Quantum-mechanical dissipation: From the weak- to the strong-coupling limit

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We study a spin-$\frac{1}{2}$ entity interacting with a set of quantum-mechanical oscillators using the Heisenberg picture. In the weak-coupling limit, the Heisenberg equation is given an analytical solution, suggesting that upon increase of the coupling the system should exhibit a critical transition to an overdamped regime, like that of a heavily damped oscillator. Thus the lifetime of the trapped state increases with increasing temperature. Under the assumption of very small tunneling frequency, another analytical solution is found, which predicts the occurrence of trapped states, whose lifetime decreases with an increase of temperature. As a result of the competition between the two trapping processes, the lifetime of the localized state is shown to be a nonmonotonic function of temperature with a minimum lifetime at a certain critical value of temperature. The predictions of this theoretical treatment are carefully checked by numerical solution of the interaction between a $\frac{1}{2}$-spin entity and a dissipative oscillator, and a very satisfactory agreement between theory and numerical solution is found over the whole range of coupling strength.

I. INTRODUCTION

This paper is located at the border between two different fields of investigations: quantum dissipation[1,2] and nonlinear Schrödinger equation.[3-5] In the former field of investigation use is made of the functional-integral approach,[1,2,6] to predict interesting effects of dissipation in quantum mechanics. The most striking effect of dissipation is perhaps the tunneling rate reduction,[1,7] reminiscent of the transition from the inertial to the overdamped regime of a classical oscillator. In the overdamped regime a classical oscillator indeed becomes critically slow upon increasing friction, and it takes it an infinite time to move from a far-from-equilibrium initial condition to the standard equilibrium state. As to the latter field of investigation,[3] we have in mind both the work of Kenkre and co-workers,[4] who applied the nonlinear Schrödinger equation[3] to study the behavior of a two-state quantum-mechanical system and predicted under which physical conditions trapped states might occur, and the related work of other authors.[5]

We approach the problem of quantum dissipation starting from a microscopic and quantum-mechanical Hamiltonian of a two-state system coupled to a bath consisting of a very large number of independent bosonic-type elementary excitations. The choice of the spectral density of the interaction between system and bath, is made in such a way as to render it possible to compare our theoretical predictions with exact numerical calculations. Thus we refer ourselves to a spectral density equivalent to the interaction with a single dissipative oscillator. The dissipation properties of this oscillator mimic the influence of an infinite number of degrees of freedom without involving quantum-mechanical spaces of extremely large dimensions, thereby permitting the use of the standard diagonalization computational routines. First, along the lines illustrated by Dekker,[8] we adopt a weak-coupling approximation which leads us to express the motion of our system under the form of a damped oscillator. If we assume the resulting equations to be valid also in the case of coupling strength of fairly large intensity, this picture leads us to a first mechanism of trapping. This is analogous to the inertialess motion of an oscillator moving very slowly from its initial nonequilibrium position. Then we adopt a different approach, based on the approximation of a weak tunneling frequency. This permits us to reveal a novel trapping mechanism which has nothing to do with the critical slowing down of an underdamped oscillator.

Section II is devoted to the illustration of a microscopic Hamiltonian under study in this paper. We also derive a formally exact equation of motion in the Heisenberg picture. In Section III this general equation of motion is studied under the basic assumption of weak coupling with the bath. Section IV is devoted to the case of a very weak tunneling frequency. In Section V the major theoretical predictions of this paper are checked via computer calculations. Concluding remarks are found in Sec. VI.

II. MODEL OF QUANTUM-MECHANICAL DISSIPATION AND THE HEISENBERG PICTURE

To study quantum-mechanical dissipation we adopt the following Hamiltonian ($\hbar=1$)

$$\mathcal{H}=-\frac{\omega_0}{2}\sigma_z+gx\sigma_x+\sum_i \omega_i b_i^\dagger b_i \ , \tag{2.1}$$

where

$$x\equiv\sum_i x_i \tag{2.2}$$

and

$$x_i \equiv \Gamma_i (b_i^\dagger + b_i) \ .$$

This is a spin-1/2 entity, with Larmor frequency ω_0, interacting with a set of oscillators. By exchanging with one another the x with the z axis, the Hamiltonian of Eq. (2.1) becomes identical to that used in the field of quantum dissipation. For the physical interpretation of this model the reader is invited to consult Ref. 1. In this field of investigation ω_0 is referred to as tunneling frequency.

To deal with this problem, following Lindenberg and West[9] we adopt the Heisenberg picture. This leads us to

$$\begin{aligned} \dot{\sigma}_x &= \omega_0 \sigma_y(t) \ , \\ \dot{\sigma}_y &= -\omega_0 \sigma_x(t) - 2gx\sigma_z(t) \ , \\ \dot{\sigma}_z &= 2gx(t)\sigma_y(t) \ , \\ \dot{x}_i &= v_i(t) \ , \\ \dot{v}_i &= -\omega_i^2 x_i(t) - 2\omega_i \Gamma_i^2 g \sigma_x(t) \ . \end{aligned} \tag{2.3}$$

Solving the last two equations we obtain

$$x_i(t) = x_i^0(t) - 2\Gamma_i^2 g \int_0^t ds \, \sin[\omega_i(t-s)]\sigma_x(s) \ , \tag{2.4}$$

where

$$x_i^{(0)} = x_i(0)\cos(\omega_i t) + \frac{v_i(0)}{\omega_i}\sin(\omega_i t) \ . \tag{2.5}$$

Equation (2.5) expresses the motion of the ith oscillator in the absence of interaction with the spin-$\frac{1}{2}$ system. Replacing Eq. (2.4) in the second and third equation of the set of (2.3) we get

$$\begin{aligned} \dot{\sigma}_x &= \omega_0 \sigma_y(t) \ , \\ \dot{\sigma}_y &= -\omega_0 \sigma_x(t) - 2g[x_0(t)\sigma_z(t)]_S \\ &\quad + 4g^2 \int_0^t ds \sum_i \Gamma_i^2 \sin\omega_i(t-s)[\sigma_x(s)\sigma_z(t)]_S \ , \\ \dot{\sigma}_z &= 2g[x_0(t)\sigma_y(t)]_S \\ &\quad - 4g^2 \int_0^t ds \sum_i \Gamma_i^2 \sin\omega_i(t-s)[\sigma_x(s)\sigma_y(t)]_S \ . \end{aligned} \tag{2.6}$$

We have adopted the following definitions:

$$[\Phi\Pi]_S \equiv (\Phi\Pi + \Pi\Phi)/2 \ , \tag{2.7}$$

$$x_0(t) \equiv \sum_i x_i^{(0)}(t) \ . \tag{2.8}$$

Equation (2.6) is formally exact. When $g=0$ the system is characterized by harmonic oscillations with frequency ω_0. Equation (2.6) describes how this ideal behavior is perturbed by the interaction between the spin of interest and its bath of oscillators. However, to gain a deeper understanding and make the physics more transparent, it is convenient to approximate Eq. (2.6) in some relevant limit conditions. Section III will be devoted to the solution in the weak-coupling limit, along the same lines as Dekker.[8]

III. WEAK-COUPLING LIMIT

We carry out a perturbation expansion of Eq. (2.6) at second order in the parameter g. All the results of this section are valid without making any particular assumption on the spectral density and so they will be expressed in general in terms of the couplings Γ_i. To be consistent with this approximation, the two-time operators $[\sigma_x(s)\sigma_z(t)]_S$ and $[\sigma_x(s)\sigma_y(t)]_S$ must be evaluated at the zeroth order. From Eq. (2.6) we get the following zeroth-order approximations:

$$\begin{aligned} [\sigma_x(s)\sigma_z(t)]_S &= \tfrac{1}{2}[[\sigma_x(0)\cos\omega_0 s + \sigma_y(0)\sin\omega_0 s]\sigma_z(0)]_S \\ &= \tfrac{1}{2}[\sigma_x(0)\sigma_z(0)]_S \cos\omega_0 s + [\sigma_y(0)\sigma_z(0)]_S \sin\omega_0 s = 0 \ , \end{aligned} \tag{3.1}$$

$$\begin{aligned} [\sigma_x(s)\sigma_y(t)]_S &= \tfrac{1}{2}[[\sigma_x(0)\cos\omega_0 s + \sigma_y(0)\sin\omega_0 s][\sigma_y(0)\cos\omega_0 t - \sigma_x(0)\sin\omega_0 t]]_S \\ &= \tfrac{1}{2}\{[\sigma_x(0)\sigma_y(0)]_S \cos(\omega_0 s)\cos(\omega_0 t) + 2\sin(\omega_0 s)\cos(\omega_0 t) - 2\cos(\omega_0 s)\sin(\omega_0 t) \\ &\quad - [\sigma_y(0)\sigma_x(0)]_S \sin(\omega_0 s)\sin(\omega_0 t)\} = \sin[\omega_0(s-t)] \ . \end{aligned} \tag{3.2}$$

To derive Eqs. (3.1) and (3.2) we have used the anticommutation rules of the Pauli matrices:

$$[\sigma_x\sigma_y]_S = [\sigma_y\sigma_z]_S = [\sigma_z\sigma_x]_S = 0 \ . \tag{3.3}$$

Note that $\sigma_x(0)=\sigma_x, \sigma_y(0)=\sigma_y, \sigma_z(0)=\sigma_z$. Incidentally, any ansatz ought to fulfill Eqs. (3.3), due to quantum-mechanical nature of the system under study.

Still focusing on Eq. (2.6), we note that $\sigma_z(t)$, appearing in the second term on the right-hand side (rhs) of the second equation and σ_y, in the first term on the rhs of the

third equation, should be evaluated at the first order in g. This yields

$$\sigma_z(t)=\sigma_z(0)+2g\int_0^t ds[x_0(s)\sigma_y(s)]_S \ , \tag{3.4}$$

$$\sigma_y(t)=\sigma_y(0)\cos(\omega_0 t)-\sigma_x(0)\sin(\omega_0 t) -2g\int_0^t ds\cos[\omega_0(t-s)][x_0(s)\sigma_z(s)]_S \ . \tag{3.5}$$

Inserting Eqs. (3.1), (3.2), (3.4), and (3.5) in Eq. (2.6), we obtain

$$\begin{aligned}
\dot\sigma_x &= \omega_0\sigma_y(t) \ , \\
\dot\sigma_y &= -\omega_0\sigma_x(t)-2g[x_0(t)\sigma_z(0)]_S \\
&\quad -4g^2\int_0^t ds[x_0(t)[x_0(s)\sigma_y(s)]_S]_S \ , \\
\dot\sigma_z &= 2gx_0(t)\sigma_y(0)\cos(\omega_0 t)-2gx_0(t)\sigma_x(0)\sin(\omega_0 t) \\
&\quad -4g^2\int_0^t ds\cos\omega_0(t-s)[x_0(t)[x_0(s)\sigma_z(s)]_S]_S \\
&\quad +4g^2\int_0^t ds\left[\sum_i \Gamma_i^2\sin(\omega_i s)\right]\sin(\omega_0 s)
\end{aligned} \tag{3.6}$$

Note that

$$\begin{aligned}
&-2g\int_0^t ds[x_0(t)[x_0(s)\sigma_y(s)]_S]_S \\
&\quad =-2g\int_0^t ds[x_0(t)[x_0(t-s)\sigma_y(t-s)]_S]_S \\
&\qquad -2g\int_0^t ds\tfrac{1}{4}[x_0(t)x_0(t-s)\sigma_y(t-s)+x_0(t)\sigma_y(t-s)x_0(t-s) \\
&\qquad\quad +x_0(t-s)\sigma_y(t-s)x_0(t)+\sigma_y(t-s)x_0(t-s)x_0(t)] \ .
\end{aligned} \tag{3.7}$$

We made the change of variable $s\to t-s$ and used the definition of Eq. (2.7). Then we average Eq. (3.6) with respect to the following initial condition:

$$\rho(0)=P_+\rho_B \ , \tag{3.8}$$

where

$$P_+\equiv\frac{(\mathcal{I}+\sigma_x)}{2}, \tag{3.9a}$$

$$\rho_B\propto\exp\left[-\beta\sum_i \omega_i b_i^\dagger b_i\right] . \tag{3.9b}$$

And $\mathcal{I}$ is the unity operator. Thus from Eq. (3.7) we have

$$\begin{aligned}
&\mathrm{Tr}\left[-2g\int_0^t ds[x_0(t)[x_0(s)\sigma_y(s)]_S]_S\right] \\
&\quad =-2g\int_0^t ds\langle[x_0(t)x_0(t-s)]_S\rangle\langle\sigma_y(t-s)\rangle \ .
\end{aligned} \tag{3.10}$$

On the left-hand side (l.h.s.) of Eq. (3.10) the factorization into an oscillator and a spin part is not an assumption. Indeed, a correlation between spin and oscillator would only appear at a perturbation order in g higher than the second.

Using similar arguments we find

$$\begin{aligned}
&\mathrm{Tr}\left[-4g^2\int_0^t ds\cos\omega_0(t-s)[x_0(t)[x_0(s)\sigma_z(s)]_S]_S\right] \\
&\quad =-4g^2\int_0^t ds\cos(\omega_0 s) \\
&\qquad\quad \times\langle[x_0(t)x_0(t-s)]_S\rangle\langle\sigma_z(t-s)\rangle \ ,
\end{aligned} \tag{3.10'}$$

and

$$\langle x_0(t)\rangle=0 \ . \tag{3.11}$$

Then, using Eqs. (3.9), (3.10), and (3.11), Eq. (3.6) yields

$$\begin{aligned}
\langle\dot\sigma_x\rangle &= \omega_0\langle\sigma_y(t)\rangle \ , \\
\langle\dot\sigma_y\rangle &= -\omega_0\langle\sigma_x(t)\rangle \\
&\quad -4g^2\int_0^t ds\langle[x_0(0)x_0(-s)]_S\rangle\langle\sigma_y(t-s)\rangle \ ,
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\dot\sigma_z(t) &= -4g^2\int_0^t ds\cos[\omega_0(s)] \\
&\quad \times\langle[x_0(0)x_0(-s)]_S\rangle\langle\sigma_z(t-s)\rangle \\
&\quad +4g^2\int_0^t ds\left[\sum_i \Gamma_i^2\sin(\omega_i s)\right]\sin(\omega_0 s) \ .
\end{aligned}$$

Clearly, the time evolution of $\sigma_y(t-s)$ and $\sigma_z(t-s)$ from $t-s$ up to t, within their respective time-convoluted expressions, must be evaluated at the zeroth order in g to have a consistent second-order calculation in g. This means

$$\langle\sigma_z(t-s)\rangle\approx\langle\sigma_z(t)\rangle \ , \tag{3.13}$$

$$\langle\sigma_y(t-s)\rangle\approx\cos(\omega_0 s)\langle\sigma_y(t)\rangle+\sin(\omega_0 s)\langle\sigma_x(t)\rangle \ . \tag{3.14}$$

Inserting Eqs. (3.13) and (3.14) in Eq. (3.12), we arrive at the final result

$$\langle \dot{\sigma}_x \rangle = \omega_0 \langle \sigma_y(t) \rangle ,$$
$$\langle \dot{\sigma}_y \rangle = -\omega_0 \langle \sigma_x(t) \rangle - 4g^2 \int_0^t ds \langle [x_0(0)x_0(-s)]_S \rangle \cos(\omega_0 s) \langle \sigma_y(t) \rangle - 4g^2 \int_0^t ds \langle [x_0(0)x_0(-s)]_S \rangle \sin(\omega_0 s) \langle \sigma_x(t) \rangle , \quad (3.15)$$
$$\langle \dot{\sigma}_z(t) \rangle = -4g^2 \int_0^t ds \cos[\omega_0(s)] \times \langle [x_0(0)x_0(-s)]_S \rangle \langle \sigma_z(t) \rangle + 4g^2 \int_0^t ds \left[\sum_i \Gamma_i^2 \sin(\omega_i s) \right] \sin(\omega_0 s) .$$

For t very large, or "macroscopic," the first two equations of Eq. (3.15) tend to

$$\langle \dot{\sigma}_x \rangle = \omega_0 \langle \sigma_y(t) \rangle ,$$
$$\langle \dot{\sigma}_y \rangle = -\omega_0 \langle \sigma_x(t) \rangle - \gamma_1 \langle \sigma_y(t) \rangle - \gamma_2 \langle \sigma_x(t) \rangle , \quad (3.16)$$
$$\langle \dot{\sigma}_z(t) \rangle = -\gamma_1 [\langle \sigma_z(t) \rangle - \langle \sigma_z \rangle_{eq}] ,$$

where

$$\gamma_1 \equiv 4g^2 \int_0^\infty ds \cos(\omega_0 s) \langle [x_0(0)x_0(-s)]_S \rangle , \quad (3.17a)$$
$$\gamma_2 \equiv 4g^2 \int_0^\infty ds \sin(\omega_0 s) \langle [x_0(0)x_0(-s)]_S \rangle . \quad (3.17b)$$

Equation (3.16) yields

$$\langle \ddot{\sigma}_x \rangle + \omega_0(\omega_0 + \gamma_2) \langle \sigma_x \rangle + \gamma_1 \langle \dot{\sigma}_x \rangle = 0 . \quad (3.18)$$

Under the approximation of small g's, the motion of our system is equivalent to the motion of a damped oscillator with frequency $\sqrt{\omega_0(\omega_0+\gamma_2)}$ and friction γ_1, provided that the "coordinate" x of that oscillator is identified with $\langle \sigma_x \rangle$.

We note from Eqs. (3.17) that the damping coefficients increase if we increase the coupling g. Of course, when the coupling is very large the second-order expressions in Eqs. (3.17) are invalidated. However, let us assume for the time being that we are allowed to increase g beyond any limit without invalidating Eqs. (3.17). Then, after reaching the critical condition

$$\gamma_1 = 2\sqrt{\omega_0(\omega_0+\gamma_2)} , \quad (3.19)$$

the decay of σ_x would change from a damped oscillation to a mere exponential behavior. Furthermore, the lifetime of the exponential decay would become larger and larger. Thus for g tending to infinity the state with positive polarization in the x direction would be a virtually stable state (trapped state).

In Sec. IV we shall develop a theory based on the assumption alone that the tunneling frequency ω_0 is small. For the sake of readers' convenience, we notice that under this assumption γ_2 becomes negligible compared to γ_1 and Eq. (3.18) becomes

$$\langle \ddot{\sigma}_x \rangle + \omega_0^2 \langle \sigma_x \rangle + \gamma \langle \dot{\sigma}_x \rangle = 0 , \quad (3.20)$$

with

$$\gamma \equiv 4g^2 \int_0^\infty ds \langle [x_0(0)x_0(-s)]_S \rangle . \quad (3.21)$$

Keeping the spin frequency much smaller than the bath frequencies will allow us to explore the effect of the coupling strength, from the weak to the large limit and to discover a trapping mechanism distinct from that of the overdamped motion described in this section. Under the weak-coupling assumption the theory of Sec. IV will be shown to lead to the result of Eq. (3.20).

IV. FROM THE WEAK- TO THE STRONG-COUPLING LIMIT

This is the central section of the present paper. The theory here developed is based on the two basic assumptions: (i) the tunneling frequency ω_0 is very small, and (ii) at the initial time $t=0$ the oscillators of the bath are placed at equilibrium in a shifted potential. Let us illustrate first the second assumption.

Let us assume that at $t=0$ the spin is in the state $|+\rangle_x$, the eigenstate of σ_x with positive eigenvalue, therefore satisfying the condition

$$\sigma_x |+\rangle_x = |+\rangle_x . \quad (4.1)$$

Then the bath oscillators experience harmonic potentials with unperturbed frequencies but with a minimum shifted by the quantity

$$\Delta x_i = -\frac{2g\Gamma_i^2}{\omega_i} , \quad (4.2)$$

the unperturbed equilibrium position being assumed to be placed at $x=0$. The meaning of assumption (ii) is that at the initial time all the oscillators are placed in a statistical equilibrium state with respect to this shifted potential and we study the ensuing evolution of the system.

It must be remarked that if we adopt the spectral density corresponding to the model studied in Sec. V this initial condition is naturally reached by the system and does not need to be assumed true. Let us explain why it is so.

The numerical simulation of Sec. V concerns the case when the spin-$\frac{1}{2}$ system interacts with a single dissipative oscillator. This oscillator reaches its equilibrium condition in a time very short compared to the time scale of the spin and the wave function of the oscillator will adapt itself very quickly to the shifted potential. Thus if the spin is placed in the state $|+\rangle_x$, the system after a transient process, which is of no interest to our macroscopic description, collapses precisely into the initial condition envisaged by assumption (ii). Actually this model belongs to the class of those studied in Sec. III but with a suitable spectral density. To understand this aspect let us consider the case of a spin interacting with a single oscillator, or *doorway oscillator,* which, in turn, interacts linearly with a set of oscillators. Under precise constraints on the coupling between the doorway oscillator and its bath, the doorway oscillator becomes a dissipative oscillator of the kind studied numerically in Sec. V. On the other hand, the spectral density associated to this model is immediately derived using the normal-modes technique. This is strongly peaked around the frequency Ω of the doorway

oscillator, with a nonvanishing width resulting from its dissipative nature. It is very important to note that this spectral density is quite different from that usually assumed in the field of quantum dissipation.[1,2] The standard spectral density[1,2] is a power law multiplied by a cutoff function. Our spectral density is mainly dictated by the purpose of making a comparison with the numerical results of Sec. V. It must be noticed, however, that Kenkre and Wu,[10] for different purposes, adopted precisely this same spectral density.

In accordance with the above remarks it is convenient to adopt the following change of variables

$$b_i=\tilde{b}_i-\frac{g\Gamma_i}{\omega_i}\ ,\qquad b_i^\dagger=\tilde{b}_i^\dagger-\frac{g\Gamma_i}{\omega_i}\ . \tag{4.3}$$

This means that we are using the new coordinates

$$\tilde{x}_i=\Gamma_i(\tilde{b}_i+\tilde{b}_i^\dagger)\ , \tag{4.4}$$

which correspond indeed to shifting the reference frame by the quantity of Eq. (4.2). In this new system of reference the Hamiltonian of Eq. (2.1) reads

$$\mathcal{H}=-\frac{\omega_0}{2}\sigma_z-2P_-g\sum_i\Gamma_i(\tilde{b}_i+\tilde{b}_i^\dagger)+\sum_i\omega_i\tilde{b}_i^\dagger\tilde{b}_i -2\sigma_x\sum_i\frac{g^2\Gamma_i^2}{\omega_i}+\sum_i\frac{g^2\Gamma_i^2}{\omega_i}\ . \tag{4.5}$$

Adopting the same Heisenberg approach that leads to Eqs. (2.6) we now obtain

$$\begin{aligned}
&\dot{\sigma}_x=\omega_0\sigma_y(t)\ ,\\
&\dot{\sigma}_y=-\omega_0\sigma_x(t)+\left[4g^2\sum_i\frac{\Gamma_i^2}{\omega_i}\right]\sigma_z(t)-2g\tilde{x}(t)\sigma_z(t)\ ,\\
&\dot{\sigma}_z=-\left[4g^2\sum_i\frac{\Gamma_i^2}{\omega_i}\right]\sigma_y(t)+2g\tilde{x}(t)\sigma_y(t)\ ,\\
&\dot{\tilde{x}}_i(t)=v_i(t)\ ,\\
&\dot{v}_i(t)=-\omega_i^2\tilde{x}_i(t)+4g\Gamma_i^2\omega_iP_-\ ,
\end{aligned} \tag{4.6}$$

where

$$P_-\equiv\frac{(\mathcal{I}-\sigma_x)}{2}\ , \tag{4.7}$$

$\mathcal{I}$ is the unity operator on the spin space, and

$$\tilde{x}(t)\equiv\sum_i\Gamma_i(\tilde{b}_i+\tilde{b}_i^\dagger)\ . \tag{4.8}$$

From the last two equations of (4.6) we obtain

$$\tilde{x}(t)=\tilde{x}^{(0)}(t)+4g\int_0^t ds\left[\sum_i\Gamma_i^2\sin\omega_i(t-s)\right]P_-(s)\ , \tag{4.9}$$

where

$$\tilde{x}^{(0)}(t)\equiv\sum_i\left[\tilde{x}_i(0)\cos(\omega_it)+\frac{v_i(0)}{\omega_i}\sin(\omega_it)\right]\ . \tag{4.10}$$

Replacing Eq. (4.10) into the second and third equation of the set of Eq. (4.6), and carrying out the quantum-mechanical averages on the system-bath distribution, we get

$$\begin{aligned}
&\langle\dot{\sigma}_x\rangle=\omega_0\langle\sigma_y(t)\rangle\ ,\\
&\langle\dot{\sigma}_y\rangle=-\omega_0\langle\sigma_x(t)\rangle+\left[4g^2\sum_i\frac{\Gamma_i^2}{\omega_i}\right]\langle\sigma_z(t)\rangle\\
&\quad-2g\langle[\tilde{x}^{(0)}(t)\sigma_z(t)]_S\rangle\\
&\quad-8g^2\int_0^t ds\sum_i\Gamma_i^2\sin[\omega_i(t-s)]\\
&\qquad\times\langle[P_-(s)\sigma_z(t)]_S\rangle\ ,\\
&\langle\dot{\sigma}_z\rangle=-\left[4g^2\sum_i\frac{\Gamma_i^2}{\omega_i}\right]\langle\sigma_y(t)\rangle+2g\langle[\tilde{x}^{(0)}(t)\sigma_z(t)]_S\rangle\\
&\quad+8g^2\int_0^t ds\sum_i\Gamma_i^2\sin[\omega_i(t-s)]\langle[P_-(s)\sigma_y(t)]_S\rangle\ .
\end{aligned} \tag{4.11}$$

Under the assumption that ω_0 is small enough, it is possible to make the following approximations

$$2g\langle[\tilde{x}^0(t)\sigma_z(t)]_S^-\rangle\approx4g^2\int_0^t dx\langle[\tilde{x}^{(0)}(t)\tilde{x}^{(0)}(t-s)]_S\rangle\langle\sigma_y(t-s)\rangle\ , \tag{4.12}$$

$$2g\langle[\tilde{x}^0(t)\sigma_y(t)]_S\rangle\approx-4g^2\int_0^t ds\langle[\tilde{x}^{(0)}(t)\tilde{x}^{(0)}(t-s)]_S\rangle\langle\sigma_z(t-s)\rangle\ .$$

These approximations are obtained by (i) replacing the formal solution of the third equation of the set of Eq. (4.6) in the second equation of this same set and the formal solution of the second equation into the third equation of the same set, and (ii) carrying out the averages with the additional assumption that system and bath factorize.

This procedure can be justified as follows. First of all, the factorization carried out at step (ii) allows us to take partially into account the correlation between system and bath. Indeed, if the same factorization were applied to the second and the third equation of (4.6), thereby fully neglecting the spin-bath correlation, the quantities of Eq. (4.12) would vanish. On the other hand, the Markovian assumption on Eqs. (4.12) yields

$$\begin{aligned}
&2g\langle[\tilde{x}^{(0)}(t)\sigma_z(t)]_S\rangle\approx\gamma\langle\sigma_y(t)\rangle\ ,\\
&2g\langle[\tilde{x}^{(0)}(t)\sigma_y(t)\rangle\approx-\gamma\langle\sigma_z(t)\rangle\ ,
\end{aligned} \tag{4.12'}$$

with

$$\gamma \equiv 4g^2 \int_0^\infty \langle [\tilde{x}^{(0)}(0)\tilde{x}^{(0)}(t)]_S \rangle dt \ . \tag{4.13}$$

We see that this is precisely the friction coefficient resulting from the weak-coupling analysis of Sec. III, supplemented by the condition of weak tunneling frequency [Eq. (3.21)].

Under the assumption that ω_0 is weak enough, the third terms on the rhs of the second and third equation of (4.1) can be neglected. Indeed we note that

$$2\langle [P_-(s)\sigma_y(t)]_S \rangle = \mathrm{Tr}(\rho_{\mathrm{bath}}^{(+)} P_+ \{ \exp(i\mathcal{H}s) P_- \exp[i\mathcal{H}(t-s)]\sigma_y \exp(-i\mathcal{H}t) + \exp(i\mathcal{H}t)\sigma_y \exp[-i\mathcal{H}(t-s)] P_- \exp(-i\mathcal{H}s) \}) \ , \tag{4.14}$$

where $\rho_{\mathrm{bath}}^{(+)}$ denotes the initial bath density matrix, assumed to be canonical with respect to the potential corresponding to the spin in the state $|+\rangle_x$, that is,

$$\rho_{\mathrm{bath}}^{(+)} \propto \exp\left[-\beta \sum_i \omega_i \tilde{b}_i^\dagger \tilde{b}_i \right] .$$

The Hamiltonian $\mathcal{H}$ consists of three parts: $\mathcal{H}_A$ which is proportional to ω_0, $\mathcal{H}_I$ the interaction part, and $\mathcal{H}_B$ the bath Hamiltonian. Under the basic condition

$$\mathcal{H}_A \ll \mathcal{H}_I + \mathcal{H}_B \ , \tag{4.15}$$

we get a Hamiltonian $\mathcal{H}$ which almost exactly commutes with P_-. We see that

$$[P_-, \mathcal{H}] = -\frac{1}{2}\left[\sigma_x, -\frac{\omega_0}{2}\sigma_z \right] = -i\frac{\omega_0}{2}\sigma_y \ , \tag{4.16}$$

which, implies a vanishing commutator in the limit of vanishing ω_0. On the other hand, under the condition

$$[P_-, \mathcal{H}] \sim 0 \tag{4.17}$$

from Eq. (4.14) we obtain

$$\langle [P_-(s)\sigma_y(t)]_S \rangle \sim 0 \ . \tag{4.18a}$$

If the label y is replaced by z in (4.14), the same arguments used above apply again and we are consequently led to

$$\langle [P_-(s)\sigma_z(t)]_S \rangle \sim 0 \ . \tag{4.18b}$$

In conclusion, from Eq. (4.11) we obtain

$$\begin{aligned}
\langle \dot{\sigma}_x \rangle &= \omega_0 \langle \sigma_y(t) \rangle \ , \\
\langle \dot{\sigma}_y \rangle &= -\omega_0 \langle \sigma_x(t) \rangle \\
&\quad + \left[4g^2 \sum_i \frac{\Gamma_i^2}{\omega_i} \right] \langle \sigma_z(t) \rangle - \gamma \langle \sigma_y(t) \rangle \ , \\
\langle \dot{\sigma}_z \rangle &= -\left[4g^2 \sum_i \frac{\Gamma_i^2}{\omega_i} \right] \langle \sigma_y(t) \rangle - \gamma \langle \sigma_z(t) \rangle \ ,
\end{aligned} \tag{4.19}$$

where γ is defined by Eq. (4.13). We note that at the second order in the coupling g, the prediction of Eq. (4.19) for the time evolution of $\langle \sigma_x(t) \rangle$, is indistinguishable from that obtained by making ω_0 tend to zero in Eq. (3.16). In fact, taking into account that Eq. (4.19) refers to the initial condition $\langle \sigma_x(0) \rangle = 1$, $\langle \sigma_y(0) \rangle = 0$, $\langle \sigma_z(0) \rangle = 0$, the third equation of (4.19) results in a deviation from $\langle \sigma_z(t) \rangle = 0$ proportional to g^2, whereas the third equation of Eq. (3.16) predicts $\langle \sigma_z(t) \rangle \sim 0$ throughout the whole time interval, in the limit $\omega_0 \to 0$. However, when this small mean value is replaced into the second of the equations of (4.19), the resulting effect on the time evolution of $\langle \sigma_x(t) \rangle$ turns out to be proportional to g^4. Effects of that order of magnitude are ignored by the weak-coupling approximation leading to Eq. (3.16). Taking also into account the considerations leading to Eq. (3.21), we can thus conclude saying that, within this weak-coupling approximation, Eqs. (4.19) and (3.16) are proved to lead to the same result.

Equation (4.19) and, more precisely, the second term on the rhs of the second equation and the first term on the rhs of the third equation, are the important result of this paper.[11] To stress the importance of these terms, let us consider the special case where the temperature T is so small that the friction terms of Eq. (4.19) are negligible. In other words, let us assume that

$$\gamma = 0 \ . \tag{4.20}$$

In this case Eq. (4.19) yields

$$\langle \sigma_x(t) \rangle = \frac{\omega_0^2 \cos[(16\Delta^2 + \omega_0^2)^{1/2} t] + 16\Delta^2}{16\Delta^2 + \omega_0^2} \ . \tag{4.21}$$

$$\langle \sigma_y(t) \rangle = -\frac{\omega_0 \sin[(16\Delta^2 + \omega_0^2)^{1/2} t]}{(16\Delta^2 + \omega_0^2)^{1/2}} \ , \tag{4.22}$$

$$\langle \sigma_z(t) \rangle = \frac{4\Delta\omega_0}{16\Delta^2 + \omega_0^2} \{ 1 - \cos[(16\Delta^2 + \omega_0^2 t)^{1/2}] \} \ , \tag{4.23}$$

where

$$\Delta \equiv g^2 \sum_i \frac{\Gamma_i^2}{\omega_i} \ . \tag{4.24}$$

Equation (4.21) remarkably coincides with the strong-coupling limit of Kenkre and Campbell.[4] It shows that the system is basically localized. According to the picture of Kenkre and Campbell,[4] the high-frequency oscillations are interpreted as resulting from the inertial motion at the bottom of one of the two minima of a double-well potential. The result of Eqs. (4.21)–(4.23) is

not confined to the strong-coupling regime. It applies also to the weak-coupling regime, and the correct expression for the free spin motion is recovered from it. We see that, upon increasing of the coupling, i.e., upon increasing of Δ, the center of the oscillations of $\langle\sigma_x(t)\rangle$ moves from $\langle\overline{\sigma_x(t)}\rangle=0$ towards $\langle\overline{\sigma_x(t)}\rangle=1$, while the oscillation frequency becomes higher and the oscillation amplitude smaller. In the limit of infinitely large coupling the time evolution of $\langle\sigma_x(t)\rangle$ becomes indistinguishable form a horizontal straight line at $\langle\overline{\sigma_x(t)}\rangle=1$.

Although this straight line is indistinguishable from that which would be rescaled increasing γ beyond any limit in Eq. (3.20), the onset of this localization process is complete different, since this cannot be interpreted as the consequence of an extremely large damping. In Sec. V we shall study the joint action of these two different localization processes.

V. COMPUTER RESULTS

Let us now study the case when the spin-$\frac{1}{2}$ entity of Eq. (2.1) interacts with only one oscillator with frequency Ω. This oscillator is dissipative. This means that its density matrix is driven by the superoperator

$$\mathcal{L}_{\text{eff}}=-i\mathcal{H}_B^X-i\mathcal{H}_I^X+\mathcal{R}\ , \tag{5.1}$$

where

$$\mathcal{H}_i=g\Gamma(b+b^\dagger)\sigma_x\ , \tag{5.2}$$

$$\mathcal{H}_B=\Omega b^\dagger b\ , \tag{5.3}$$

The superoperator $\mathcal{R}$ is defined in this way:

$$\langle\mathcal{R}\rho^{(0)}\rangle_{n,n}=\delta_L(-w_n\rho^{(0)}_{n,n}+T_{\text{down}}\rho^{(0)}_{n+1,n+1}+T_{\text{up}}\rho^{(0)}_{n-1,n-1})\ . \tag{5.4}$$

The transition coefficients are suitably balanced to ensure that, in the absence of the coupling with the system, the dissipative oscillator reaches the canonical equilibrium distribution

$$\rho^{(0)}\propto\exp\left[-\frac{\mathcal{H}_B}{k_BT}\right]\ , \tag{5.5}$$

Thus

$$w_n\equiv T_{\text{up}}+T_{\text{down}} \tag{5.4'}$$

and

$$T_{\text{up}}\equiv\exp\left[-\frac{\Omega}{2k_BT}\right] \tag{5.4''}$$

$$T_{\text{down}}\equiv\exp\left[\frac{\Omega}{2k_BT}\right]\ . \tag{5.4'''}$$

Furthermore a transverse relaxation process is included. This is defined by

$$(\mathcal{R}\rho^{(0)})_{n,n'}=-\delta_T\rho^{(0)}_{n,n'},\ n\neq n'\ . \tag{5.6}$$

The relation with the parameters of (4.19) is easily established and is given by

$$\Delta=\frac{g^2}{2\Omega^2}\ , \tag{5.7}$$

$$\gamma\sim\frac{8g^2\Gamma^2}{\delta_T}\text{cotanh}\left[\frac{\Omega}{2k_BT}\right]\ . \tag{5.8}$$

Equation (5.8) is well satisfied in the region

$$\delta_T\geq\delta_L\ , \tag{5.9}$$

relevant to our numerical calculations. Eq. (5.8) means that although the friction γ is reduced upon decreasing temperature, it does not vanish at $T=0$. In the weak-temperature region the quantum mechanical fluctuations become dominant and the friction γ mainly depends on them.

Figure 1 shows the time behavior of $\langle\sigma_x(t)\rangle$ at different values of the coupling parameter g. We see that in the weak-coupling regime $\langle\sigma_x(t)\rangle$ exhibits damped oscillations around the standard equilibrium $\langle\sigma_x(t)\rangle=0$. Upon increase of g, as suggested by Eq. (3.20), a transition to the overdamped motion takes place (see curve 2). However, the early decay of this curve is characterized by fast oscillations around the slower process of exponential relaxation. These fast oscillations are caused by the terms $4\Delta\langle\sigma_z(t)\rangle$ and $-4\Delta\langle\sigma_y(t)\rangle$ appearing in Eq. (4.19). As illustrated by Eq. (4.20), the increase of the frequency 4Δ results in oscillations of increasing frequency and decreasing amplitude around a mean value closer and closer to $\langle\sigma_x(t)\rangle=1$. This is a contribution to localization distinct from the transition to the overdamped regime predicted by Eq. (3.20). Curve 2 of Fig. 1 results from the joint action of these two distinct processes.

Note that in the scale of Fig. 1, the predictions of Eq.

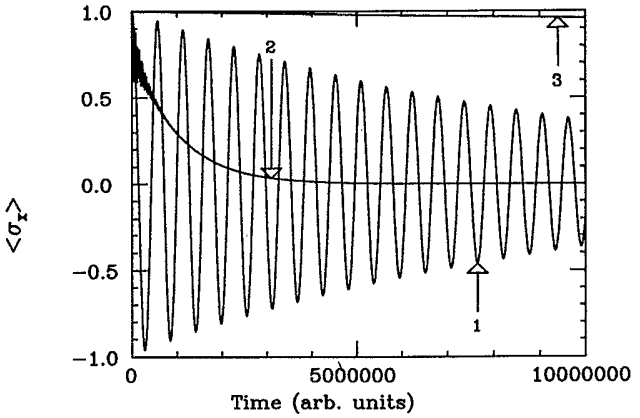

FIG. 1. $\langle\sigma_x(t)\rangle$ vs time at different values of the coupling g. The fixed parameters are as follows: $\omega_0=\Omega=0.01$, $\Gamma=1$, $\delta_T=\delta_L=1$, $k_BT=5\times10^{-3}$. The lines 1, 2, and 3 correspond to $g=2\times10^{-4}, 10^{-3}, 8\times10^{-3}$, respectively.

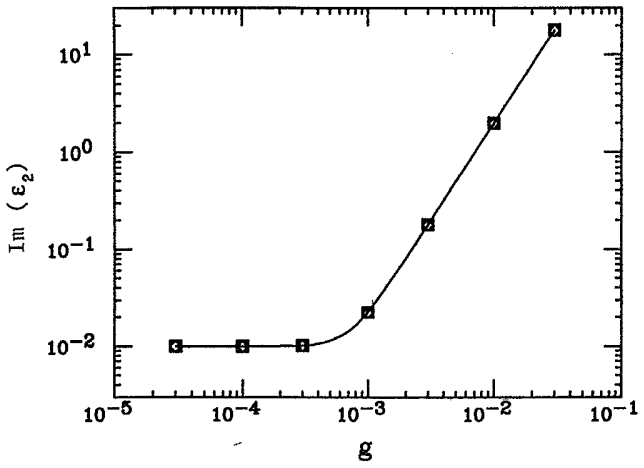

FIG. 2. Imaginary part of the eigenvalue ε_2 as a function of the coupling g at two different values of T. We considered the two cases $k_BT/\omega_0=0.5$ (squares) and $k_BT/\omega_0=1$ (diamonds). In accordance with the theory leading to Eq. (4.19) this oscillation frequency turns out to be temperature independent. The fixed parameters are as follows: $\omega_0=\Omega=0.01$, $\Gamma=1$, $\delta_T=\delta_L=1$. The solid line is the theoretical prediction.

(4.19) are virtually indistinguishable from the results of numerical calculations. To make this important aspect more transparent we have compared the three eigenvalues associated to the system of Eq. (4.19) ε_1 (real eigenvalue), $\varepsilon_2=\alpha+i\beta$, and $\varepsilon_3=\alpha-i\beta$ to the three lowest eigenvalues of the superoperator $\mathcal{R}$ defined by Eqs. (5.4) and (5.6), excluding the trivial zero eigenvalue. We found a very satisfactory agreement between the theoretical and the numerical eigenvalues, which is illustrated by Figs. 2 and 3.

To make more transparent the occurrence of two distinct trapping processes, we have also plotted the lifetime of the localized state as a function of the temperature T.

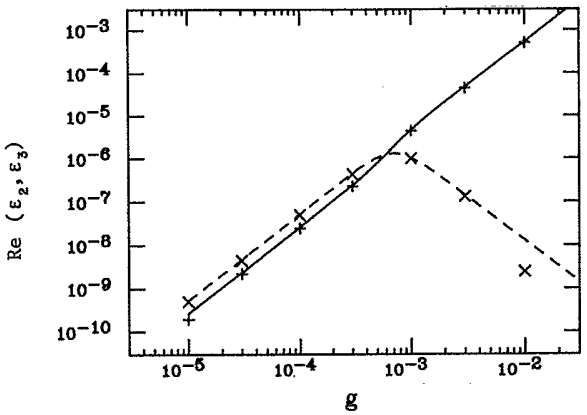

FIG. 3. The eigenvalue $\varepsilon_1(X)$ and the real part of the eigenvalues ε_2 and $\varepsilon_3(+)$ as a function of the coupling g. The fixed parameters are as follows: $\omega_0=\Omega=0.01$, $\Gamma=1$, $\delta_T=\delta_L=1$, $k_BT=5\times10^{-3}$. The lines denote the corresponding theoretical predictions.

The lifetime of the trapped state is defined by

$$\tau_{TS}=\int_0^\infty ds\langle\sigma_x(s)\rangle\ . \tag{5.10}$$

Calculating the Laplace transform of the function $\langle\sigma_x(t)\rangle$ from Eq. (4.19), we get the explicit result

$$\tau_{TS}=\frac{\gamma^2+16\Delta^2}{\omega_0^2\gamma}\ . \tag{5.11}$$

Using Δ and γ provided by Eqs. (5.7) and (5.8), respectively, we get the explicit expression

$$\tau_{TS}=\frac{8g^2\Gamma^2}{\omega_0^2\delta_T}\left[\operatorname{cotanh}\left[\frac{\Omega}{2k_BT}\right]+\frac{\delta_T^2}{16\Omega^4\Gamma^4}\tanh\left[\frac{\Omega}{2k_BT}\right]\right]\ . \tag{5.12}$$

This analytical result, fully supported by our computer calculations, is illustrated in Fig. 4. We see from Fig. 4 that in the low-temperature region the lifetime of the trapped state is very large, but is not infinitely large. This is so because of the quantum-mechanical fluctuations which keep finite the mean square value of $\tilde{x}^{(0)}$ even at zero temperature. When the thermal fluctuations become predominant over the quantum-mechanical fluctuations, at around $k_BT/\omega_0=1$, the rate of destruction of the trapped state become more consistent. At higher temperatures the process of trapping stemming from the friction γ becomes predominant and this determines a fairly abrupt change in the temperature dependence of τ_{TS}. We see indeed that in the high-temperature region the lifetime of the trapped state is an increasing function of temperature. The transition from the former to the latter

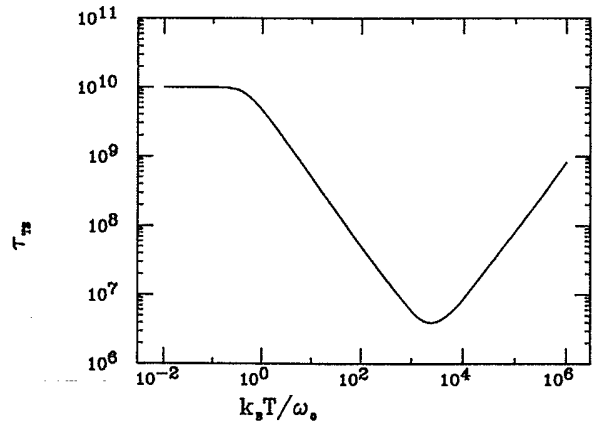

FIG. 4. The lifetime of the trapped state τ_{TS} as a function of temperature. The parameters of the system are as follows: $\omega_0=\Omega=0.01$, $\Gamma=1$, $\delta_T=\delta_L=1$, $g=10^{-3}$.

kind of behavior takes place at the temperature T_T defined by

$$T_T=\frac{\Omega}{2k_B}\left[\text{cotanh}^{-1}\left[\frac{\delta_T}{4\Omega^2\Gamma^2}\right]\right]^{-1} . \quad (5.13)$$

Under the physical condition of this section, this becomes

$$T_T\sim\frac{\delta_T}{8\Omega\Gamma^2k_B} . \quad (5.14)$$

At $T>T_T$ the trapping mechanism illustrated in Sec. III is predominant on that of Sec. IV and in the region $T<T_T$ the latter mechanism is predominant over the former one.

VI. CONCLUDING REMARKS

We have mentioned in the introduction that this paper is related to both the subject of nonlinear Schrödinger equation and the field of quantum-mechanical dissipation. The nonlinear Schrödinger equation proposed by Davydov,[12] which is the object of intense discussion concerning its microscopic derivation,[13,14] has been used by Kenkre and co-workers to derive fundamental spectroscopic properties in the field of condensed matter.[4,5] It is remarkable that the nonlinear dimer of Kenkre, or better the dissipative and stochastic version of it[16] is rederived from our general result of Eq. (2.6) by making the ansatz

$$\langle[\sigma_x(s)\sigma_z(t)]_S\rangle=\langle\sigma_x(s)\rangle\langle\sigma_z(t)\rangle , \quad (6.1)$$

$$\langle[\sigma_x(s)\sigma_y(t)]_S\rangle=\langle\sigma_x(s)\rangle\langle\sigma_y(t)\rangle . \quad (6.2)$$

We note that this ansatz unfortunately contravenes the important conditions

$$\langle[\sigma_x(t)\sigma_y(t)]_S\rangle=0 \quad (6.3)$$

$$\langle[\sigma_x(t)\sigma_z(t)]_S\rangle=0 , \quad (6.4)$$

which, in turn, are imposed by the quantum-mechanical properties of Eq. (3.3). On the other hand, let us discuss what happens when the Hamiltonian of Eq. (2.1) is replaced by

$$\mathcal{H}=-\frac{\omega_0}{2}S_z+gxS_x+\sum_i\omega_ib_i^\dagger b_i . \quad (6.5)$$

For $S\to\infty$ the quantum-mechanical spin becomes classical and the ansatz of Eq. (5.1) might be correct. When $S=\frac{1}{2}$, the quantum-mechanical nature of the spin makes it necessary to fulfill Eqs. (6.3) and (6.4). The arguments used to derive Eqs. (4.18a) and (4.18b) means that the correct ansatz is, in this case, given by

$$\langle[\sigma_x(s)\sigma_y(t)]_S\rangle\sim\langle\mathcal{J}(s)\sigma_y(t)\rangle\sim\langle\sigma_y(t-s)\rangle , \quad (6.6)$$

$$\langle[\sigma_x(s)\sigma_z(t)]_S\rangle\sim\langle\mathcal{J}(s)\sigma_z(t)\rangle\sim\langle\sigma_z(t-s)\rangle . \quad (6.7)$$

This ansatz rigorously fulfills the quantum-mechanical requirements of Eqs. (6.3) and (6.4). In fact, as it is clear from Eq. (4.14), the quantum-mechanical averages are computed over a quantum-mechanical density matrix implying the spin to be in the state $|+\rangle_x$, thereby resulting in $\langle\sigma_y(0)\rangle=\langle\sigma_y(0)\rangle=0$. In spite of the fact that we are obliged to adopt an ansatz different from that which would recover the dissipative and stochastic version of the Kenkre dimer[16] in the high-coupling limit and at very small temperatures our prediction, given by Eq. (4.21), coincides exactly with the prediction of Kenkre and Campbell.[4]

Thus our work would seem to lend support to the discrete nonlinear Schrödinger equation used by Kenkre and co-workers. Unfortunately, the agreement with their appealing nonlinear picture is lost upon either decreasing the interaction strength or raising the temperature. If we keep the temperature very low and we decrease the interaction strength, we recover the standard weak-coupling description without finding any significant departure from a "trigonometric behavior." In the presence of dissipation our picture never departs from a behavior described in terms of damped circular functions of time. This is reflected in the dependence of the lifetime of the trapped state on temperature [see Eq. (5.12) and Fig. (4)]. This conflicts with the Arrhenius-like behavior stemming from the nonlinear nature of the Kenkre dimer.[16] The reader can appreciate this aspect by comparing Eq. (5.12) and Fig. 4 to Eq. (4.10) and Fig. 2 of Ref. 16. If the localized stationary states are to be interpreted[16] as a signature of nonlinear evolution (polaronic or solitonic behavior), then the result of this paper, rather than reinforcing doubts on the existence of these processes,[13] raises the intriguing question of their correct dependence on temperature. For the time being, we limit ourselves to remarking that the choice of the initial conditions we have adopted is certainly suitable for the weak-coupling region as well as for the strong-coupling region. This is so because in the former case the shift of Eq. (4.2) is so small that the novel reference framework is almost indistinguishable from the "natural" system of reference associated to the free oscillator. Finally, we must remark that, the fast convergence of the numerical calculations and the excellent agreement with the predictions of Eq. (4.19) throughout the whole dominion of coupling, implies our picture to apply to the whole range of coupling strength. Equation (4.19) is based on the linear superposition of the trapping process, that leading to Eqs. (4.21)–(4.23), and the dissipation process with damping, γ. We must frankly admit that it is not yet quite clear to us how this can be justified in the region of intermediate couplings, where the spin system significantly departs from the initial condition $\langle\sigma_x(0)\rangle=1$ and the damping process is not yet predominant. Thus we conclude our comments pertaining to the field of the nonlinear Schrödinger equation[12,4,10,16] saying that the predictions of this equation are confirmed in the high-coupling limit (and, of course, in the weak-coupling limit). Further studies should be devoted to the region of intermediate couplings.

Within the field of quantum-mechanical dissipation, we would like to stress that our approach is distinct from those found in the relevant literature. The theory developed by Dekker[8] is basically a weak-coupling expansion. The approach of Aslangul, Pottier, and Saint-James,[17] which recovers the well-known results of Legget

and co-workers,[1] relies on the use of the polaronic transformation, which is distinct from our change of reference system. The discussion of the possible relations between our results and those of the above mentioned groups is difficult because we are certainly using a spectral density quite different from those used there. Our is the same spectral density as that used by Kenkre and coworkers.[10,16] The same remarks apply to the comparison between our results and those of Chvosta,[18] based on the use of the generalized master equation. As to our treatment of the weak-coupling regime, resulting in the damped oscillator of Eq. (3.20), we must emphasize that our approach is very close to that of Harris and Silbey.[19] We would like also to quote the significant work of Silbey and Harris.[20] This is an earlier application of the polaron transformation used to recover a damped oscillator with the same structure as that of Eq. (3.20), but with an oscillation frequency and a friction fully renormalized.

The model we have used to carry out our computer calculations belongs to the class of "reduced" models.[21] According to the strategy of the so called "reduced" model theory,[22] at least a qualitative understanding of non-Markovian processes can be obtained by replacing a real bath (usually involving an overwhelming number of degrees of freedom) with a few auxiliary variables. In classical stochastic physics the foundation of linear Fokker-Planck equations is well understood, and attention has been recently focused on nonlinear stochastic processes.[22] In this case one assumes that the variable of interest is coupled, via a nonlinear interaction, to a single auxiliary variable, which is assumed to be coupled to its bath in such a way as to replace the microscopic picture with a standard fluctuation-dissipation process. In the quantum-mechanical case here under study, we have introduced a spin interacting with only but one dissipative oscillator. Let us bear in mind that we are trying to make more transparent the physical effects of the breakdown of the weak-coupling approximation in quantum mechanics. To carry out the corresponding computer calculations we are then forced to introduce a reduced model, where the dissipation from a bath with an infinite number of degrees of freedom is replaced by the coupling with only one oscillator driven by a *standard dissipation* mechanism.

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