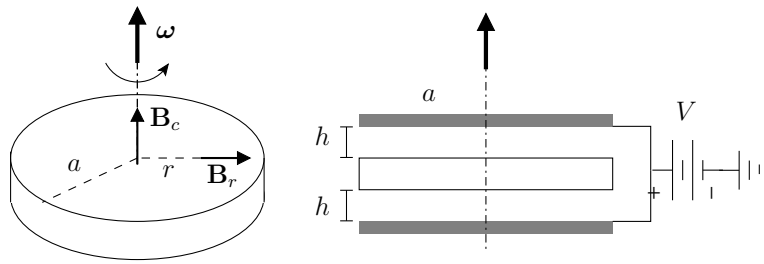
The Rowland experiment



This experiment by Henry A. Rowland (1876) aimed to show that moving charges generate magnetic fields. A metallic disk or radius a and thickness $\ll a$ is electrically charged and kept in rotation with period $T = 2\pi/\omega$.

a) To charge the disk up, it is placed between two capacitor plates which are separated by $h \simeq 1$ cm and kept at a potential $V_0 = 10^4$ V with respect to the disk. Find the surface charge density on the disk surfaces.

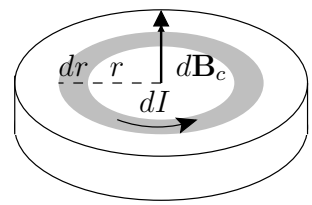
b) Caclulate the magnetic field $\mathbf{B}_c$ near the center of the disk and the magnetic field component $\mathbf{B}_r$ parallel and near to the surface, as a function of the distance r from the axis. Typical experimental values were $a = 10$ cm e $T = 10^{-2}$ sec.

c) The field $\mathbf{B}$ generated by the disk at the more convenient point is measured by orienting the apparatus in order that $\mathbf{B}$ is perpendicular to the Earth's magnetic field $\mathbf{B}_E$ of strength $B_E \simeq 5 \times 10^{-5}$ T and measuring the rotation of a magnetic needle when the disk rotates. Find the rotation angle.

Solution

a) Neglecting boundary effects, the electric field $\mathbf{E}_0$ in the region between the disk and the electrodes is uniform and perpendicular to the plane surfaces. Since the disk is at a potential V_0 with respect to both plates which are at a distance $h/2$, the field value is $E_0 = V_0/(h/2) = 2V_0/h$. The field is zero inside the disk, thus the surface charge density σ on both sides is $\sigma = \varepsilon_0 E_0 = 2\varepsilon_0 V_0/h$. Being $\varepsilon_0 = 8.85 \times 10^{-12}$ in SI units we find $\sigma = 2\varepsilon_0 \times 10^6$ V/m $= 1.77 \times 10^{-5}$ C/m^2.

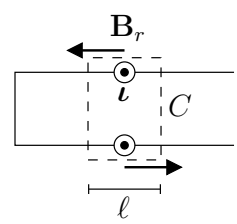
b) In order to evaluate $\mathbf{B}_c$, we imagine to divide the disk in rings of radius r (with $0 < r < a$) and thickness dr. Thus, each ring contains the charge $\mathrm{d}q = \sigma \mathrm{d}S = 2\pi\sigma r \mathrm{d}r$. Due to the rotation of the disk, every ring is equivalent to a coil with a current intensity $\mathrm{d}I = q/T$ that generates at its center a magnetic field of modulus $\mathrm{d}B = \mu_0 \mathrm{d}I/2r$ and perpendicular to the coil plane. The total field is given by the integral



$$B_c = 2\int_0^a \frac{\mu_0 \mathrm{d}I}{2r} = \frac{2\pi}{T}\mu_0 \sigma a \ , \tag{1}$$

where the factor of 2 accounts for the contribution of both surfaces of the disk.

In order to evaluate $\mathbf{B}_r$ we use Ampere's law applied to the path C shown in the figure. The path is placed at a distance r from the axis with the sides parallel to the surface having length $\ell \ll r$, to that $\mathbf{B}_r$ is approximately constant along the sides. The contribution of the perpendicular sides to the line integral cancel each other, so



$$\oint_c \mathbf{B} \cdot d\mathbf{l} \simeq 2B_r \ell \ , \tag{2}$$

where the antisymmetry of $\mathbf{B}_r$ with respect to the midplane has been used. The rotation of the disk leads to a surface current $\boldsymbol{\iota} = \sigma \mathbf{v}_\phi = \sigma\omega r \boldsymbol{\phi}$, so that the total current flowing through C is $I_c = 2\iota\ell = 2\sigma\omega r$. By posing $2B_r\ell = \mu_0 I_c$ we obtain

$$B_r = \mu_0 \sigma \omega r = \frac{2\pi}{T}\mu_0 \sigma r \ . \tag{3}$$

We thus obtain that, for $r = a$, the maximum value of $B_r = B_c$.

Inserting the result of point **a)** we estimate the field strength as

$$B = \frac{2\pi}{T}\mu_0 a \left(2\varepsilon_0 \frac{V}{h}\right) = \frac{4\pi}{T}\frac{Va}{hc^2} = 1.4 \times 10^{-9} \text{ Tesla} \ . \tag{4}$$

c) The rotation angle is given by $\tan\theta = B/B_T$, then

$$\theta \simeq \frac{B}{B_T} = 2.8 \times 10^{-5} \text{ rad} = 1.6 \times 10^{-3} \text{ deg} \ . \tag{5}$$

Thus, a very small angle of rotation is expected, so the experiment required exceptional care to measure it.