term $(\mathcal{D}\phi'^{\,\dagger})(\mathcal{D}\phi')$ produces the gauge boson masses

$$M_{W^\pm}^2 = \frac{g^2\, v^2}{4}; \quad M_Z^2 = \frac{v^2}{4}\, (g^2 + g'^{\,2}); \qquad (5)$$

(iii) the physical gauge bosons are the charged $W^{\pm}$, and two neutral vector bosons described by the fields

$$Z_\mu = \cos\theta_W \, A_{3\,\mu} - \sin\theta_W \, B_\mu; \qquad A_\mu = \sin\theta_W \, A_{3\,\mu} + \cos\theta_W \, B_\mu,$$

where the mixing angle

$$\theta_W = \tan^{-1}\frac{g'}{g} \quad (\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}})$$

is known as the Weinberg angle. The massless A_μ field describes the photon.

2.1 Fermi interactions and neutral currents

The fermions interact with gauge bosons through the charge and neutral currents

$$\mathcal{L} = \frac{g}{2}\,(J_{-\,\mu}\,W_+^\mu + J_{+\,\mu}\,W_-^\mu) + \mathcal{L}^{n.c.}, \tag{6}$$

$$\mathcal{L}^{n.c.} = g\,J_\mu^3\,A^{3\,\mu} + \frac{g'}{2}\,J_\mu^Y\,B^\mu = e\,J_{em}^\mu\,A_\mu + \frac{g}{\cos\theta_W}\,J_\mu^0\,Z^\mu, \tag{7}$$

where

$$J_{+\,\mu} = \sum \bar\psi_L\,\gamma_\mu\,\tau^+\,\psi_L = \frac{1}{2}\sum \bar\psi\,\gamma_\mu\,\tau^+\,(1-\gamma_5)\,\psi \equiv \frac{1}{2} J_{+\,\mu}^{V-A}; \tag{8}$$

corresponds to the standard charged current, and

$$J_\mu^0 = J_\mu^3 - \sin^2\theta_W\,J_\mu^{em}, \tag{9}$$

is the neutral current to which the Z boson is coupled ($J_\mu^3 = \frac{1}{2}\sum\bar\psi_L\,\gamma_\mu\,\tau^3\,\psi_L$ and J_μ^{em} is the electromagnetic current). The model thus predicts the existence of neutral current processes, mediated by the Z boson, such as $\nu_\mu\,e \to \nu_\mu\,e$ or $\bar\nu_\mu\,e \to \bar\nu_\mu\,e$, with cross section of the same order of that for the charged current process, $\bar\nu_e\,e \to \bar\nu_e\,e$, but with a characteristic L-R asymmetric couplings depending on the Weinberg angle. By Eq.(9) appropriate ratios of cross sections, such as $\sigma(\nu_\mu\,e \to \nu_\mu\,e)/\sigma(\bar\nu_\mu\,e \to \bar\nu_\mu\,e)$ can be used to measure $\sin^2\theta_W$.

The exchange of heavy W bosons generate an effective current-current interaction at low energies

$$\mathcal{L}_{eff}^{c.c} = -\frac{g^2}{2\,M_W^2}\, J_{-\,\mu}\, J_+^{\mu},$$

the well-known Fermi-Feynman-Gell-Mann Lagrangian $-\frac{G_F}{\sqrt{2}}\, J^{\dagger}_{V-A\,\mu}\, J^{\mu}_{V-A}$, with

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\,M_W^2}.$$

This means that the Higgs vacuum expectation value must be taken to be

$$v = 2^{-1/4}\, G_F^{-1/2} \simeq 246\,\mathrm{GeV}. \tag{10}$$

2.2 Masses

It is remarkable that *all* known masses of the elementary particles - except perhaps those of the neutrino masses - are generated in GWS theory through the spontaneous breakdown of $SU_L(2) \times U(1)$ symmetry, through the Higgs vacuum expectation value VEV, Eq.(3), Eq.(10). The boson masses are given by (4) and (5). Note that the relation

$$\rho = \frac{M_W^2}{M_Z^2\, \cos^2\theta_W} = 1 + O(\alpha),$$

reflects an acccidental $SO(3)$ symmetry present (note the $SO(4)$ symmetry of the Higgs potential in the limit $\alpha \to 0$, before the spontataneous breaking) in the model, called custodial symmetry. This is a characteristic, model-dependent feature of the minimal model, not necessarily required by the gauge symmetry. This relation is well met experimentally, although a quantitative discussion requires the choice of the renormalization scheme (including the definition of $\sin\theta_W$ itself) and check of consistency with various other data.

The fermions get mass through the Yukawa interactions, Eq. (2); the fermion masses are arbitrary parameters of the model and cannot be predicted within the GWS theory. An important feature of this mechanism is that the coupling of the physical Higgs particle to each fermion is *proportional* to the mass of the latter. This should give a clear, unambiguous experimental signature for the Higgs scalar of the minimal GWS model.

The recent discovery of nonvanishing neutrino masses requires the theory to be extended. Actually, there is a natural way to incorporate such masses in the standard GWS model, by a minimal extension. As the right-handed neutrinos, if they exist, are entirely neutral with respect to the $SU_L(2)\times U(1)$ gauge symmetry, they need not its breaking to have mass. In other words, ν_R may get Majorana masses, $\sim M_R\,\nu_R\,\nu_R$, by some yet unknown mechanism, much larger than those of other fermions (such a mechanism is quite naturally present in some grand unified models). If now the Yukawa couplings are introduced as for the quarks and for the down leptons, then the Dirac mass terms result upon condensation of the Higgs field, and the neutrino mass matrix would take the form, for one flavor (in the space of $(\nu_L, \bar\nu_R)$):

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}. \tag{11}$$

If the Dirac masses are assumed to be of the same order of those of the quarks and if the right-handed Majorana masses M_R are far larger, *e.g.*, of the order of the grand unified scale, $O(10^{16}\,\mathrm{GeV})$, then diagonalization of the mass matrix would give for the physical masses of the left-handed neutrinos, $\sim \frac{m_D^2}{M_R} \ll m_D$, much smaller than other fermion masses, quite naturally ("see-saw" mechanism).

2.3 CKM quark-mixing

As there is a priori no reason why the weak-interaction eigenstates should be equal to the mass eigenstates, the Yukawa couplings in Eq. (2) are in general nondiagolal matrices in the flavor. Suppose that the the weak base for the quarks is given in terms of the mass eigenstates (in which quark masses are made diagonal), by unitary transformations

$$u_{L\,i} = \sum_j V_{ij}^{up}\,\tilde u_{L\,j}; \quad d_{L\,i} = \sum_j V_{ij}^{down}\,\tilde d_{L\,j},$$

then the interaction terms with $W^\pm$ bosons (6) can be cast in the form [2]

$$\mathcal{L}^{W-exc} = \bar u_L^i\,\gamma^\mu\,W_\mu^+\,U_{ij}^{(CKM)}\,d_L^j + \bar d_L^k\,\gamma^\mu\,W_\mu^-\,U_{k\ell}^{(CKM)\,\dagger}\,u_L^\ell, \tag{12}$$

where $U_{ij}^{CKM} \equiv (V^{up\,\dagger} \cdot V^{down})_{ij}$ is called Cabibbo-Kobayashi-Maskawa (CKM) matrix. It can be parametrized in terms of three Euler angles and one phase

$$
\begin{aligned}
U &= \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\, e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\, e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\, e^{i\delta_{13}} & c_{12}s_{23} - s_{12}c_{23}s_{13}\, e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}
\end{aligned}
\tag{13}
$$

where $c_{12} = \cos\theta_{12}$, $s_{23} = \sin\theta_{23}$, etc. The requirement that charge-current weak processes are all described by these matrix elements, satisfying the unitarity relation,

$$\sum_{\ell} U_{i\ell}^{CKM}\, U_{\ell k}^{CKM\,\dagger} = \delta_{ik}, \tag{14}$$

gives a very stringent test for the validity of the model.

2.4 CP violation

CP (product of charge conjugation and parity transformation) invariance is an approximate symmetry of Nature. Although it is known to be broken by very tiny amounts only, the exact extent and the nature of CP violation can have far-reaching consequences.

CP violation has first been discovered by Cronin and Fitch (BNL, 1964) in the K-meson system; more precise information on the nature of CP violation from the neutral kaon decays has been obtained more recently (2000) in NA48 (CERN) and KTeV (Fermilab) experiments. CP violation has been established in the B-meson systems as well, very recently (2002), by Babar experiments at SLAC and Belle experiments at KEK.

Through the so-called CPT theorem, CP invariance (or violation) is closely related to the T (time reversal invariance) symmetry. Also, CP non invariance is one of the conditions needed in the cosmological baryon number generation (baryogenesis).

In the Glashow-Weinberg-Salam theory, with three families of quark flavors (six quarks), there is just one source of CP violation: the phase δ_{13} appearing in the CKM matrix Eq. (13). For $\delta \neq 0, \pi$, W-exchange interactions (12) induce CP violation. The earlier and more recent experimental data on $K^0 - \bar{K}^0$ mixing and $K_{L,S}$ decay data appear to be compatible with the CKM mechanism for CP violation, but a quantitative comparison with the standard model remains somewhat hindered by the difficulty of estimating certain strong interaction effects. The recent confirmation of CP violation in B systems is made in the context of a global fit with the standard model predictions such as the "unitarity triangle" relations, *e.g.*,

$$1 + \frac{U_{ud}\, U_{ub}^*}{U_{cd}\, U_{cb}^*} + \frac{U_{td}\, U_{tb}^*}{U_{cd}\, U_{cb}^*} = 0, \tag{15}$$

(Eq. (14)), and by combining data from kaon deays, charmed meson decays, B meson decay and mixings, etc., and is a part of direct tests of the GWS model, with nonvanishing CP violation CKM phase (Eq. (16) and Fig. 1).

Recent evidence for nonzero neutrino masses and mixings, opens the way to possible CP violation in the leptonic processes as well.

Finally, within the standard model including strong interactions, there is one more source of CP violation: the so-called θ (vacuum) parameter of QCD.

2.5 B and L nonconservation

Another set of approximate symmetries in Nature are the baryon and lepton number conservations. In the electroweak theory, these global symmetries are exact to all orders of perturbation theory. Nonperturbative effects (a sort of barrier penetration in gauge field space) however violate both B and L; the combination $B - L$ is conserved even nonperturbatively though. The nonperturbative electroweak baryon number violation is an extremely tiny effect, the amplitude being proportional to the typical tunneling factor $e^{-2\,\pi/\alpha}$, but the process is unsuppressed at finite temperatures as might have been experienced by the universe at some early stage after Big Bang.

B or L nonconservation can also arise naturally at high energy scales, if the electroweak theory is embedded as the low-energy approximation in a grand unified model. The experimental lower limit of proton lifetime, $\tau_P \geq 10^{32}$ yrs, from Kamiokande experiments, however severely restricts acceptable models of this type (the simplest $SU(5)$ model is already ruled out).

On the other hand, cosmological baryogenesis requires sufficient amount of baryon number violation, at least in some stage of cosmological expansion. Detailed analyses suggest that the standard electroweak transition might not in itself explain the baryon number $n_P/n_\gamma \sim 10^{-10}$ observed in the present universe. Recent observations of neutrino oscillations suggest the right-handed Majorana type neutrino masses to be present, which violate the lepton number L. In such a case it might be possible that the correct amount of baryon number excess would be generated, through the leptogenesis.

2.6 Global fit

Various relations exist at the tree level among the masses, scattering cross sections, decay rates, various asymmetries, etc., which can be read off or calculated from the formulas given in Section 2. These quantities receive corrections at higher orders, and the experimental checks of these modified relations provide precision tests of the model on the one hand, and possibly a hint for new physics, if there is any discrepancy with the prediction. Very often the amplitudes of interest receive important contributions due to strong interactions, which are difficult to estimate.

The basic parameters of the model, apart from the Higgs mass, and fermion masses and mixing parameters, can be taken to be (i) the fine structure constant, $\alpha = 1/137.03599911(46)$; (ii) the Fermi constant $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2} (which can be determined from the muon lifetime), and the Z-boson mass, $M_Z = 91.1876 \pm 0.0021$ GeV (observed directly at LEP). M_W and $\sin^2\theta_W$ are then calculable numbers, in terms of these quantities, and depending on m_t (measured independently by CDF and D$\emptyset$ experiments at Fermilab) and on the unknown M_H.

Such precision tests of the GWS model are being made, combining the analyses of various decay rates and asymmetries in B-meson systems at B factories and in colliders, production and decays of Z and W bosons, elastic $\nu\, e$ or $\bar{\nu}\, e$ scatterings, elastic $\nu\, p$ or or $\bar{\nu}\, p$ scatterings, deep inelastic lepton nucleon (or deuteron) scatterings, the muon anomalous magnetic moment, atomic parity violation experiments, etc.

An overall fit to the data gives an excellent agreement, with the input parameters

$$M_H = 113^{+56}_{-40}\,\text{GeV}, \quad m_t = 176.9 \pm 4.0\,\text{GeV}, \quad \alpha_s(M_Z) = 0.1213 \pm 0.0018.$$

For instance (in GeV)

$$M_W = 80.390 \pm 0.018 \qquad \text{vs} \qquad 80.412 \pm 0.042 \quad \text{(exp. value (LEP))};$$

$$\Gamma_Z = 2.4972 \pm \pm 0.0012 \qquad \text{vs} \qquad 2.4952 \pm 0.0023 \quad \text{(exp. value)}.$$

For $\sin^2\theta_W$ (defined in the so-called $\overline{MS}$ scheme) all data give consistently the value

$$\sin^2\theta_W = 0.23120 \pm 0.00015,$$

(a slightly larger value is reported by an $\nu\, N$ experiment at Fermilab).

The unitarity-triangle tests of the standard model and determination of CKM matrix have already been mentioned. The results of global fit can be summarized in Fig. 1, and by the angles

$$s_{12} = 0.2243 \pm 0.0016, \quad s_{23} = 0.0413 \pm 0.0015, \quad s_{13} = 0.037 \pm 0.0005,$$

$$\delta_{13} = 60^o \pm 14^o. \tag{16}$$

For the muon anomalous gyromagnetic ratio $(g-2)$, the experimental data

$$a_\mu^{exp} = \frac{g_\mu - 2}{2} = (1.1165920.37 \pm 0.78) \times 10^{-9},$$

is to be compared with the theoretical prediction

$$a_\mu^{th} = (1.1165918.83 \pm 0.49) \times 10^{-9},$$

which is slightly smaller $(1.9\,\sigma)$, where the largest theoretical uncertainty comes from the two loop hadronic contribution $a_\mu^{had} \simeq (69.63 \pm 0.72) \cdot 10^{-9}$ (the QED corrections to $O(\alpha^5)$ are included).

For further details of the analyses and the present status of experimental tests of the electroweak theory, see the reviews by J. Erler and P. Langacker, and by F.J. Gilman et. al., cited in Further Reading (most of numbers cited here come from these two reviews).

3 Need for extension of the model

In spite of such an impressive experimental confirmation, there are reasons to believe that the electroweak theory, in its standard minimal form, is not a complete story. As already mentioned, neutrino oscillations, predicted earlier by Pontecorvo, have recently been experimentally confirmed, giving uncontroversial evidence for nonvanishing neutrino masses and their mixing. This is a clear signal that the theory must be extended. If the mass is instead taken in the form of Eq. (11) but with three neutrinos families, the diagonalization in general yields a mixing for the light neutrinos, as for the quarks. Some of the experimental data on the neutrinos are summarized in Table 7.

Also, the Higgs sector of the theory (the part of the interactions responsible for spontaneous breaking $SU_L(2) \times U(1) \to U_{EM}(1)$) is still largely untested. The theory predicts a physical scalar particle, the Higgs particle, of unknown mass. The present-day expectation for its mass, which combines the experimental lower-limit and an indirect upper limits following from the analysis of various radiative corrections, is

$$114\,(\text{GeV}) < m_H < 250\,(\text{GeV}).$$

This particle should be observable either in the Tevatron at Fermilab or in the coming LHC experiments at CERN; negative results would force upon us a substantial modification of the electroweak theory.

Last, but not least, there are a few theoretical motivations for an extension of the model to be considered necessary. First, the structure of the GWS theory is not entirely determined by the gauge principle. The form of the Higgs self interactions, as well as their number and the Yukawa couplings of the Higgs scalar to the fermions, are unconstrained by any principle, and the particular, minimal form assumed by Weinberg and Salam is yet to be confirmed experimentally.

Also, the theory is not really a unified gauge theory: $SU_L(2)$ and $U(1)$ gauge couplings are distinct. One possibility is that the $SU(3)_{QCD} \times SU_L(2) \times U(1)$ theory of the standard model is actually a low-energy manifestation of a truly unified gauge theory - grand unified theory (GUT) - defined at some higher mass scale. The simplest version of GUT models based on $SU(5)$ or $SO(10)$ gauge groups, have however a difficulty with the proton decay rates, and with the coupling-constant unification itself. Supersymmetric GUTs appear to be more acceptable both from the coupling-constant unification and from the proton lifetime constraints.

A more subtle, but perhaps more severe theoretical problem is the so-called naturalness problem. At the quantum level, due to the quadratic divergences in the scalar mass, the structure of the theory turns out to be quite peculiar. If the ultraviolet cutoff of the theory is taken to be the Planck mass scale, $\Lambda_{UV} \sim m_{Pl} \sim 10^{19}\,\mathrm{GeV}$, at which gravity becomes strongly coupled, the theory at Λ_{UV} would have to possess parameters which are fine-tuned with an excessive precision. The problem is known also as a "hierarchy" problem.

A way to avoid having such a difficulty is to introduce supersymmetry. In a supersymmetric version of the standard theory - in fact there are phenomenologically well-acceptable models such as the MSSM (minimal supersymmetric standard model) - this problems is absent due to the cancellation of bosonic and fermionic loop contributions typical of supersymmetric theories. As a result, the properties of the theory at low-energies are much less sensitive to those of the theory at the Planck mass scale. Experiments at LHC (≥ 2008, CERN) should be able to produce a whole set of new particles associated with supersymmetry, if this is part of the physical law beyond TeV energies.

At a deeper level, however, the hierarchy problem in a more general sense persists, even in supersymmetric models: why masses of order of $O(100\mathrm{GeV})$ at all, in a theory with a natural cutoff of the order of the Planck mass? Furthermore, if the masses of the neutrinos turn out to be of order of $O(10^{-3}-10^{0})\,\mathrm{eV}$, we are left with the problem of understanding the large disparities *among* the quark and lepton masses, spanning the range of more than 13 orders of magnitudes: another "hierarchy" problem.

It is also possible that the spacetime the physical world lives in is actually higher dimensional: the usual four-dimentional Minkowski spacetime times either compactified or uncompactified *extra dimensions*. In theories of this type, some of the difficulties mentioned above might find a natural solution. It is to be seen, whether a consistent theory of this type can be constructed, which correctly account for the properties of the universe we inhabit.

Keywords/See also

Gauge theory
Yang-Mills theory
Spontaneous symmetry breaking
Higgs mechanism
Renormalization theory
Grand unified theories
Quantum electrodynamics
Quantum chromodynamics
Standard model of particle physics
Chirality
Supersymmetry
Superstring theory
Hierarchy problem
Extra dimensions

Further reading

A short but comprehensive introduction to the Weinberg-Salam theory is found in: J. C. Taylor (1976), Gauge theories of weak interactions, Cambridge University Press; see also, E.S. Abers and B.W. Lee (1973), Gauge Theories, Physics Reports C9, 1-141; G. 't Hooft and M. Veltman (1973), Diagrammar, CERN Yellow Report 73-9.

A reprint collection, Gauge theories in the twentieth century, Editor, J.C. Taylor, Imperial College Press (2001), contains many of fundamental papers, *e.g.*, on Yang Mills theories (by C.N. Yang, R.L. Mills, R. Shaw), on spontaneous symmetry breaking and its application to gauge theories (by Y. Nambu, J. Schwinger, P.W. Anderson, P.W. Higgs, F. Englert, R. Brout, T.W.B. Kibble) and on renormalization of Yang-Mills theories and application to the electroweak theory (L.D. Faddeev, V.N. Popov, G. 't Hooft).

For up-to-date review on precision tests of the GWS theory, and details of the analyses, see J. Erler and P. Langacker (2004), Electroweak Model and Constraints on New Physics, in S. Eidelman, et. al., Review of Particle Physics.

Particle Data Group, Phys. Lett. B592, 1-1109, [http://pdg.lbl.gov/2005], and references cited therein.

For a recent review on neutrino experiments, see J. Shirai (2005), Neutrino Experiments: Review of Recent Results, Nuclear Physics B (Proc. Suppl.) 144, 286-296. For theory on neutrinos, see M. Fukugita and T. Yanagida (2003), Physics of Neutrinos and Applications to Astrophysics, Springer-Verlag Berlin.

For the unitarity triangle test of the GWS model and determination of CKM matrix elements, see:

CKM fitter Group (A. Bret et. al.) (2005), CP Violation and the CKM Matrix: Assesing the Impact of the Asymmetric B Factories, Eur. Phys. J. C41, 1-131 [hep-ph/0406184];

UTfit Collaboration (M. Bona, et. al.) (2005), The 2004 UTfit Collaboration Report on the Status of the Unitarity Triangle in the Standard Model, JHEP 0507, 028-059, [hep-ph/0501199];

and a review:

F.J. Gilman, K. Kleinknecht and B. Renk, The Cabibbo-Kobayashi-Maskawa Quark-Mixing Matrix, in S. Eidelman, et. al. (PDG), Review of Particle Physics. Particle Data Group, Phys. Lett. B592, 1-1109, [http://pdg.lbl.gov/2005], and references cited therein.

References

[1] S. Weinberg (1967), Theory of Leptons, Phys. Rev. Lett. 19, 1264-1266; A. Salam (1968), Weak and electromagnetic interactions, in "Elementary Particle Theory", ed. N. Svartholm, Almqvist Forlag AB, 367; S.L. Glashow, J. Iliopoulos and L. Maiani (1970), Weak Interactions with Lepton-Hadron Symmetry, Phys. Rev. D2, 1285-1292;

[2] M. Kobayashi and T. Maskawa (1972), CP Violation in Renormalizable Theory of Weak Interaction, Progr. Theor. Physics, 49, 652-657.

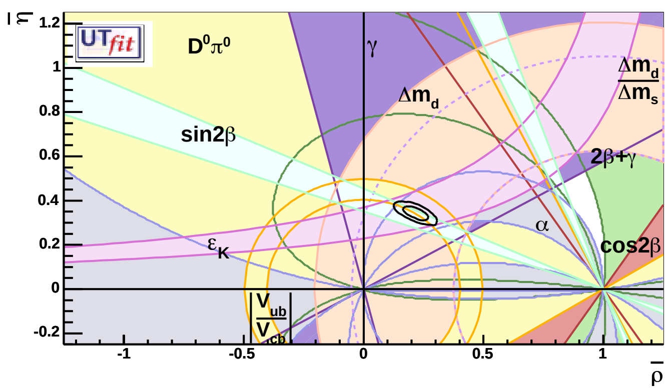

Figure 1: Unitarity triangle test (Eq.(15)) taken from UTfit collaboratio (2005), JHEP 0507, 028-059. The small ellipses represent 68% and 95% probability zones for the apex corresponding to $-\frac{U_{ud}U^*_{ub}}{U_{cd}U^*_{cb}}$.

4 Tables

Quarks and Their Charges

Quarks	$SU_L(2)$	$U_Y(1)$	$U_{EM}(1)$
$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \begin{pmatrix} t_L \\ b'_L \end{pmatrix}$	$\underline{2}$	$\frac{1}{3}$	$\begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$
$u_R, \quad c_R, \quad t_R$	$\underline{1}$	$\frac{4}{3}$	$\frac{2}{3}$
$d_R, \quad s_R, \quad b_R$	$\underline{1}$	$-\frac{2}{3}$	$-\frac{1}{3}$

Table 1: The primes indicate that the mass eigenstates are different from the states transforming as multiplets of $SU_L(2) \times U_Y(1)$. They are linearly related by Cabibbo-Kobayashi-Maskawa mixing matrix.

Leptons and Their Charges

Leptons	$SU_L(2)$	$U_Y(1)$	$U_{EM}(1)$
$\begin{pmatrix} \nu'_{e\,L} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu'_{\mu\,L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu'_{\tau\,L} \\ \tau_L \end{pmatrix}$	$\underline{2}$	-1	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$e_R, \quad \mu_R, \quad \tau_R$	$\underline{1}$	-2	-1

Table 2: The primes indicate again that the mass eigenstates are in different from the states transforming as multiplets of $SU_L(2) \times U_Y(1)$, as required by the observed neutrino oscillations.

Higgs doublet

Higgs doublet	$SU_L(2)$	$U_Y(1)$	$U_{EM}(1)$
$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	$\underline{2}$	1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Table 3: Higgs doublet scalars and their charges

Quark Masses

u (MeV)	c (GeV)	t (GeV)	d (MeV)	s (MeV)	b (GeV)
$1.5 - 4$	$1.15 - 1.35$	174.3 ± 5.1	$4 - 8$	$80 - 130$	$4.1 - 4.4$

Table 4:

Leptons Masses

ν_e (eV)	ν_μ (MeV)	ν_τ (MeV)
< 3	< 0.19	< 18.2
e (MeV)	μ (MeV)	τ (MeV)
$0.51099892 \pm 4 \cdot 10^{-8}$	$105.658369 \pm 9 \cdot 10^{-6}$	1776.99 ± 0.26

Table 5:

Gauge Boson Masses

photon	gluons	$W^\pm$ (GeV)	Z (GeV)
0	0	80.425 ± 0.038	91.1876 ± 0.0021

Table 6:

Neutrino Mass Square Differences and Mixing

ν_e ν_μ ν_τ
$\Delta_{12}\, m^2 = (6-9) \cdot 10^{-5}$ eV^2 $\Delta_{23}\, m^2 = (1-3) \cdot 10^{-3}$ eV^2

Table 7: Solar neutrinos and reactor (SNO, SuperKamiokande, KamLAND) experiments give the first results. Atmospheric neutrino data and the long baseline experiment (SuperKamiokande, K2K) provide the second. The mixing angle relevant to the solar and reactor neutrino oscillation is large, $\tan^2\theta_{12} \sim 0.40^{+0.10}_{-0.07}$, while the one related to the atmospheric neutrino data is maximal, $\sin^2 2\theta_{23} \sim 1$. Cosmological considerations give $\sum m_{\nu_i} < O(1\,\mathrm{eV})$.