

Comment on "Stochastic Resonance in Bistable Systems"

In a recent paper,¹ Gammaitoni *et al.* described an important experimental and theoretical study of stochastic resonance² (SR) in a system simulating a Brownian particle which moves in a symmetric double-well potential. In particular, the remarkable enhancement by additive white noise of the signal-to-noise ratio R for weak periodic forcing of the system was investigated.

We would like to comment that a qualitative and quantitative understanding of the latter phenomenon, and of the dependence of R on relevant parameters, can be gained through an analysis of the power spectral density $Q(\omega)$ of the system in the *absence* of periodic forcing, which is already known (see Ref. 3, and references therein). SR is connected directly with a pronounced narrow peak in $Q(\omega)$ at $\omega=0$ resulting from noise-induced interwell transitions and with the corresponding peak in the susceptibility of the system $\chi(\omega)$. Such peaks are a universal feature⁴ of bistable systems in the range of parameters where stationary populations of the stable states are close in magnitude.

In the presence of the weak periodic force $A \cos(\Omega t)$, the averaged coordinate of the symmetric system¹ $\langle x(t) \rangle = A \operatorname{Re}[\chi(\Omega) \exp(-i\Omega t)]$. Therefore in the power spectrum there arises a δ -shaped spike at frequency Ω [because, according to the principle of the decay of correlations, $\langle x(t)x(0) \rangle \rightarrow \langle x(t) \rangle \langle x(0) \rangle$ as $t \rightarrow \infty$; see also Ref. 5]. If R is defined⁶ as the ratio of the strength of this spike to $Q(\Omega)$, then

$$R = \frac{1}{4} A^2 |\chi(\Omega)|^2 / Q(\Omega), \quad \chi(\omega) \equiv \chi'(\omega) + i\chi''(\omega) \quad (1)$$

[where $R(A, D)$ of Ref. 1 is equal to $R/\pi\Delta V_{\text{expt}}$]. For the "quasithermal" system under consideration, $\chi(\omega)$ can be expressed easily via the fluctuation-dissipation theorem in terms of $Q(\omega)$ and of the noise-intensity parameter D corresponding¹ to the temperature of the Brownian motion:

$$\chi'(\omega) = \frac{2}{D} \int_0^\infty d\omega_1 [\omega_1^2 / (\omega_1^2 - \omega^2)] Q(\omega_1), \quad (2)$$

$$\chi''(\omega) = (\pi\omega/D) Q(\omega).$$

When only the peak caused by interwell transitions is taken into account in $Q(\omega)$, Eqs. (1) and (2) and Eq. (22) of Ref. 3 result in $R = \frac{1}{2} \pi A^2 x_m^2 \mu_K / D^2$, where x_m is the equilibrium value of the coordinate and the transition probability $\mu_K \propto \exp(-\Delta V/D)$ (ΔV is the height of the potential barrier; we suppose that $\Delta V \gg D$). This expression coincides with Eq. (5) of Ref. 1 for small A and $D/\Delta V$, and it gives explicitly the dependence of R on D . We stress, however, that within the domain of linear-response theory it is, in principle, valid for *arbitrary* Ω/μ_K , not only for small Ω/μ_K as implied in Ref. 1. In practice, the limitations on the range of applicability

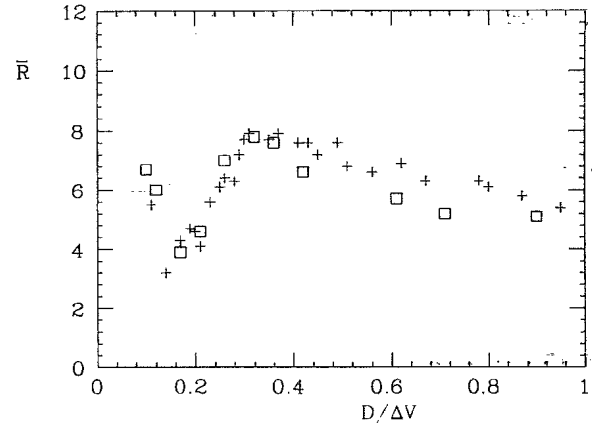


FIG. 1. Comparison of experimental (squares) and calculated (crosses) values of $\bar{R} = 1.54 \times 10^3 R$, as a function of reduced noise intensity $D/\Delta V$, for an electronic model of the system $\ddot{x} = -\gamma\dot{x} + x - x^3 + A \cos(\Omega t) + V(t)$, with $\Omega = 0.06952$, $A = 0.1$, $\gamma = 0.25$, $\langle V(t) \rangle = 0$, and $\langle V(t)V(t') \rangle = 2\gamma D \delta(t-t')$.

come from the other contributions to $Q(\omega)$ discussed in Ref. 3 and it is the latter that give rise to the deviations in R observed in Ref. 1 for $\Omega/2\pi = 500$ Hz.

Measured values of R (squares) are compared with those calculated from $Q(\omega)$ (crosses) in Fig. 1. The calculation contains no adjustable parameters.

We acknowledge valuable discussions with C. J. Lambert and R. J. Watts-Tobin.

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Received 16 October 1989

PACS numbers: 05.40.+j, 02.50.+s

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