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Spectrum of k-string tensions in SU(N) gauge theories *

L. Del Debbio[a], H. Panagopoulos[b], P. Rossi[a], E. Vicari[a]

[a] Dipartimento di Fisica, Università di Pisa, and INFN, I-56127 Pisa, Italy
[b] Department of Physics, University of Cyprus, Nicosia CY-1678, Cyprus

We compute, for the four-dimensional SU(4) and SU(6) gauge theories formulated on a lattice, the string tensions σ_k related to sources with Z_N charge k, using Monte Carlo simulations. Our results are compatible with $\sigma_k \propto \sin k\pi/N$, and show sizeable deviations from Casimir scaling.

1. Introduction

Nonabelian gauge theories are the building blocks of the field-theoretical description of fundamental interactions. It is therefore essential to achieve a deep understanding of their physical content and a better quantitative knowledge of their testable predictions. It is widely believed that nonabelian gauge theories admit a reinterpretation in terms of effective strings; describing their properties is an issue of the utmost importance. In particular one would like to understand if these strings belong to some wider group of objects ("universality class") whose properties may eventually be studied by different approaches and techniques. The study of the string tensions between static charges in representations higher than the fundamental one and for different values of N may shed new light on the nature of the confining strings, helping to identify the most appropriate models of the QCD vacuum and to select among the various confinement hypotheses. A static source carrying charge k with respect to the center Z_N is confined by a k-string with string tension σ_k ($\sigma_1 \equiv \sigma$ is the string tension related to the fundamental representation). The k string is the lightest state propagating in the k-charged channel, and is related to the antisymmetric representation of rank k. If $\sigma_k < k\,\sigma$, then a string with charge k is stable against decay to k strings of charge one. Charge conjugation implies $\sigma_k = \sigma_{N-k}$. Therefore $SU(3)$ has only one independent string tension determining the large distance behavior of the potential for $k \neq 0$. One must consider larger values of N to look for distinct k-strings. In particular for $N \geq 4$ one may consider the ratio

$$R(k,N) \equiv \sigma_k/\sigma. \qquad (1)$$

2. Models and their predictions

Some different conjectures on the behavior of $R(k,N)$ have been discussed in the recent literature. We briefly discuss a few of them before presenting the results of our numerical simulations.

2.1. Casimir scaling

According to this hypothesis (see Refs. [2–6]):

$$R(k,N) = C(k,N) \equiv \frac{k(N-k)}{(N-1)} \qquad (2)$$

This formula is exact in two-dimensional SU(N) gauge theories. In four dimensions it is satisfied by the strong-coupling limit of the lattice Hamiltonian formulation of SU(N) gauge theories, and by the small-distance behavior of the potential between two static charges in different representations, as shown by perturbation theory up to two loops.

The main objections to Casimir scaling come from the absence of a mechanism preserving Casimir scaling from small distance (essentially perturbative, characterized by a Coulombic potential) to large distance (characterized by a string tension for sources carrying Z_N charge). Moreover, Casimir scaling does not survive the next-to-leading order calculation of the ratios

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$R(k,N)$ in the strong-coupling lattice Hamiltonian approach [7].

2.2. Sine formula

Another interesting hypothesis is that the k-string ratios $R(k,N)$ may reveal a universal behavior within a large class of asymptotically free theories characterised by the $SU(N)$ symmetry [1]. Accordingly, the k-string ratios should be

$$R(k,N) = S(k,N) \equiv \frac{\sin(\pi k/N)}{\sin(\pi/N)}. \tag{3}$$

Indeed, this result is obtained for the $\mathcal{N}=2$ supersymmetric SU(N) gauge theory softly broken to $\mathcal{N}=1$ [8,9]. The same result has been derived in the context of M-theory, and extended to the case of large breaking of the $\mathcal{N}=2$ supersymmetric theory [9]. Moreover, it is suggested by a (rather speculative) M-theory approach to QCD.

The same formula emerges for the spectrum of the bound states in the two-dimensional $SU(N)\times SU(N)$ chiral models, which are matrix-valued, asymptotically free, and present interesting analogies with the four-dimensional gauge theories (see e.g. Refs. [10,11]). For these models the spectrum is obtained from the exact S-matrix, derived using essentially the Bethe Ansatz.

The main objection to this proposal is essentially the weakness of the hypotheses on which the universality assumption is based.

3. Results from Monte Carlo simulations

We performed numerical Monte-Carlo simulations of four-dimensional lattice $SU(4)$ and $SU(6)$ gauge theories using the Wilson formulation. Our results were obtained from very high statistics runs for $SU(4)$ (2-4$\times 10^6$ sweeps on $12^3\times 24$ and $16^3\times 32$ lattices). The statistics for $SU(6)$ was approximately 10 times smaller. The reader is referred to Ref. [7] and a forthcoming paper for the details of the analysis and for comparison with related work [6].

In order to compute the k-string tensions, we consider correlators of strings in the appropriate representations:

$$F_k(t) = \sum_{x_1,x_2} \langle \chi_k[P(0,0;0)]\chi_k[P(x_1,x_2;t)]\rangle \tag{4}$$

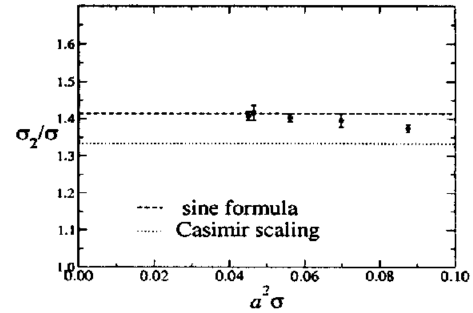

Figure 1. k-string ratio σ_2/σ for $SU(4)$.

where

$$P(x_1,x_2;t) = \Pi_{x_3} U_3(x_1,x_2,x_3;t) \tag{5}$$

$$\chi_2[P] = \mathrm{Tr}P^2 - (\mathrm{Tr}P)^2 \tag{6}$$

$$\chi_3[P] = 2\mathrm{Tr}P^3 - 3\mathrm{Tr}P^2\mathrm{Tr}P + (\mathrm{Tr}P)^3 \tag{7}$$

We use standard smearing techniques to improve the overlap with the lightest propagating state. The k-string tensions are determined from the asymptotic decay of the correlators, that, for a k-loop of size L, is [12,6]:

$$F_k(t) \sim \exp - \left(\sigma_k L - \frac{\pi}{3L}\right) t, \tag{8}$$

where the $O(1/L)$ correction is conjectured to be universal and is related to the flux excitations described by a free bosonic string [13]. The choice of the fit-range is a delicate matter: correlations at short time distances are affected by heavier state contributions, while at long time distances the signal is obscured by the statistical noise. A systematic error related to the choice of the fit-range is therefore unavoidable. To keep it under control, we perfomed a careful analysis, especially in the case of SU(4), where the high statistic of the simulations provided good estimates of the correlators up to relatively large distances.

Results for $R(k,N)$ are shown for $N=4,6$ in Figs. 1 and 2 respectively, and plotted versus σ. The tension ratios show a satisfactory scaling behavior for the coupling values chosen for the simulation. Therefore we did not find necessary to fit the dependence of our result on the lattice spacing. Our estimates are essentially obtained from the results at the largest β-values (smallest σ values), see Ref. [7]. The size of scaling violations can be inferred from the data at lower values of

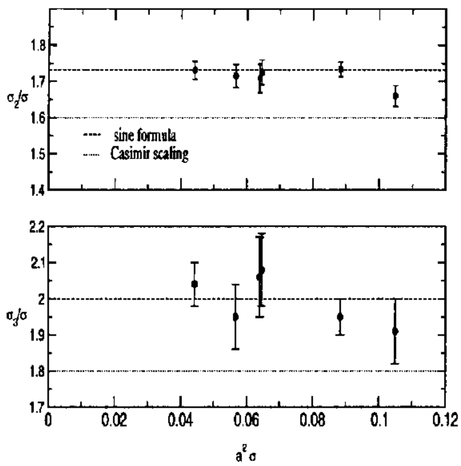

Figure 2. k-string ratios for $SU(6)$.

the coupling; the indication is that they are comparable with the error quoted below.

Our results for the ratios are (the SU(4) estimate is still preliminary):

$$R(2,4) = 1.405 \pm 0.015 \quad (9)$$

$$R(2,6) = 1.72 \pm 0.03 \quad (10)$$

$$R(3,6) = 1.99 \pm 0.07 \quad (11)$$

We mention the result $R(2,4) = 1.357(29)$ reported in Ref. [6], which is marginally consistent with ours. We have also explored correlators in the *symmetric* rank-2 representation, finding no evidence for stable bound states, as expected.

Figure 3 compares our MC results with the above-mentioned hypotheses of spectrum. We claim that $SU(4)$ and $SU(6)$ results show substantial agreement with the sine formula and evidence of disagreement with Casimir scaling. Indeed the sine formula (3) predicts $S(2,4) = \sqrt{2} = 1.414...$, $S(2,6) = 1.732...$, and $S(3,6) = 2$ respectively, while the Casimir scaling predictions are $C(2,4) = 4/3$, $C(2,6) = 8/5$ and $C(3,6) = 9/5$. Considering our results alltogether, we can state that the sine formula is verified within an accuracy of approximately 1%. This result should be relevant for the recent debate on confinement models. Of course our numerical results do not prove that the sine formula holds exactly. But they put a stringent bound on the size of the possible corrections. On the other hand, our results appear rather conclusive on the existence

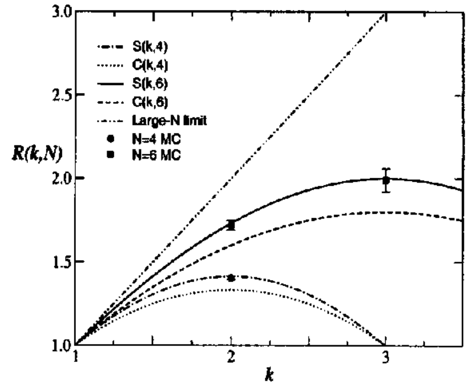

Figure 3. Comparison of the various hypotheses for the k-string ratios with our MC results.

of deviations from the Casimir scaling. However, Casimir scaling may still be considered as a reasonable approximation, since the largest deviations we observed were about 10%.

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