Mathematics as a social language: a (past and present) phenomenology

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Abstract

It is argued that the exchange between the formal and the social uses of mathematics has often been mediated by recreational mathematics. Many examples are offered, taken by ancient and modern history.

Actually an alternative title of this essay might have been "Mathematics BETWEEN formal language and social language". Indeed we would just like to argue the thesis that, in the continuous exchange between the two levels of the language, the transition from formal to social is often conveyed by what is sometimes called “recreational mathematics”

As a matter of fact mathematics arises from the very beginning as a social language. Arithmetic is a necessary tool for establishing the terms of exchange for the products of hunting, and later on for crops and livestock, but it is also needed in order to measure time and space (the days requested to walk from one place to another, the lunar and seasonal cycles). Geometry (as the name itself says) has its first motivation in land distribution, and then also in the increasing complexity of building (dams, palaces and pyramids!).

The first split between the social uses and their formalization can be symbolically identified with the day when Thales "measures" the height of the Great Pyramid by the shadow of his stick. Empirical knowledge turns into a formal theorem even before Pythagoras (and the Pythagoreans), and not by chance philosophy is born at the same time, and Plato in the *Phaedo*, will soon combine mathematical and philosophical knowledge, using the proof of Pythagoras’ theorem as evidence for his theory of mind and knowledge.

Mathematics as a formal language was born and then developed in classical Greece, but we cannot ignore the existence of a very different tradition, that of the Bible and the Kabbalah, and later on that of early medieval Christianity. Jews and Christians turned mathematics into a sacred language, starting from the words of the *Book of Wisdom* (11:21): *Omnia in mensura et et pondere disposuisti.*

This approach justified the study of mathematics, but restricted it to an elite, that of the clergy admitted to the interpretation of sacred texts, and in the early Middle Ages the main use of mathematics was the *Computus*, the complicated method that had to be used in order to fix the date of Easter. The method of *Computus was* almost definitively coded by Bede the Venerable (773-735 AD).

But already Augustine (354-430 AD) warns on the dangers of a “lay” conception of mathematics: "*The good Christian should beware of mathematicians and all those who make empty prophecies. There is a danger that mathematicians have made a pact with the devil to darken the spirit and deliver the man to hell* "(*De Genesi ad Litteram*, Book II, xviii, 157). The bishop of Hippo probably had in mind above all the astrologers, but so be it.

In order to go back to a more social "conception" of the mathematical language is was necessary to switch to a playful attitude, which took place for the first time in the post-classical Western world, with Alcuin of York (735-804), a pupil Archbishop Ecgbert, who in turn was a pupil of Bede.

Alcuin re-established the “public” school at the Court of Charlemagne and to this purpose he wrote the “*Propositiones ad acuendos Iuvenes*”, a collection of 53 soluble problems involving (relatively) simple mathematical reasoning. The themes were not particularly original: for some a trace already exists in the Rhind Papyrus, a Egyptian text of the sixteenth century BC, and among the propositions one can also find the classic problem of "saving both ways".

But the interest of the text is mainly in the "revolutionary" (for the time) idea that mathematics could be useful in order to 'sharpen' the intellect of young people, and it could become a social language to the extent that, through the schools, its methods and content would be shared by many.

The medieval school involved two levels of education, the *Trivium* (lower lever) devoted to grammar, logic and rhetoric, and the *Quadrivium* (advanced level). All the disciplines of *Quadrivium* (arithmetic, geometry, music and astronomy) are of a mathematical nature, but this fact does not mean that the transfer of this kind of knowledge from the cloister to the public street was seen with favour by political and religious institutions: any attempt to vulgarization was fought tenaciously.

When Gerbert of Aurillac (947-1003), Pope in the year 1000 by the name of Sylvester II, attempted to introduce Arabic numerals, the abacus and the astrolabe, his work was mystified, attributing it a diabolic doctrine, and Gerbert himself was presented as a wizard, a notion gaining widespread fame thanks to imaginative legends told by William of Malmesbury, a twelfth-century chronicler. We shall not belabor on these legends (including the creation of a mechanical talking head), but they are full evidence of the hostility created by any attempt to make mathematics “popular”.

Things went slightly better for Leonardo Pisano, also known as Leonardo Fibonacci (1175-1250), who with his *Liber Abaci* (1202) managed to introduce the use of the Arabic numerals to a much wider audience. We must stress that even in this case the transition to the social dimension went through mathematical games. Let’s recall the "competition" between Leonardo and users of Roman numerals (and of the related algorithms) at the Palermo Court od Emperor Frederick II. Let’s also recall the many problems that Fibonacci tackled in his book, including the most famous problem of rabbit reproduction, which gave rise to the introduction of the numbers today known as "Fibonacci numbers", linked also to classic idea of the "golden section".

Nevertheless, at the end of the thirteenth century in Florence the prohibition was still enforced of using Arabic numbers in bank transactions, probably because of the supposed competitive advantage that those who were able to use them would have had against those who did not know the techniques.

A quite different fate, it is worth noting, had mathematics in the Arab and oriental world. We shall not dwell on the legend about the birth of chess or on the popularity of Mathematics in India (still true in present times, just think of Ramanujan’s story), nor on the production and dissemination of astrolabes and astronomy texts (essential for the proper orientation of prayer towards Mecca, which reminds us of the sacred value of mathematics).

Part of this culture is perhaps reflected in the complex astronomical and numerological architecture of the *Comedy* (at least so Asin Palacios thinks and argues in his “*Dante and Islam*”). Numerology is already an integral part of *Vita Nova*, all played on the constant return of the number nine. But it is still a secret code, a *trobar clus* speaking to the cognoscenti, as the contemporary *Ars Magna* by Ramon Llull, (later renamed *ars combinatoria* by Leibnitz), which is also clearly influenced by Islamic culture of Spain. And we see that in Dante and Llull the recreational dimension, is completely lacking. But we find it again in the real social dynamics of their contemporaries. The "abacus treaties" written in the Late Middle Ages are rich in mathematical games: as Giovanni de Danti wrote in 1370 "*tractaremo de certe materie più per dare dilecto che per utilità che crediamo trare d’esse* " (we shall treat some issues more in order to give pleasure than for the uses we think they may have).

With the Renaissance (and with the revival of Platonism) we see mathematics regain a “social” role when it becomes a tool for painting, with the studies on the perspective of Piero della Francesca. We may also recall the drawings by Leonardo for the *De Divina Proportione* by Luca Pacioli on the golden section (notice that Luca writes also a booklet on games, *De Viribus quantitatis*). Geometry is essential for architecture (and in the recreational context think of the *Ex ludis* by Leon Battista Alberti). But we see the real triumph of this “social” reinterpretation of the role of mathematics in its identification with the language of nature, proposed by Galilei and soon to become the “common sense” of modern physics.

It is also a great season for "recreational" mathematics, now in close relationship with the calculation of chances, so important in card games that are more and more common, but also in games with dice, which have very ancient history. And we must not overlook the role of "scientific" astrology, which in the sixteenth century, by Nostradamus and Cardano, got a social credibility not to be anachronistically

interpreted in the light of modern knowledge (and we must remember that even Galileo did not disdain to calculate horoscopes, for a fee!).
With the successive generations, those of Descartes, Mersenne, Fermat and Pascal, even “formal” mathematics became a kind of game: number theory was developed, and it was seen by its practitioners as a pure intellectual entertainment. But in the mean time the way was paved for the extraordinary developments that soon followed, with the Newtonian and Leibnitzian “*Calculus*”, and with the "social" uses of mathematics, in the eighteenth century, in the context of the new economic science associated with the industrial revolution. In this context let’s recall also the paradox of St. Petersburg, which was devised by Bernoulli in the early eighteenth century and played a significant role in the development of economic theory with the introduction of the utility function.
Actually mathematics seems to accompany all the great political and social revolutions, oscillating always among the (esoteric) formal developments and (exoteric) recreational and practical applications.
We must now come to the twentieth century, and the last pieces of our argument.
Game theory was born "officially" in 1944 by Von Neumann, and developed later by Nash although, as we have seen the birth of the theory should be backdated at least to the time of Blaise Pascal.
Certainly, the development of financial economy has contributed more than ever to the growth the social dimension of mathematics, with an impact perhaps equaled only by the birth and the growth of computer science. Today we have on our tablets a game called Fibonacci game (along with many other math-based games); computers are now world champions in chess and go (which seems to be the hardest game ever invented); but especially we have a new mathematical mass folklore, a new real mythology.
A good example of this statement can be found in the television series Numb3rs, in which a young talented mathematician makes good use of its knowledge in order to solve intricate criminal cases. In this case it is proper to say that the medium is the message: if mathematics can be the subject of fiction serial, then it acquires the nature of serial fiction, and its"recreational" dimension becomes meta-recreation to the extent that it becomes a show.